A new integrated group decision making framework with linguistic interval fuzzy preference relations

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Abstract

The high complexity of socio-economic environments often makes it difficult for a single decision maker (DM) to consider all important aspects of decision problems. Therefore, a group decision making (GDM) process is often preferred by organizations. Moreover, during the decision process, DMs may have difficulties in the prioritization of alternatives. Linguistic interval fuzzy preference relation is a strong tool which can deal with uncertainty in case of subjective and vague information. For this reason, this paper develops a new GDM approach based on integrated linguistic interval fuzzy preference relation and analytic network process. To demonstrate the applicability of the proposed approach, an illustrative example is presented.
Keywords: Group decision making, Linguistic interval fuzzy preference, Analytic network process, Consensus model.

1. Introduction

As the complexity of the socio-economic environment is much increased today, many organizations make use of a group of decision makers (DMs – experts) instead of a single DM to accomplish the given tasks successfully [1-3].The group decision making (GDM) consists of multiple individuals interacting to reach a decision [4]. Each DM may have unique motivations or goals and may approach the decision process from a different angle, but have a common interest in reaching eventual agreement on selecting the ‘‘best’’ option(s). To do this, experts have to express their preferences by means of a set of evaluations over a set of alternatives. Recently, linguistic preference relations used by DMs to express their linguistic preferences when comparing decision alternatives have been investigated in the literature [5-10].

Moreover, at modeling of real life situations, DMs may not be able to discriminate explicitly the degree to which an alternative is better than another or may not estimate his/her preference with only one label. In such cases, linguistic interval fuzzy preference relations is useful for adequately modeling the uncertainty and imprecision in decision making processes [11].

While dealing with linguistic interval fuzzy preference relations of DMs, the solution procedure should also consider the dependence and interactions among decision criteria. The increasing complexity and uncertainty of the socio-economic environment makes it less possible to assume all criteria as independent. Hence, this work proposes a new integrated GDM approach based on the analytic network process (ANP) [12] method with linguistic interval fuzzy preference relations [11]. As linguistic interval fuzzy preference relations are not widespread currently, the main contribution of this paper is the integration of linguistic interval fuzzy preferences into the ANP model.

The paper is organized as follows. Section 2 presents the literature survey briefly. Section 3 describes the methodology adopted in the paper and characterizes the novel computational procedure. Section 4 includes an implementation of the proposed evaluation framework through an illustrative example. Section 5 concludes the paper.

2. Literature Survey

In decision making problems preference relation is the most well-known and widely used representation of information. It is an advantageous tool to model decision processes, when there is a necessity to combine experts’ preferences into a new form of group preferences [13-15].The linguistic information can be defined as a variable whose values have the form of words, phrases or sentences rather than numbers in a natural language [16-17]. Moreover, linguistic variables help to model problems in a qualitative ways which is typical in human communication for representing qualitative concepts such as ‘‘importance’’ or ‘‘significance” [16].

In this view, fuzzy set theory is assessed in complex situations which include imprecise information in DMs assessments. In other words, experts could have some difficulties for estimating their preference degrees with exact numerical values. Under these circumstances, fuzzy linguistic approach is used in order to capture all data, manage linguistic information and provide better solution [18-19].

In the GDM process, aggregating each DM’s decision information is the key [1, 6]. Literature on linguistic fuzzy preference relations mainly focus on operators in aggregation processes. Since the ordered weighted averaging (OWA) operator was first generated by Yager [21] in 1988, many aggregation operators such as the linguistic weighted ordered weighted averaging (LOWA) operator [5, 22] and linguistic ordered weighted geometric averaging (LOWGA) operator [8] have been developed.

On the other hand, interval fuzzy preference relations are useful tools to describe experts' preferences in GDM under uncertainty. The concept of interval fuzzy preference relation was first introduced by Xu [23], based on interval fuzzy preference relation. In this study a priority method was given to determine the weights of objects, and a possibility degree formula is used to rank and select the given objects. In addition to this, some authors investigated similarity measures, aggregation and priority methods of interval fuzzy preference relations. For example, Xu [24] defined the concept of compatibility degree of two interval fuzzy preference relations, and showed the compatibility relationship among individual interval fuzzy preference relations and collective interval fuzzy preference relation. Herrera et al. [25] developed an aggregation process for combining interval fuzzy preference relations.

In GDM problems, DMs' opinions may differ substantially. Therefore, it is necessary to develop a consensus process in an attempt to obtain a solution of consensus [6]. Classically, consensus is defined as the full and unanimous agreement of all the experts (DMs') regarding all the possible alternatives. Recently, Tapia Garcia et al. [11] propose a new consensus model for GDM problems with linguistic interval fuzzy preference relations. They use two kinds of consensus measures to guide the consensus reaching processes, which are consensus degrees (to evaluate the agreement of all the experts) and proximity degrees (to evaluate the agreement between the experts' individual preferences and the group preference). Besides, the consensus process discusses how to obtain the maximum degree of consensus or agreement among a set of experts. Therefore, this process is necessary to obtain a final solution with a certain level of agreement among experts [26]. Then, both measures on the three levels of representation of linguistic interval fuzzy preference relations are computed, which are: level of pair, level of alternative and level of relation.

Year	Ref.	Methodology	Operator	Area	Type
2004	[27]	Linguistic interval	FN- IOWA	-	Illustrative Example
2008	[28]	Linguistic interval	WC- OWA	-	Illustrative Example
2008	[29]	Interval fuzzy pref. relation.	-	-	Illustrative Example
2008	[30]	Interval fuzzy pref. relation	-	-	Illustrative Example
2011	[31]	Uncertain linguistic variables, interval probability	-	Investment	Illustrative Example
2011	[32]	Interval fuzzy pref., Quadratic prog. model	-	Military	Illustrative Example
2012	[33]	Fuzzy linguistic with 2-tuples, consensus	-	-	Illustrative Example
2012	[11]	Linguistic interval,consensus	LOWA	-	Illustrative Example
2013	[34]	Interval intuitionistic uncertain linguistic variables	IVIULWGA IVIULOWG	Developing rural area	Illustrative Example
2013	[35]	2-tuple linguistic information	IVTWG,IVTOWG GIVTWA GIVTOWA	-	Illustrative Example

Table 1. Several studies make use of GDM with linguistic interval fuzzy preferences

Then, an automatic feedback mechanism is applied to guide experts in the consensus reaching process and substitute their considerations in the DM process [26].

Several authors have previously studied the GDM methodology with linguistic interval fuzzy preferences. Table 1 lists a sample of those studies.

3. Computational procedure of the proposed approach

The general view of the proposed GDM approach which is based on integrated linguistic interval fuzzy preference relation and analytic network process is given in Figure 1.

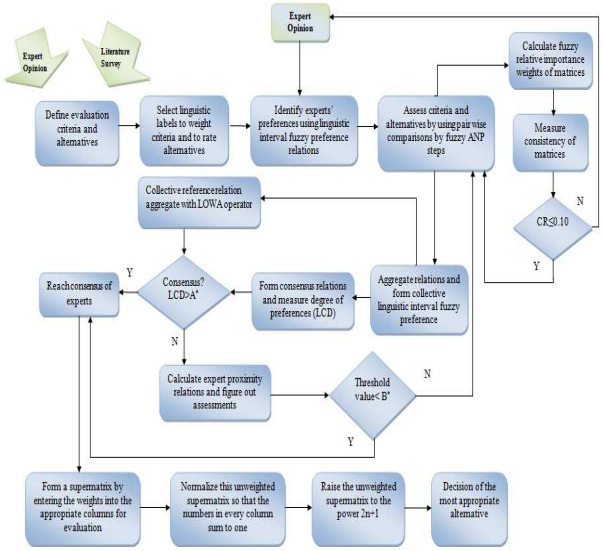

Fig 1. A general view of the proposed integrated methodology (A^* and B^* are threshold values)

The computational steps of the proposed approach are as follows:

Step 1: Construct a committee of experts (DMs), determine the alternatives and develop the network structure for evaluation.

Step 2: Design and select a comparison scale to weight criteria set and to rate alternatives.

Trapezoidal fuzzy intervals are often used in practice. The reason of their popularity is that trapezoidal membership functions are better than triangular membership functions with respect to representing the problem more realistically [36]. Moreover, according to Herrera and Herrera-Viedma [26], [37] it is considered that linear trapezoidal membership functions are good enough to capture the vagueness of these linguistic assessments [37-39]. Table 2 gives the nine linguistic label set with their respective associated semantics to express the preferences.

Step 3: Construct and evaluate the pair-wise comparison matrices with linguistic interval fuzzy preference relations.

s_8 = C	Certain	(1.00, 1.00, 0.00,0.00)
s_7= EL	Extremely likely	(0.98, 0.99, 0.05, 0.01)
s_6 = ML	Most likely	(0.78, 0.92, 0.06, 0.05)
s_5= MC	Meaningful chance	(0.63, 0.80, 0.05,0.06)
s_4 = IM	It may	(0.41, 0.58, 0.09, 0.07)
s_3 = SC	Small chance	(0.22, 0.36, 0.05, 0.06)
s_2 = VLC	Very low chance	(0.10, 0.18, 0.06,0.05)
s_1 = EU	Extremely unlikely	(0.01, 0.02, 0.01, 0.05)
s_0 = I	Impossible	(0.00, 0.00, 0.00,0.00)

Table 2. Corresponding linguistic terms for evaluation [10]

Step 4: Aggregate of linguistic interval fuzzy preference relations [11].

The Linguistic Ordered Weighted Averaging (LOWA) is used as an operator in order to aggregate nonweighted ordinal linguistic information [11],[40]. Suppose that $\{a_1, \ldots, a_m\}$ be a set of labels to be aggregated, then the LOWA operator, ϕ, is defined as

$\phi(a_1,..a_2) = W.B^T = C^m\{\mathbf{W_k}, \mathbf{b_k}, \mathbf{k = 1}, .. \mathbf{m}\}$

$= w_1 \odot b_1 \oplus (1 - w_1) \odot C^{m-1}\{\beta_h, b_h, h = 2, .. m\}$ (1)

where $W=\{W_1,\ldots,W_m\}$, is a weighting vector, such that, $W_i \in [0,1]$ and $\Sigma_i w_i=1$; $\beta_h = W_h / \Sigma_2^m w_k$, $h = 2, .. m$ andB is the associatedordered label vector. Each element $b_i \epsilon B$ is the ith largest label in the collection $a_1, \ldots, a_m$. C^m is the convex combination operator of m labels and if m = 2, then it is defined as

$C^2\{W_i, b_i, i = 1,2\} = w_1 \odot s_j \oplus (1 - w_1) \odot s_i = s_k, s_j, s_i$

$S(j \geq i)$ (2)

such that k = min {T,i + round(w_1. (j - i))}, where round is the usual round operation, and

$b_1 = s_j, b_2 = s_i$. If $w_j = 1$ and $w_i = 0$ with $i \neq j$, then the convex combination is defined as:

$C^m\{W_i, b_i, k = 1, \ldots, m\} = b_j$. (3)

U, indicates the global preference between every ordered pair of alternatives according to the majority experts 'opinions. For example, a possibility to obtain U in the case of the linguistic interval fuzzy preference relations it would be as follows:

$U = (U_{ij})$ for i,j=1,…,n with

$U_{ij} = U[p_{ij}^-, p_{ij}^+] = [\phi_-(p_{ij}^{k-}), \phi_+(p_{ij}^{k+})]$

$= [\min_k(p_{ij}^{k-}), \max_k(p_{ij}^{k+})]$ for k=1,…,n (4)

With $w_- = \{0, \ldots, 0, 1\}$ in ϕ_- and $w_+ = \{1, 0, \ldots, 0\}$ in ϕ_+

Obtain exploitation phase and select the more preferable value. Calculate its dominance degree $px_i$from the collective linguistic interval fuzzy preference relation as:

$Px_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} \left(s(p_{ij}^-) + s(p_{ij}^+)\right)$ (5)

In such a way, we obtain a classification of the alternatives: if $s(px_i) > s(px_j)$ then x_i is preferable to x_j and therefore, the alternative x_i is the recommended solution.

Step 5: Conduct consensus and proximity measures of the model. Consensus indicators are computed by the help of the following steps [11]:

Firstly, we compute the consensus relations of each expert e^k, called C^k, with respect to

$C^k = (C_{ij}^k)$with

$\left(\left|s(p_{ij}^{k-}) - s(p_{ij}^-)\right| + \left|s(p_{ij}^{k+}) - s(p_{ij}^+)\right|\right)/T$

For i,j=1,…,n (6)

Secondly, we define the linguistic global consensus degree, LCD, as,

$$\sum_{i=1}^{n} LCD_i = 1 - \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{k=1}^{m} C_{ij}^k / ((n^2 - n)m)$$

$$= CD. \quad (7)$$

Now, we continue with the process in order to calculate the proximity measures. Firstly, we calculate the expert proximity relations, called F_k, with respect to the collective preference relation U as:

$F^k = F_{ij}^k$ with

$$F_{ij}^k = \left(s(p_{ij}^{k-}) - s(p_{ij}), s(p_{ij}^{k+}) - s(p_{ij})\right)$$

$$= (f_{ij}^{k-}, f_{ij}^{k+}) \quad (8)$$

For i,j =1,…,n and $p_{ij} = \phi_Q(p_{ij}^-, p_{ij}^+)$ and $s(p_{ij}) = n$

If $p_{ij} = s_n$.

Then, we define the proximity measure of the expert e^k on a preference p_{ij} as:

$PM_{ij}^k = \left(|f_{ij}^{k-}| + |f_{ij}^{k+}|\right)/2T$ (9)

After that, we should define the proximity measure of the expert e^k in an alternative x_i as

$$PM_i^k = \sum_{\substack{j=1 \\ j \neq i}}^{n} PM_{ij}^k/(n-1) \quad (10)$$

Then, we have to define the global proximity measure of the expert e^k as:

$$PM^k = \sum_{i=1}^{n} PM_i^k/(n) \quad (11)$$

Step 6: Checking consistencies and doing the feedback process. The feedback mechanism helps to guide the change of the expert's opinions using with proximity matrices F^k [26],[41-43].To reach a consensus, this mechanism provides a good control of consistencies and if there are inconsistencies, then the expert's preferences should be revised.

Feedback mechanism is accomplished in two phases:

- Identification phase,
- Recommendation phase.

Identification phase: It is necessary to compare the global consensus degree LCD and a consensus threshold A, previously fixed. Then, if LCD > A or LCD = A the consensus process will stop; on the other hand, if LCD < A, a new consensus round must be applied. Firstly, the pairs of alternatives with a consensus degree smaller than a threshold value A defined at level of pairs of alternatives, $CD_{ij} < A$, are identified. Secondly, we identify the experts who will be required to modify the identified pairs of alternatives. To do that, we use the expert proximity measures PM^k and PM_i^k, and also we fix a value threshold B. The experts that are required to be modified are preferences whose $PM^k > B$.

Recommendation phase: In this phase we recommend expert changes of their preferences according to some rules to change the opinions.

Step 7: Form the supermatrix of ANP model. A supermatrix is a partitioned matrix, where each

submatrice is composed of a set of relationships between two clusters. After establishing the supermatrix, we normalize it so that the numbers in every column add up to one. To derive the overall priorities of elements, we need to multiply the submatrices until the columns stabilize and become identical in each block of submatrices. It is necessary to raise the unweighted supermatrix to the power of 2n + 1 where n is an arbitrary large number. Raising a matrix to powers gives the long term relative influences of the elements on each other, and this new matrix is called the limit supermatrix [12].

Step 8: Determine the most appropriate alternative.

4. An illustrative example: Partner selection for customized product development

To illustrate our approach, we give an illustrative example on the partner selection problem for customized product development.

Step 1: There are three DMs and four possible partner alternatives (P1, P2, P3, P4).The evaluation system consists of three main dimensions: Customization Strategies, Performance and Partner Evaluation Criteria. The evaluation network structure, interactions and interdependency relationships of evaluation elements are depicted in Figure 2.

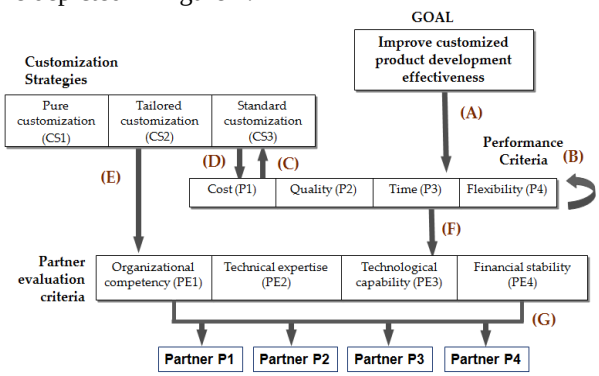

Figure 2.Network model of evaluation structure.

The notation for supermatrix is in Table 3 where relations forcomparisons are presented with letters and I is the identity matrix.

	Goal	PC	CS	PEC	ALTS
Goal	0	0	0	0	0
Performance criteria (PC)	A	B	D	0	0
Customization strategies(CS)	0	C	0	0	0
Partner evaluation c. (PEC)	0	F	E	0	0
ALTS	0	0	0	G	I

Table 3. General submatrix notation for supermatrix.

Step 2: The linguistic evaluation scale has already been discussed in Section 3.

Step 3: The importance degrees of criteria and alternatives are considered and DMs' preferences using linguistic interval fuzzy preference relations are given in Tables4, 5, 6 respectively.

Assessment	(P1)	(P2)	(P3)	(P4)
Cost (P1)	-	[VLC, MC]	[SC, ML]	[SC, ML]
Quality (P2)	[VLC, IM]	-	[SC, MC]	[VLC, ML]
Time (P3)	[MC, EL]	[MC, C]	-	[SC, EL]
Flexibility(P4)	[EU, SC]	[VLC, ML]	[SC, MC]	-

Table 4. First DM's evaluation of performance criteria

Assessment	(P1)	(P2)	(P3)	(P4)
(P1)	-	[VLC, MC]	[SC, ML]	[EU, SC]
(P2)	[VLC, IM]	-	[SC, EL]	[EU, IM]
(P3)	[VLC, MC]	[SC, EL]	-	[SC, ML]
(P4)	[VLC, MC]	[VLC, ML]	[VLC, IM]	-

Table 5. Second DM's evaluation of performance criteria

Assessment	(P1)	(P2)	(P3)	(P4)
(P1)	-	[SC, IM]	[IM, EL]	[MC, C]
(P2)	[VLC, SC]	-	[MC, C]	[IM, ML]
(P3)	[SC, MC,]	[SC, EL]	-	[IM, MC]
(P4)	[VLC, IM]	[SC, ML]	[EU, SC]	-

Table 6. Third DM's evaluation of performance criteria

Step 4: Aggregation of collective linguistic interval fuzzy preference relations.
Using the previous aggregation tool and Eq. (1)-(4) we obtain U as shown in Table 7.

U	(P1)	(P2)	(P3)	(P4)
(P1)	-	[SC, IM]	[IM, EL]	[Ml, C]
(P2)	[SC, MC]	-	[MC, ML]	[SC, IM]
(P3)	[SC, IM]	[IM, ML]	-	[IM, MC]
(P4)	[VLC, IM]	[EU, IM]	[EU, SC]	-

Table 7. Collective linguistic interval fuzzy preference relation of performance criteria

Step 5: Conduct consensus and proximity measures of the model: Consensus relations using Eq. (5-6) is calculated and first DMs consensus matrices are as follows:

$$C^1 = \begin{pmatrix} - & 1/4 & 1/4 & 5/8 \\ 1/4 & - & 3/8 & 3/8 \\ 5/8 & 3/8 & - & 3/8 \\ 1/4 & 3/8 & 1/2 & - \end{pmatrix}$$

From calculation using Equation (17), global consensus degree (CD) is obtained as 0.697 (or CD=69.7 %). Then we fix a consensus threshold value A is equal to 0.7. This means that if CD is less than 0.7; it seems not to be acceptable and the DMs should perform the pair wise comparison again. However, to be able to sure that DM's judgment is acceptable, we have to measure the DM proximity relations called F^k with the help of Eq. (7).As an example the F^k for first DM $e^1$is:

$$F^1 = \begin{pmatrix} - & (-2,1) & (-3,0) & (-4,-1) \\ (-2,0) & - & (-3,-1) & (-2,2) \\ (1,3) & (0,3) & - & (-2,2) \\ (-2,0) & (-1,3) & (1,3) & - \end{pmatrix}$$

The proximity measures for DMs e^k in each alternative x_i are calculated using Eq. (8)-(11), and the results are:

$PM_1^1 = 0.229$ $PM_1^2 = 0.333$ $PM_1^3 = 0.146$
$PM_1^2 = 0.208$ $PM_2^2 = 0.188$ $PM_2^3 = 0.167$
$PM_1^3 = 0.229$ $PM_3^2 = 0.208$ $PM_3^3 = 0.146$
$PM_1^4 = 0.208$ $PM_4^2 = 0.208$ $PM_4^3 = 0.146$

And,

$$PM^1 = 0.219;\ PM^2 = 0.234;\ PM^3 = 0.151.$$

Step 6: Checking consistencies and do feedback process. DMs should change their preferences in order to achieve appropriate agreement degree. If we fix a threshold value to 0.15, those DMs should change their assessments, especially DM1 and DM2.So, DMs focus on new assessments which are given in Tables 8, 9 and 10 respectively.

Assessment	(P1)	(P2)	(P3)	(P4)
(P1)	-	[IM, EL]	[MC, EL]	[ML, C]
(P2)	[SC, MC]	-	[SC, ML]	[VLC, IM]
(P3)	[VLC, IM]	[SC, MC]	-	[IM, MC]
(P4)	[I, EU]	[EU, SC]	[EU, SC]	-

Table 8.First DM's renewed evaluation of pref.criteria

Assessment	(P1)	(P2)	(P3)	(P4)
(P1)	-	[MC, C]	[ML, C]	[EL, C]
(P2)	[IM, ML]	-	[IM, EL]	[VLC, MC]
(P3)	[SC, MC]	[IM, ML]	-	[IM, ML]
(P4)	[I, VLC]	[EU, IM]	[VLC, IM]	-

Table 9. Second DM's renewed evaluation of pref. criteria

Assessment	(P1)	(P2)	(P3)	(P4)
(P1)	-	[MC, EL]	[ML, EL]	[ML, C]
(P2)	[IM, MC]	-	[SC, EL]	[VLC, IM]
(P3)	[SC, IM,]	[SC, ML]	-	[IM, MC]
(P4)	[I, VLC]	[EU, SC]	[VLC, SC]	-

Table 10. Third DM's renewed evaluation of pref. criteria

Then, we obtain the following collective linguistic interval fuzzy preference relation U as given in Table 11.

U	(P1)	(P2)	(P3)	(P4)
(P1)	-	[MC, EL]	[ML, EL]	[EL, C]
(P2)	[IM, MC]	-	[IM, ML]	[VLC, IM]
(P3)	[SC, IM]	[IM, MC]	-	[IM, MC]
(P4)	[I, EU]	[EU, SC]	[VLC, SC]	-

Table 11. Collective linguistic interval fuzzy preference relation of performance criteria

In this case, global consensus degree is CD=0.91319and the threshold ratio is less than CD which means acceptable. Finally, collective fuzzy preference relations matrix (U) is suitable to indicate dominance degrees of each weight which is: px_1= 0.40; px_2= 0.25; px_3= 0.25; px_4= 0.10.

Step 7: Form supermatrix: An unweighted supermatrix is formed by including priority vectors in the related columns.

Step 8: Determine the suitable partner alternative. Afterwards by normalizing the unweighted supermatrix and raising it to the power of 5, the weighted supermatrix is attained. The final ranking is given in Table 12. The obtained result shows that Alternative C has the highest score among all alternatives and it can be identified as the most suitable partner for improving the customized product development effectiveness.

Alternative	Weight
A	0.265
B	0.226
C	0.274
D	0.235

Table 12. Final ranking in customized product development partner selection problem

5. Concluding remarks

In some evaluation and decision situations, DMs have difficulty to express some preferences by means of exact preference degrees. In order to represent their preferences accurately, this study supported an approach that enables a combined consensus model in GDM with linguistic interval fuzzy preference relations. In addition, for dealing with the dependency of evaluation criteria, ANP is capable of incorporating many interrelationships of factors into the decision model. For this reason, an integrated methodology based on fuzzy ANP and incomplete preference relations was proposed in this study. An illustrative customized product development partner evaluation case was used to exemplify the proposed framework. To our knowledge, no previous work has investigated this new integrated approach. As the proposed approach is new and applied to partner selection process as an illustrative example, it might be applied to other GDM problems and can be used for real life applications.

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