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The Scenario of Appearance of Periodic and Chaotic Modes in a Model with Control over Biosynthetic Processes

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Abstract

Chaotic regimes of the open dynamic biosystem are considered, and it is established that their development and the scenario for appearance of the strange attractor are depended on the bifurcation parameter.

Biochemical processes involving microorganisms, that grow with consumption of glucose in a flow bioreactor, demonstrate a complicated oscillating and chaotic behaviour [1, 2]. On Fig.1, the scheme is presented of hydrolysis of cellulose C by the polyenzymic complex including three enzymes (endogluconase E, cellobiase E_1, and exoglucosidase E_2). By G_2, G and X we denote cellobiose, glucose, and microorganisms, respectively.

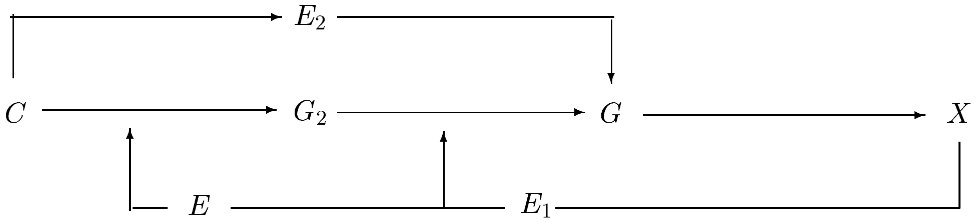


Fig.1

Starting from this scheme, we develop the model, which absorbs concrete representations concerning the related biochemical processes, find and investigate the conditions, under which the system under study reveals a peculiar stochastic behaviour.

The model involves the process of synthesis of enzymes, fermentation of cellulose, growth of microorganisms, and is based on the following system [3] of kinetic equations in dimensionless concentrations of species taking part in hydrolysis:

$$\frac{dC}{dt} = \sigma - l\frac{E}{L+E+\sigma_2 G}\,\frac{C}{1+C} - l_2\frac{E_2}{1+E_2}\,\frac{C}{1+C} - \alpha C, \tag{1}$$

$$\frac{dG_2}{dt} = k\frac{E}{L+E+\sigma_2 G}\,\frac{C}{1+C} - l_1\frac{E_1}{L_1+E_1}\,\frac{G_2}{1+G_2+G} - \alpha_1 G_2, \tag{2}$$

$$\frac{dG}{dt} = k_1\frac{E_1}{L_1+E_1}\,\frac{G}{1+G_2+G} + k_2\frac{E_2}{1+E_2}\,\frac{C}{1+C} - 2\mu\frac{GX}{\mu_2+G+\mu_3 x^2+\mu_1 x}, \tag{3}$$

$$\frac{dX}{dt} = 2\mu\frac{GX}{\mu_2+G+\mu_3 x^2+\mu_1 x} - \mu_0 X, \tag{4}$$

$$\frac{dE}{dt} = E_0\frac{C^2}{\beta+C^2}V\left[1-\frac{G_2+MG}{N+G_2+MG}\right] - \alpha_4 E, \tag{5}$$

$$\frac{dE_1}{dt} = E_{10}\frac{G_2^2}{\beta_1+G_2^2}V\left[1-\frac{G}{N+N_1+G}\right] - \alpha_5 E_1, \tag{6}$$

$$\frac{dE_2}{dt} = E_{20}\frac{C^2}{\beta_2+C^2}V\left[1-\frac{G_2+MG}{N_2+G_2+MG}\right] - \alpha_6 E_2. \tag{7}$$

The equations given describe the biosynthesis of enzymes by microorganisms and control over it by a substrate C and product G. The temporal dependence of the amount of cellulose C supplied into a bioreactor is determined by the parameter σ. The multiplier

$$V = \frac{X}{1+X}\,\frac{\exp\left(-\alpha_3\frac{dX}{dt}\right)}{1+\exp\left(-\alpha_3\frac{dX}{dt}\right)}$$

defines a delay of the biosynthetic processes that is related to the activation of biosynthesis by a substrate and the inhibition it by products of hydrolysis.

On study and discussion of the peculiarities of the model, it is convenient to separate all the involved parameters into two groups. The first consists of so-called inner parameters that are determined by biological features of the modelled system and respond to the evolution of substrate and microorganisms

$$\begin{aligned} &l = 0.1, \quad l_1 = 0.1, \quad l_2 = 0.07,\\ &\mu = 0.16, \quad \mu_1 = 0.7, \quad \mu_2 = 4.4, \quad \mu_3 = 7.0,\\ &\sigma_2 = 9.75, \quad L = 5.0, \quad L_1 = 35.0 \end{aligned} \tag{8}$$

as well as to the biosynthesis of enzymes

$$\begin{aligned} &E_0 = 5.9, \quad E_{10} = 5.3, \quad E_{20} = 3.9,\\ &\beta = 1.05, \quad \beta_1 = 5.25, \quad \beta_2 = 1.78, \quad \alpha_3 = 655,\\ &N = 0.92, \quad N_1 = 0.83, \quad N_2 = 0.50, \quad M = 6.30. \end{aligned} \tag{9}$$

The outer parameters

$$\begin{aligned} &0 < \sigma < 0.01, \quad \mu_0 = 0.017, \quad \alpha = 0.00001,\\ &\alpha_1 = 0.00007, \quad \alpha_4 = 0.004, \quad \alpha_5 = 0.03, \quad \alpha_6 = 0.002 \end{aligned} \tag{10}$$

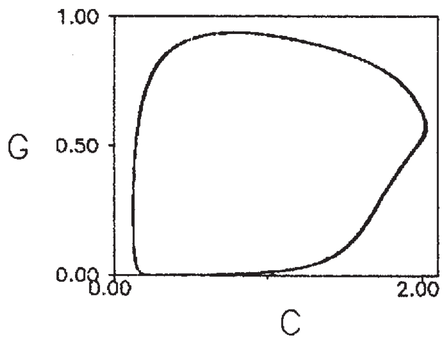
are related to the flow-type conditions of the open biosystem.

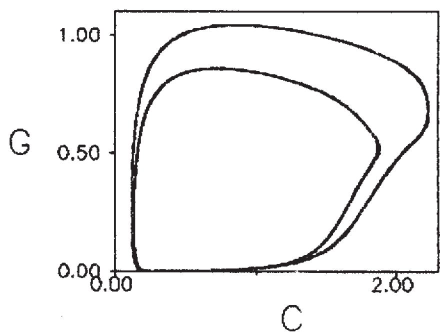
To determine the run of biochemical processes, we should possess data charterizing the system at the initial moment $t = 0$ (i.e., the initial state). It is worth noting that the main influence on the system is produced by current but not initial conditions, since the biosystem reveals the property to forget own previous dynamic states. Due to this circumstance, the system can be shown to enter a peculiar limiting mode after some time of functioning. The new "initial" conditions are found as

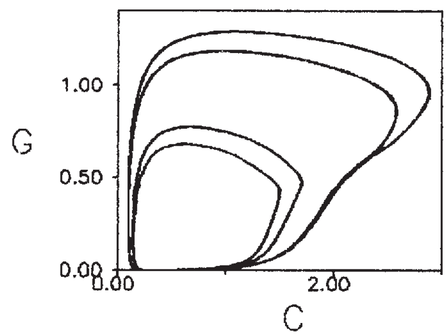
$$\begin{aligned} &C = 0.998, \quad G_2 = 0.852, \\ &G = 0.537, \quad X = 0.57, \\ &E = 0.53, \quad E_1 = 0.46, \quad E_2 = 0.514. \end{aligned} \tag{11}$$

The given kinetic equations (1)–(7), values of parameters (8)–(10), and initial conditions (11) constitute a Cauchy problem for the system of nonlinear differential equations. The dynamic system considered is characterized by the presence of various regulatory mechanisms which are responsible for different evolutionary regimes of its functioning, e.g., the process of adaptation of the biosystem to external conditions.

In what follows we will consider, for simplicity, the dynamics of the biosystem as a function of the rate σ of supply of cellulose into a bioreactor. First, we have found by numerical calculations that for σ in the range 0.001–0.00027, all the characteristics asymptotically approach stationary values. In this case, by means of the Poincare mapping the state of the system is represented by a point on the phase plane. The subsequent decrease in σ up to 0.000262 reveals the appearance of periodic oscillations. Respectively, the attractor of the system is presented by a limiting cycle, whose projection σ onto the plane (C, G) is depicted in Fig.2.







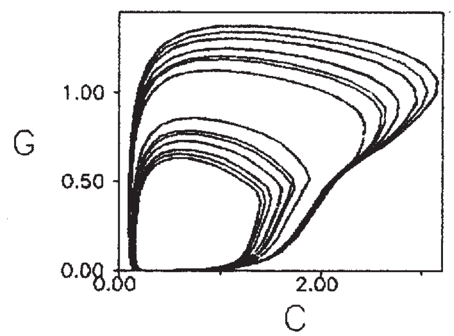


Fig.2 Projection $1 * 2^0$ of the limiting cycle. Fig.3 Projection $1 * 2^1$ of the limiting cycle.
Fig.4 Projection $1 * 2^2$ of the limiting cycle. Fig.5 Plane projection of the strange attractor.

For values of σ in the interval 0.000261–0.000257, the transition is happened from the mode of a limiting cycle to the mode of a strange attractor as a result of realization of an infinite sequence of bifurcations with a doubling of periods of the limiting cycles.

In Figs.3–5, three points of bifurcation with respect to the parameter σ are demonstrated (σ=0.000261, 0.000258, 0.000257) that belong to the induced scenario of bifurcations with a doubling of the period.

Thus, the appearance of chaotic modes and a strange attractor is demonstrated for the bifurcation parameter σ to be in a rather narrow interval. The latter fact appears to be not suprising if we regard nonlinearity and complexity of the system under study (cf., e.g., [4]).

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