Delay Dependent Memory Robust Model Predictive Control for Uncertainty Polytopic Time-delay Systems

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***Abstract*—In this article, a delay dependent memory robust model predictive control (RMPC) algorithm was proposed for uncertain polytopic state delay systems with input constraints. The state delay is time-varying with an upper and lower bound. We minimize a cost function by minimizing its upper bound for the proposed RMPC algorithm. A new sufficient condition, in accordance with LMI, is presented for cost monotonicity. We show that the memory feedback controller obtained by the RMPC approach stabilizes the closed-loop system and reduces the conservativeness. Finally, the effectiveness of the proposed algorithm is illustrated by a numerical example.**

Keywords- delay dependent; robust model predictive control; uncertain polytopic systems; time delay; memory state feedback control

I. INTRODUCTION

Model predictive control (MPC) is a control strategy, which can cope with many constraints, such as input constraint, state constraint and time delay etc. It has been widely used in many practical applications and also studied by the research community [1-5].

Since the system model is actually an approximation of the actual system, it is essential to be robust to uncertainties and disturbances. The classes of model uncertainty that are most adopted in the robust MPC literatures are the polytopic uncertainty [6] and the multiplant uncertainty [7]. As the polytopic description of model uncertainty is more general than the multi-plant description, it becomes important to clearly define what sort of uncertainty can be tolerated by the robust controller. Kothare proposed an RMPC strategy employing linear matrix inequalities for coping with model uncertainty and constraints on the controlled and output variables [6]. In [8] it was assumed that the time-varying parameters could be measured online and developed with feedback control for improving performance. Gautam presented nominal and min-max RHC strategies for a class of linear systems with a polytopic system and bounded disturbances by employing an uncertainty-based dynamic control policy [9]. Jones and Morari proposed an algorithm to exploit approximate explicit control, which trade off complexity against approximation error, to the optimal cost function and barycentric interpolation [10].

By nature, the industrial processes, especially in network conditions, continually experience uncertainties and time-delays. As is well known, uncertainties and delays are the main causes of performance deterioration and even system instability, it is essential to study systems with both uncertainties and delays. In recent years, the investigation of RMPC for these systems has been attracting more and more attention in the MPC literature [11-15]. Jeong and Park presented MPC algorithm for uncertain polytopic time-varying systems with input constraints and state delay [11]. A fascinating feature of this algorithm is that the delay is unknown but with a known upper bound. [13] put the ideas of Jeong and Park [11] into detail and then extended relevant results from types of system delays [12, 13]. [14] proposed the sufficient condition for the cost monotonicity and robust MPC algorithm for linear parameter varying system by relaxation matrices. However, most of the existing results treated with time-varying delay of the form $0 \le d \le d_M$ In some practical situations, the delayed systems are stable with some nonzero delay, such as the wireless network systems. And latency-free is impossible in practical situations. Therefore, it is essential to investigate the stability of systems with nonzero lower bound of time-delay in many practical cases. On the other hand, one designed the controllers based solely on the information coming from the current system state, but did not take into account the impact of delay state. As a result, it is quite conservative for designing the controller.

Motivated by these considerations, we proposed a new delay dependent memory RMPC for uncertain polytopic time varying systems with input constraint and state delays. The delay is unknown but with a known upper and lower bound. The infinite-horizon min-max optimization problem is formulated as a minimization of the upper bound of cost function to design a memory state feedback MPC law. At the end, we derive sufficient conditions, which guarantee the asymptotic stability of the closed loop system, including not only the input condition but also a new condition for the cost monotonicity.

II. PROBLEM STATEMENTS

Consider the uncertain polytopic time-varying discrete time system with delayed state given by [11] as following form

$$\begin{aligned} &x(k+1) = A(k)x(k) + A_d(k)x(k-d) + B(k)u(k) \\ &x(k) = \Phi(k), \qquad k \in [-d_M, 0] \\ &\text{s.t. } -u_M \le u(k) \le u_M,\ u_M > 0 \ \text{ for } k \in [0,\infty) \end{aligned} \tag{1}$$

where $x(k) \in \mathbf{R}^n$ is the state, $u(k) \in \mathbf{R}^m$ is the control and $\Phi(k) \in \mathbf{R}^n$ is the initial condition.

the assumptions about the system is made as following:

A1: there is an interval upper bound between two successive measurements.

A2: d is an unknown constant representing state delay, but being $d_m \le d \le d_M$ with the known upper and lower bound d_M , d_m .

A3: the system matrices $\left[A(k)\ \ A_d(k)\ \ B(k)\right]$ is unknown but lie within a polytope Ω , that is to say,

$$\left[A(k)\ \ A_d(k)\ \ B(k)\right] \in \Omega \equiv \mathrm{Co}\left\{\left[A^l\ \ A_d^l\ \ B^l\right], l = 1,\ldots,p\right\} \tag{2}$$

where the convex hull is denoted as *Co* and $\left[A^l\ \ A_d^l\ \ B^l\right]$, $l = 1,\ldots,p$ is the vertices of the convex hull and precisely known.

A4: the state is available at every sampling instant k.

Remark1: in many practical applications, it is feasible that the measurement $x(k-d)$ is received at a time instant k. Because the delays are time varying and A1, it is possible that the controllers may receive delay state $x(k-d)$ at a time k, which does not provide new information. It implied that it is reasonable for the unknown and bounded time delay.

To design a memory feedback controller

$$u(k+i|k) = K_1(k)x(k+i|k) + K_2(k)x(k+i-d|k) \tag{3}$$

by the MPC strategy to stabilize (1), the optimization problem are considered at each time k.

$$\min_{u(k+i|k),i\ge 0} \ \max_{\left[A(k)\ \ A_d(k)\ \ B(k)\right]\in\Omega} J(k) \tag{4}$$

subject to

$$J(k) = \sum_{i=0}^{\infty}\left(\left\|x(k+i|k)\right\|_R^2 + \left\|u(k+i|k)\right\|_S^2\right) \tag{5}$$

$$x(k+i+1|k) = A(k)x(k+i|k) + A_d(k)x(k+i-d|k) + B(k)u(k+i|k) \tag{6}$$

$$-u_M \le u(k+i|k) \le u_M, \quad i \in [0,\infty) \tag{7}$$

where $R = R^T > 0$ and $S = S^T > 0$, $x(k+i|k)$ and $u(k+i|k)$ denote predicted the state and the input, respectively. $x(k|k) = x(k)$, $x(k-i|k) = x(k-i)$ for $i \ge 1$.

Let us present the following lemma,

Lemma 1[16]: Let M, N be real constant matrices and P be a positive matrix of compatible dimensions

Then

$$M^T PN + MPN^T \le \varepsilon M^T PM + \varepsilon^{-1} N^T PN$$

hold for any $\varepsilon > 0$.

III. MAIN RESULTS

Consider a delay-dependent quadratic function

$$V(x(k+i|k)) = \sum_{v=1}^{3} V_v(x(k+i|k)) \tag{8}$$

with $V_1(x(k+i|k)) = x^T(k+i|k)P(k)x(k+i|k)$,

$$V_2(x(k+i|k)) = \sum_{j=1}^{d} x^T(k+i-j|k)Q(k)x(k+i-j|k),$$

$$V_3(x(k+i|k)) = \sum_{l=d_m}^{d_M-1}\sum_{j=1}^{l} x^T(k+i-j|k)Q(k)x(k+i-j|k)$$

where $P(k) = P^T(k) > 0$, $Q(k) = Q^T(k) > 0$.

At time k, suppose the inequality is obtained

$$V(x(k+i+1|k))V(x(k+i|k)) \le \left\|x(k+i|k)\right\|_R^2 + \left\|u(k+i|k)\right\|_S^2 \tag{9}$$

which is used to derive the sufficient condition of the objective monotonicity.

A. The Cost Monotonicity

In order to propose stabilization RMPC algorithm, we first derive a new sufficient condition for cost monotonicity. The condition is presented in the following Theorem 1.

Theorem 1: the inequality (9) is satisfied for $\left[A(k)\ \ A_d(k)\ \ B(k)\right] \in \Omega$, if there exist $Y_1(k)$, $Y_2(k)$, $Q_1(k) = Q_1^T(k) > 0$ and $Q_2(k) = Q_2^T(k) > 0$, satisfying the following conditions:

$$\begin{bmatrix} -Q_1(k) & * & * & * & * \\ (1+\varepsilon)M_1 & -(1+\varepsilon)Q_1(k) & * & * & * \\ (1+\varepsilon)S^{\frac{1}{2}}Y_1(k) & 0 & -(1+\varepsilon)I & * & * \\ R^{\frac{1}{2}}Q_1(\mathrm{k}) & 0 & 0 & -I & * \\ rQ_1(\mathrm{k}) & 0 & 0 & 0 & -rQ_2(k) \end{bmatrix} \le 0 \tag{10}$$

$$\begin{bmatrix} -Q_2(k) & * & * \\ (1+\varepsilon^{-1})M_2(k) & -(1+\varepsilon^{-1})Q_2(k) & * \\ (1+\varepsilon^{-1})S^{\frac{1}{2}}Y_2(k) & 0 & -(1+\varepsilon^{-1})I \end{bmatrix} \le 0 \tag{11}$$

where $Q_1(k) = P^{-1}(k)$, $Q_2(k) = Q^{-1}(k)$, $Y_1(k) = K_1(k)Q_1(k)$, $Y_2(k) = K_2(k)Q_2(k)$, $M_1 = A^{(l)}Q_1(k) + B^{(l)}Y_1(k)$, and $M_2 = A_d^{(l)}Q_2(k) + B^{(l)}Y_2(k)$ with $l = 1,\ldots,p$.

Proof: For notational simplicity, let us define $x = x(k+i|k)$, $x_d = x(k+i-d|k)$, and $x_j = x(k+i-j|k)$. Using $V(x)$ defined in (8), we have

$$\begin{aligned} \Delta V_1 = {} & x^T(\bar{A}^T(k)P(k)\bar{A}(k) - P(k))x + x^T\bar{A}^T(k)P(k)\bar{A}_d(k)x_d \\ & + x_d^T\bar{A}_d^T(k)P(k)\bar{A}(k)x + x_d^T\bar{A}_d^T(k)P(k)\bar{A}_d(k)x_d \end{aligned} \tag{12}$$

$$\Delta V_2 \le x^T Q(k)x - x_d^T Q(k)x_d + \sum_{j=d_m}^{d_M-1} x_j^T Q(k)x_j \tag{13}$$

And

$$\Delta V_3 = (d_M - d_m)x^T Q(k)x - \sum_{j=d_m}^{d_M-1} x_j^T Q(k)x_j \tag{14}$$

where $\bar{A}(k) = A(k) + B(k)K_1(k)$, $\bar{A}_d(k) = A_d(k) + B(k)K_2(k)$.

So, the following inequality can be obtained

$$\Delta V \le x^T(\bar{A}^T(k)P(k)\bar{A}(k) - P(k) + Q(k))x + x^T\bar{A}^T(k)P(k)\bar{A}_d(k)x_d + x_d^T\bar{A}_d^T(k)P(k)\bar{A}(k)x + x_d^T(\bar{A}_d^T(k)P(k)\bar{A}_d(k) - Q(k))x_d \quad (15)$$

where $r = d_M - d_m + 1$.

Applying the state feedback $u(k) = K_1(k)x + K_2(k)x_d$, and considering lemma 1, we can obtain the inequality as following form

$$\begin{bmatrix} x \\ x_d \end{bmatrix}^T \begin{bmatrix} (1+\varepsilon)(\bar{A}^T(k)P(k)\bar{A}(k) + K_1^T(k)SK_1(k) + R - P(k) + rQ(k) & 0 \\ 0 & (1+\varepsilon^{-1})(\bar{A}_d^T(k)P(k)\bar{A}_d(k) + K_2^T(k)SK_2(k) - Q(k)) \end{bmatrix} \begin{bmatrix} x \\ x_d \end{bmatrix} \quad (16)$$

After pre-and post-multiplying $diag(Q_1(k), Q_2(k))$ in the left hand sides of (16), respectively, and applying the Schur complement, then we obtain

$$\begin{bmatrix} -Q_1(k) & * & * & * & * \\ (1+\varepsilon)\bar{M}_1 & -(1+\varepsilon)Q_1(k) & * & * & * \\ (1+\varepsilon)S^{\frac{1}{2}}Y_1(k) & 0 & -(1+\varepsilon)I & * & * \\ R^{\frac{1}{2}}Q_1(\mathrm{k}) & 0 & 0 & -I & * \\ rQ_1(\mathrm{k}) & 0 & 0 & 0 & -rQ_2(k) \end{bmatrix} \le 0 \quad (17)$$

$$\begin{bmatrix} -Q_2(k) & * & * \\ (1+\varepsilon^{-1})\bar{M}_2 & -(1+\varepsilon^{-1})Q_2(k) & * \\ (1+\varepsilon^{-1})S^{\frac{1}{2}}Y_2(k) & 0 & -(1+\varepsilon^{-1})I \end{bmatrix} \le 0 \quad (18)$$

where $\bar{M}_1 = A(k)Q_1(k) + B(k)Y_1(k)$,and $\bar{M}_2 = A_d(k)Q_2(k) + B(k)Y_2(k)$. Since (17), (18) are affine. The inequalities (17), (18) are satisfied for all $[A(k)\ \ A_d(k)\ \ B(k)] \in \Omega$ if and only if there exist $Q_1(k) = Q_1^T(k) > 0$, $Q_2(k) = Q_2^T(k) > 0$, $Y_1(k) = K_1(k)Q_1(k)$, and $Y_2(k) = K_2(k)Q_2(k)$ satisfying (10), (11), respectively. This proof is completed.

B. The proposed RMPC algorithm

To obtain the upper bound of the objective function, we obtain $V(X(\infty|k)) - V(X(k|k)) \le -J(k)$ i.e. $-V(X(k|k)) \le -J(k)$, by summing (9) from $j = 0$ to $j = \infty$.

So we have

$$\max_{[A(k)A_d(k)B(k)]\in\Omega, i\ge 0} J(k) \le V(x(x|x)) \le \gamma(k) \quad (19)$$

where $\gamma(k)$ is the nonnegative upper bound of V.

Then, the original min-max problem (4) can be turned into minimization of the upper bound of the original cost function,

$$\min_{P(k),Q(k)} \gamma(k) \quad (20)$$

subject to (8), (10)

Thus, we complete the algorithm with theorem 2

Theorem 2: For the time-delay system (1), if there exist matrices $Q_1(k) = Q_1^T(k) > 0$, $Q_2(k) = Q_2^T(k) > 0$, $Y_1(k)$, $Y_2(k)$, $G(k)$, a scalar $\gamma(k) > 0$ and a given scalar ε such that the following optimization problem is solvable

$$\min_{Q_1(k),Q_2(k),\gamma(k),Y_1(k),Y_2(k),G(k)} \gamma(k)$$

subject to

$$\begin{bmatrix} 1 & * & * & \cdots & * & * & * & * & * & * & * \\ x(k|k) & Q_1(k) & * & \cdots & * & * & * & * & * & * & * \\ rx(k-1|k) & 0 & rQ_2(k) & \cdots & * & * & * & * & * & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ rx(k-d_m|k) & 0 & 0 & \cdots & rQ_2(k) & * & * & * & * & * & * \\ (r-1)x(k-d_m-1|k) & 0 & 0 & \cdots & 0 & (r-1)Q_2(k) & * & * & * & * & * \\ \vdots & \vdots & \vdots & \cdots & \vdots & 0 & \ddots & * & * & * & * \\ (d_M-d+1)x(k-d|k) & 0 & 0 & \cdots & 0 & 0 & 0 & (d_M-d+1)Q_2(k) & * & * & * \\ (d_M-d-1)x(k-d-1|k) & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & (d_M-d-1)Q_2(k) & * & * \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x(k-d_M+1|k) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_2(k) \end{bmatrix} \ge 0 \quad (21)$$

$$\begin{bmatrix} -Q_1(k) & * & * & * & * \\ (1+\varepsilon)H_1 & -(1+\varepsilon)Q_1(k) & * & * & * \\ (1+\varepsilon)S^{\frac{1}{2}}Y_1(k) & 0 & -(1+\varepsilon)I & * & * \\ R^{\frac{1}{2}}Q_1(\mathrm{k}) & 0 & 0 & -I & * \\ rQ_1(\mathrm{k}) & 0 & 0 & 0 & -rQ_2(k) \end{bmatrix} \le 0 \quad (22)$$

$$\begin{bmatrix} -Q_2(k) & * & * \\ (1+\varepsilon^{-1})H_2 & -(1+\varepsilon^{-1})Q_2(k) & * \\ (1+\varepsilon^{-1})S^{\frac{1}{2}}Y_2(k) & 0 & -(1+\varepsilon^{-1})I \end{bmatrix} \le 0 \quad (23)$$

$$\begin{bmatrix} G(k) & * & * \\ Y_1^T(k) & Q_1(k) & * \\ Y_2^T(k) & 0 & Q_2(k) \end{bmatrix} \ge 0, G_{ii} \le u_{Mi},\ i = 1, 2, \cdots, m \quad (24)$$

where $G_{ii}(k)$ is the ith diagonal entry of $G(k)$ and u_{Mi} is the ith element of u_M , $H_1 = A^{(l)}Q_1(k) + B^{(l)}Y_1(k)$, $H_2 = A_d^{(l)}Q_1(k) + B^{(l)}Y_2(k)$, $l = 1, \ldots, p$ .Then the MPC law $u(k+i|k) = Y_1(k)Q_1^{-1}(k)x(k+i|k) + Y_2(k)Q_2^{-1}(k)x(k+i-d|k)$, $i \in [0, \infty)$, minimizes the upper bound $V(x(k|k))$ of the robust performance index.

Proof: Minimisation of $V(x(k|k))$ is equivalent to

$$\min_{\gamma(k),Q_1(k),Q_2(k)} \gamma(k)$$

$$s.t.$$

$$V(x(k|k)) = x^T(k|k)P(k)x(k|k) + \sum_{j=1}^{d} x^T(k-j|k)Q(k)x(k-j|k) + \sum_{l=d_m}^{d_M-1}\sum_{j=1}^{l} x^T(k-j|k)Q(k)x(k-j|k) \le \gamma(k) \quad (25)$$

Let us define $P(k) = \gamma(k)Q_1^{-1}(k)$, $Q(k) = \gamma(k)Q_2^{-1}(k)$. Then by the Schur complement, the conditions (22) and

(23) are derived with performing some procedure as in Theorem 1 and from the constraints of (25), respectively.

an invariant ellipsoid is denoted as following

$$\mathbf{\Gamma} = \left\{ z \in \mathbf{R}^{n(d_M+1)} \middle| z^T \Phi^{-1} z \le 1 \right\} \tag{26}$$

where $\Phi = diag[Q_1(k), Q_2(k), \ldots, (d_M - d - 1)^{-1} Q_2(k), (d_M - d + 1)^{-1} Q_2(k), \ldots, (r-1)^{-1} Q_2(k), \ldots, r^{-1} Q_2(k)]$,

$z = [x^T(k+i|k), x^T(k+i-d_M|k), \ldots, x^T(k+i-d+1|k), x^T(k+i-d|k), \ldots, x^T(k+i-d_m+1|k), \ldots, x^T(k+i-1|k)]^T$.

It is proved that

$$\begin{aligned} &\max_{i\ge 0} \left| (u(k+i|k))_j \right|^2 \\ &= \max_{i\ge 0} \left| \begin{pmatrix} Y_1(k) Q_1^{-1}(k) x(k+i|k) \\ + Y_2(k) Q_2^{-1}(k) x(k+i-d|k) \end{pmatrix}_j \right|^2 \le \max_{z\in\mathbf{\Gamma}} \left| (Y\Phi^{-1} z)_j \right|^2 \end{aligned} \tag{27}$$

where $Y = \begin{bmatrix} Y_1(k) & Y_2(k) & 0 & \cdots & 0 \end{bmatrix}$.

By the Cauchy-Schwarz inequality, we obtain $\max_{z\in\mathbf{\Gamma}} \left| (Y\Phi^{-1} z)_j \right|^2 \le \left\| (Y\Phi^{1/2})_j \right\|_2^2 = (Y\Phi^{-1} Y^T)_{jj}$. Then there exist a symmetric matrix G such that

$$\begin{bmatrix} G & Y \\ * & \Phi \end{bmatrix} \ge 0,\ G_{jj} \le u_{Mj},\ j = 1, \ldots, p \tag{28}$$

Form definitions of Y and Φ, it is easily proved that (28) is equivalent to (15). This completes the proof.

C. Feasibility and the Closed-Loop Stability

In addition, we prove the feasibility and closed-loop stable of the proposed MPC algorithm.

Theory 3: If the prime problem (4) is feasible, then the proposed control law (3) can asymptotically stabilizes the system.

Proof: Support that a control sequence is feasible at time k, then at next time, the control sequence is as following forms

$$u(k+i+1|k+1) = u^*(k+i+1|k),\ \ i \ge 0 \tag{29}$$

where $u^*(k+i+1|k)$ is a solution of the prime problem at k. the constraint (7) is satisfied at time k+1, which implies that the problem is feasible at time k+1. Then, the optimization problem is feasible at all-time instants t>k.

The optimal values are $P^*(k)$, $Q^*(k)$ and $P^*(k+1)$, $Q^*(k+1)$ at time k and k+1, respectively. Consider a quadratic function

$$\begin{aligned} V(x(k|k)) &= x^T(k|k) P^*(k) x(k|k) + \sum_{j=1}^{d} x^T(k-j|k) Q^*(k) x(k-j|k) \\ &+ \sum_{l=d_m}^{d_M-1} \sum_{j=1}^{l} x^T(k-j|k) Q^*(k) x(k-j|k) \end{aligned} \tag{30}$$

Since $P^*(k+1)$, $Q^*(k+1)$ are optimal, while $P^*(k)$, $Q^*(k)$ are only feasible at time k+1, then

$$\begin{aligned} V^*(x(k+1|k+1)) &\le x^T(k+1|k+1) P^*(k) x(k+1|k+1) \\ &+ \sum_{j=1}^{d} x^T(k-j+1|k) Q^*(k) x(k-j+1|k) \\ &+ \sum_{l=d_m}^{d_M-1} \sum_{j=1}^{l} x^T(k-j+1|k) Q^*(k) x(k-j+1|k) \end{aligned}$$

Besides, it follows from (11) that $V(x(k+1|k)) - V(x(k|k)) \le 0$ for any $\left[A(k)\ A_d(k)\ B(k) \right] \in \Omega$. Then it is obtained as following

$$V^*(x(x(k+1|k+1)) \le V^*(x(x(k|k)) \tag{31}$$

In conclusions, the objective function is a monotonically decreasing and bounded function. That is a say, the proposed algorithm asymptotically stabilizes the closed-loop system.

IV. Numerical Examples

In this section, a backing up control of a computer-simulated truck-trailer example [17] was presented to illustrate the performance of the proposed method. Using the Euler first-order approximation and sampling time $T = 0.1$ sec. the system is

$$x_{k+1} = A(\alpha) x_k + A_d(\alpha) x_{k-d} + B(\alpha) u_k$$

with $\Omega \in \left\{ \left[A^1\ A_d^1\ B^1 \right], \left[A^2\ A_d^2\ B^2 \right] \right\}$.

where $A^1 = \begin{bmatrix} 1.0509 & 0 & 0 \\ -0.0509 & 1 & 0 \\ 0.0509 & -0.4 & 1 \end{bmatrix}$, $A_d^1 = \begin{bmatrix} 0.0218 & 0 & 0 \\ -0.0218 & 0 & 0 \\ 0.0218 & 0 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1.0509 & 0 & 0 \\ -0.0509 & 1 & 0 \\ 0.0810 & -0.6366 & 0 \end{bmatrix}$, $A_d^2 = \begin{bmatrix} 0.0218 & 0 & 0 \\ -0.0218 & 0 & 0 \\ 0.0347 & 0 & 0 \end{bmatrix}$

$B^1 = B^2 = \begin{bmatrix} -0.1429 \\ 0 \\ 0 \end{bmatrix}$.

Simulation parameters are as follows:

$d = 4$, $S = 1$, $\varepsilon = 0.0064$, and $R = diag(10\,10\,10)$.

The angle difference between the truck and the trailer is denoted as x_1. x_2 is the angle of the trailer, x_3 is the y-coordinate of the rear end of the trailer, and u denotes the steering angle. For the sake of comparison, we show simulation with CMPC-UDS [11], the proposed robust MPC algorithm and the proposed robust MPC without input constraint in this example.

Fig. 1 presents a comparison of the set point tracking performance. The proposed RMPC algorithm shows better set-point tracking performance and faster response compared to those of CMPC-UDS.

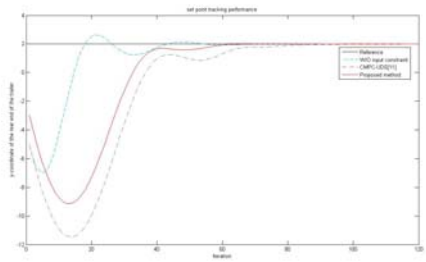

FIG.1 COMPARISON OF THE SET POINT TRACKING PERFORMANCE

Although the proposed MPC without input constraint achieves better performance than two others', these

properties are achieved at the cost of larger input, as showed in Fig.2.

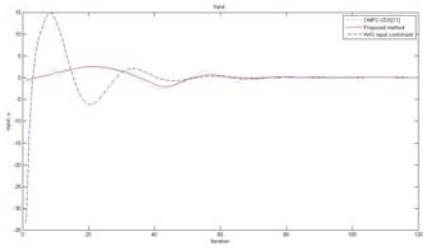

FIG.2 COMPARISON OF THE CONTROL INPUT

It is not feasible in many practical cases. Moreover, the convergence of the control input from the proposed algorithm is faster, smoother and smaller amplitude than these of CMPC-UDS. Fig.3 shows comparison of the upper bound of objective function among three situations. It is evident that the upper bound of the proposed method is smaller than that obtained with the method in CMPC-UDS.

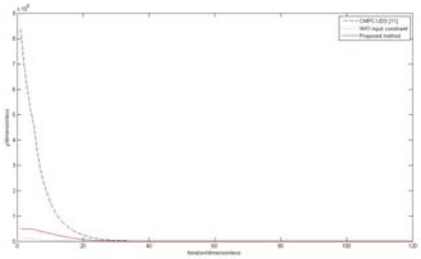

FIG.3 COMPARISON OF THE UPPER BOUND OF THE OBJECTIVE FUNCTION

V. CONCLUSIONS

The paper proposed a improved delay-dependent RMPC algorithm for delay uncertain polytopic systems. State delay with an upper and lower bound is unknown and time varying. Minimizing the upper bound of the objective function, we solved the original optimization problem. Moreover, we designed the memory feedback controller from the min-max optimization problem. With the newly proposed sufficient condition of the cost monotonicity, the less conservative MPC algorithm asymptotically stabilized the polytopic delayed system with input constraints. A numerical example demonstrates its effectiveness.

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