Book Review by P G L Leach

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N H Ibragimov: *Elementary Lie Group Analysis and Ordinary Differential Equations*, John Wiley, New York, 1999, 347 pages.

This book is divided into three parts – Introduction to Differential Equations, Fundamentals of Lie Group Analysis and Basic Integration Methods – of somewhat variable quality. Given the author's reputation and that of his mentor, L V Ovsyannikov, it is not surprising that the central section is of high quality. One is surprised that the two other sections are more than a little inferior by comparison. The presentation of the text is marred by numerous typographical errors which suggests very poor proof reading by the author and/or publisher. For example in (1.33) the force of attraction between the sun and a planet is written as F instead of the conventional $\mathbf{F}$ and is taken to be in the direction of the position vector instead of the the opposite direction as is the general practice. The interesting collection of symbols following (9.6) on p 210 seems to be out of context. Other mistakes, such as the statement that an expression containing two arbitrary constants is the general solution of a third order ordinary differential equation (p 303), are harder to dismiss so charitably.

Illustrative examples using the equations of gas dynamics, an area of the author's expertise, are well-handled, but the examples based on problems in Mechanics give one too much pause for thought. The use of the CGS system of units now some forty years after the adoption of SI units (p 15) is curious. The implication on p 86 that light beams are visible in a vacuum is indicative. On p 114 we find the claim that in 1983 the author discovered the so-called hidden symmetries which give the Laplace-Runge-Lenz vector using Noether's Theorem. They were already presented by the French writer, Jean-Marc Lévy-Leblond, in 1971 (*Amer J Phys* **39** (1971) 502-506). Again the concept of approximate symmetries is not new as the concept was commonroom gossip twenty years ago, possibly as a consequence of a number of papers on applications in Cosmology (for example Richard A Matzner, *J Math Phys* **9** (1968) 1063-1066 and 1657-1661; Arthur Komar *Phys Rev* **127** (1962) 1411-1418 and **129** (1963) 1873-1876; Alan Spero and Ralph Baierlein *J Math Phys* **18** (1977) 1330-1340 and **19** (1978) 1324-1334) . The proof of Noether's Theorem on p 238 is restricted to point symmetries even though on p 114 generalised symmetries were used in the context of Noether's Theorem. The Remark to justify the introduction of the so-called divergence relation makes one wonder why the author did not follow Noether's own derivation and thereby be able to avoid such sophistries. In the discussion of the Kepler Problem (pp 241-3) the Lie point symmetries are incorrectly given, Kelper's Third Law is incorrect and no mention is made of the discoverers of 'Laplace's vector', Jacob Herman and Jacob Bernoulli. On p 243 it is stated that there is no conserved quantity associated with the rescaling symmetry which is the basis of Kepler's Third Law. Three were given by Gorringe and Leach in 1991 (V M Gorringe and P G L Leach, *Quæstiones Mathematicæ* , **14** (1991) 277-289).

The book does have some attractive features. The notes give a good idea of work on symmetry by the Russian school and the author in particular as well as providing many interesting historical notes. It is a pity that this feature was not augmented by a bibliographic listing. The historical survey presented in Chapter 5 must be the high point of the book. All in all one is disappointed that more care was not taken with the preparation of the manuscript especially as the author's first language is Russian and not English in which the use of the definite article generally does not follow the usage in the former. Finally one is reminded of Rutherford's reply to Bohr when the latter sent a manuscript for comment. 'The English language does not consist only of 'thus' and 'hence'.'

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