The Research of Some Single-Machine Scheduling Problems

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Abstract—In this paper, we develop a new scheduling model with learning effects where the actual processing time of a job is not only a function of the total normal processing times of the jobs already processed, but also a function of the job's scheduled position. We show that the single-machine scheduling problems to minimize make span and total complete time are still polynomially solvable under the proposed model. We further show that the single-machine scheduling problems minimizing the weighted sum of completion times and minimizing the maximum lateness are also polynomially solvable under certain conditions.

Keywords—Scheduling; position-dependent learning effect model; time-dependent learning effect mode

I. INTRODUCTION

Scheduling problems have received considerable attention since the middle of the last century. However, most researchers assume that job processing times are known and fixed during the whole processing process. Recent empirical studies in several industries have demonstrated that unit costs decline as companies produce more of a product and gain knowledge or experience. This phenomenon is well-known as the “learning effect” in the literature (see,e.g., [1,2]).

Biskup (see,e.g., [2]) was the first to investigate the learning effect in scheduling problems. He assumed that the time needed to perform an operation decreases by the number of repetitions., the processing time of a job is a function of the job position in a sequence. In the classical position-dependent learning effect model, the actual processing time p_{ir} of job J_i when it is scheduled in the r th position in a processing sequence is defined as

$$p_{ir} = p_i r^a \quad (1)$$

where a is a non-positive learning index and p_i denotes the normal processing time of job J_i . Under this assumption, the learning effect of a job only depends on the number of jobs that are scheduled in front of it. It implies the learning rates of all jobs are all the same. There are several studies in this context. Biskup showed that single-machine scheduling problems with a learning effect still remain polynomially solvable if the objective is to minimize the deviation from a common due date or to minimize the sum of flow times.

Kuo and Yang (see,e.g., [3]) considered a single-machine scheduling problem with a time-dependent learning effect. There are n jobs to be

processed on a single machine. Each of them is available at time zero. Let p_i be the normal (sequence-independent) processing time of job J_i in a sequence and $p_{[k]}$ be the normal processing time of a job if scheduled in the k th position in a sequence. The normal processing time of a job is incurred if the job is scheduled first in a sequence. The processing times of the following jobs are smaller than their normal processing times because of the learning effect. To introduce the notion that the more you practice, the better you learn, we define a time-dependent learning effect as follows. Let p_{ir} be the actual processing time of J_i if it is scheduled in position r in a sequence. Then

$$p_{ir} = p_i \left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a \quad (2)$$

where $a(a \leq 0)$ is a constant learning index. Kuo and Yang proved that the total completion time and minimization problem is still polynomially solvable under the single-machine.

Based on the two models of Biskup, Kuo and Yang, in this paper, we develop a new scheduling model with learning effects where the actual processing time of a job is not only a function of the total normal processing times of the jobs already processed, but also a function of the job's scheduled position. Based on this new learning model, we need search that the single-machine scheduling problems to minimize makespan and total complete time are still polynomially solvable under the proposed model. We also need search that the single-machine scheduling problems minimizing the weighted sum of completion times and minimizing the maximum lateness are also polynomially solvable under certain conditions.

II. SOME SINGLE MACHINE PROBLEMS

There are given a single machine and n independent and non-preemptive jobs that are immediately available for processing. The machine can handle one job at a time and preemption is not allowed. Let p_i be the normal processing time of job J_i and $p_{[k]}$ be the normal processing time of a job if it is scheduled in the k th position in a sequence. Associated with each job J_i ($i=1,2,\dots,n$) is a weight $w_{[k]}$, a due date $d_{[k]}$ and

a completion date $C_{[k]}$. Let p_{jr} be the actual processing time of J_j if it is scheduled in the r th position in a sequence, then

$$p_{jr} = p_j (\alpha a^{\sum_{k=1}^{r-1} \ln p_{[k]}} + \beta) r^b \quad (3)$$

where $\alpha \geq 0, \beta \geq 0, 0 \leq a \leq 1, b < 0$ are some indexes which we can make it ourselves. For the sake of convenience, we make the above learning model as LE . Obviously, in the model (4), the actual processing time of a job depends not only on a function of the total normal processing times of the jobs already processed, but also a function of the job's scheduled position. The learning effect will be more obvious if a job's scheduled position is later. For a given feasible scheduling, $C_{\max} = \max_{1 \leq i \leq n} \{C_i\}$ and $\sum C_j$ denote the makespan and total completion time, $\sum w_j C_j$ and $L_{\max} = \max_{1 \leq j \leq n} \{L_j\}$ denote total weighted completion time and the maximum delay. Besides, $L_j(S) = C_j(S) - d_j$. In the sequel, we will adopt the three-field notation scheme $\alpha | \beta | \gamma$.

III. SOME SINGLE MACHINE PROBLEMS

First, we give a lemma. it is useful for the following theorems.

Lemma

1.

$$(u-1)(x+\beta) + v(xa^y(\ln u - u) + \beta(1-u)) \geq 0 \quad (4)$$

$$\text{If } \begin{matrix} u \geq 0 < v \leq 1 & x \geq 0 \\ y \geq 0 < \alpha \leq 1 & \beta > 0 \end{matrix}$$

$$F(u) = (u-1)(x+\beta) +$$

Proof. If $v(xa^y(\ln u - u) + \beta(1-u))$.

Then the first derivative about u is

$$\begin{aligned} F'(u) &= x + \beta + v \left(xa^y \left(\frac{1}{u} - 1 \right) - \beta \right) \\ &= x \left(1 - \left(1 - \frac{1}{u} \right) va^y \right) + \beta(1-v) \geq 0 \end{aligned}$$

$$\text{where } 0 \leq \left(1 - \frac{1}{u} \right) va^y \leq 1. \text{ We have } F(u)$$

progressive increases about u , therefore $F(u) \geq F(1) = 0$.

First, we study the single machine makespan minimization problem under model(3), denoted by $1 | LE | C_{\max}$. And we show that an optimal schedule for the problem can be obtained by the Shortest Processing Time first (SPT) rule. The result is stated as follows.

Theorem 1. For the problem $1 | LE | C_{\max}$, there exists an optimal schedule in which the jobs are ordered according to the SPT rule.

Proof. Suppose that there exists an optimal schedule $S' = (\pi, J_j, J_i, \pi')$ with $P_i < P_j$,

where π and π' denote the partial sequences of S' . Let S be a schedule with jobs J_j and J_i of S mutually exchanged, that is, $S = (\pi, J_i, J_j, \pi')$. In addition, we assume that there are $r-1$ jobs in π . Let $C_j(S)$ and $C_j(S')$ respectively denote the complete times of job J_j in S and S' and let $C_i(S)$ and $C_i(S')$. We will show that the interchange of jobs J_j and J_i does not influence the objective value. The repeated implementation of this argument will lead to the optimality of the SPT rule for the problem $1 | LE | C_{\max}$. To further simplify the notation, let t denote the completion time of the last job π . Besides, the completion time of the last job π' does not influence the two schedules. Then from the Eq. (3) the completion of job J_i and J_j in S are

$$C_i(S) = t + p_i \left(\alpha a^{\sum_{k=1}^{r-1} \ln p_{[k]}} + \beta \right) r^b$$

and

$$C_j(S) = t + p_j \left(\alpha a^{\sum_{k=1}^{r-1} \ln p_{[k]}} + \beta \right) r^b +$$

$$p_j \left(\alpha a^{\sum_{k=1}^{r-1} \ln p_{[k]} + \ln p_i} + \beta \right) (r+1)^b$$

also in S' are

$$C_j(S') = t + p_j \left(\alpha a^{\sum_{k=1}^{r-1} \ln p_{[k]}} + \beta \right) r^b$$

and

$$C_i(S') = t + p_i \left(\alpha a^{\sum_{k=1}^{r-1} \ln p_{[k]}} + \beta \right) r^b +$$

$$p_i \left(\alpha a^{\sum_{k=1}^{r-1} \ln p_{[k]} + \ln p_j} + \beta \right) (r+1)^b$$

Thus we have

$$C_i(S') - C_j(S) \neq$$

$$p_i r^b \left(\left(\frac{p_j}{p_i} - 1 \right) \left(\alpha a^{\sum_{k=1}^{r-1} \ln p_{[k]}} + \beta \right) \right. \\ \left. + \frac{(r+1)^b}{r^b} \left[\alpha a^{\sum_{k=1}^{r-1} \ln p_{[k]}} \left(a^{\ln p_j} - \frac{p_j}{p_i} a^{\ln p_i} \right) \right. \right. \\ \left. \left. + \beta \left(1 - \frac{p_j}{p_i} \right) \right] \right) \\ \text{Let}$$

$$u = \frac{p_j}{p_i}, v = \frac{(r+1)^b}{r^b}, x = \alpha a^{\sum_{k=1}^{r-1} \ln p_{[k]}}, y = \ln p_i \text{ clearly}$$

$$u \geq 1, 0 \leq \frac{(r+1)^b}{r^b} \leq 1, x \geq 0, y \geq 1.$$

Thus, by **Lemma 1** we have

$$C_i(S') - C_j(S) =$$

$$p_i r^b ((u-1)(x+\beta)$$

$$+ v [x(a^{\ln u + y} - u a^y) + \beta(1-u)]) \geq 0$$

Consequently, $C_j(S) \leq C_i(S')$. this completes the proof.

Now we study the single machine total complete time problem under model(3), denoted by $1|LE|\sum C_j$. And we show that an optimal schedule for the problem can be obtained by the Shortest Processing Time first (SPT) rule. The result is stated as follows.

Theorem 1. For the problem $1|LE|\sum C_j$, there exists an optimal schedule in which the jobs are ordered according to the SPT rule.

Proof. The marks in theorem 1 are used in here. According to the proof of theorem 1, we just need to prove $C_i(S) + C_j(S) \leq C_j(S') + C_i(S')$. Because of

$$P_i < P_j, \text{ the equation } C_i(S) - C_j(S') = \\ r^b \left(\alpha a^{\sum_{k=1}^{r-1} \ln p_{[k]}} + \beta \right) (P_i - P_j) \leq 0$$

is right clearly, that is, $C_i(S) \leq C_j(S')$. besides, the proof of $C_j(S) \leq C_i(S')$ is ready in theorem 1. then we have $C_i(S) + C_j(S) \leq C_j(S') + C_i(S')$.

This completes the proof.

The single-machine scheduling problems minimizing the weighted sum of completion times, that is, $1||\sum w_j C_j$ is a very important part. In the classical scheduling, an optimal schedule for the problem can be obtained by the Weighted Shortest Processing Time first (WSPT) rule. But the conclusion is not always right under model (3). However, under certain conditions the problems can be solved in polynomial time under model (3).

Theorem 3. For the problem $1||\sum w_j C_j$, if jobs have reversely agreeable weights, that is,

$p_i \leq p_j \Rightarrow w_i \geq w_j$ for all jobs J_i and J_j , then there exists an optimal schedule in which the jobs are ordered in non-decreasing order of $\frac{w_j}{p_j}$ (WSPT rule).

Proof. The marks in theorem 1 are also used in here. Supposing that here is an optimal schedule $S' = (\pi, J_j, J_i, \pi')$ which follow the WSPT rule, that is,

there exist job J_i and job J_j with $\frac{w_j}{p_j} \leq \frac{w_i}{p_i}$. Note that

the weights and the normal processing time of the all jobs and are reverse. Thus, we have $p_i \leq p_j$ and $w_i \geq w_j$.

We get $C_i(S) \leq C_j(S')$ and $C_j(S) \leq C_i(S')$ from theorem 1 and theorem 2 respectively. We can have

$$w_j C_j(S') + w_i C_i(S') - w_i C_i(S) - w_j C_j(S) \\ \geq w_i C_j(S) + w_j C_i(S) - w_i C_i(S) - w_j C_j(S) \text{ th} \\ = (C_i(S) - C_j(S))(w_i - w_j) \geq 0,$$

at is, $w_i C_i(S) + w_j C_j(S) \leq w_j C_j(S') + w_i C_i(S')$ This completes the proof.

Next, we will show that the single machine problem to minimizing the maximum lateness under model(3), denoted by $1|LE|\sum L_{\max}$, can be solved if the job processing times and the due dates are agreeable, that is, $d_i \leq d_j$ implies $p_i \leq p_j$ for all jobs J_i and J_j .

Theorem 4. If the normal processing times and the due dates of job are agreeable, namely, that $d_i \leq d_j$ implies $p_i \leq p_j$ is true for jobs J_i and J_j , then the Earliest Due Date first rule (EDD rule) is optimal for the problem $1|LE|\sum L_{\max}$.

Proof. The marks in theorem 1 are still used in here. We suppose that job J_i and J_j satisfy $d_j \leq d_i$ in optimal scheduling $S' = (\pi, J_j, J_i, \pi')$. The normal processing times and the due dates of job are agreeable, then we have $p_i \leq p_j$. Besides, we get $C_i(S) \leq C_j(S')$ and $C_j(S) \leq C_i(S')$ from theorem 1 and theorem 2 respectively. By definition, the lateness of jobs J_i and J_j in S and jobs J_i and J_j in S' are respectively

$$L_j(S) = C_j(S) - d_j$$

$$L_i(S) = C_i(S) - d_i$$

$$L_i(S') = C_i(S') - d_i$$

$$L_j(S') = C_j(S') - d_j$$

$L_j(S) \leq L_i(S')$ can be obtained from $C_j(S) \leq C_i(S')$ and $d_i \leq d_j$. Also, since $C_i(S) \leq C_j(S')$, we have

$L_i(S) \leq L_i(S')$.Then
 $\max\{L_i(S), L_j(S)\} \leq \max\{L_j(S'), L_i(S')\}$ This completes the proof.

IV. CONCLUSIONS

In this paper, we develop a new scheduling model with learning effects where the actual processing time of a job is not only a function of the total normal processing times of the jobs already processed, but also a function of the job's scheduled position. We show that the single-machine scheduling problems to minimize makespan and total complete time are still polynomially solvable under the proposed model. We further show that the single-machine scheduling problems minimizing the weighted sum of completion times and minimizing the maximum lateness are also polynomially solvable under

certain conditions. These results are significant extensions of some of the existing results on learning effects in the literature. We believe that the general learning effect model offered here will turn out to be more useful in the theory and applications of scheduling.

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