

Book Reviews by F Calogero

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Four books published by Birkhäuser are reviewed.

I M Gelfand, M M Kapranov and A V Zelevinsky: *Discriminants, Resultants, and Multidimensional Determinants*. Birkhäuser, Basel Boston Berlin, 1994. Approximately 452 pages. Hardcover (DM 158,00).

This book reviews classical results, and connects them to modern developments. The main hero from the past is A. Cayley: a 1948 paper of his, *On the Theory of Elimination*, reproduced as Appendix B, is credited as having introduced “*several fundamental concepts of homological algebra: complexes, exactness, Koszul complexes, and even the invariant now sometimes called the Whithead torsion or Reidemeister-Franz torsion of an exact complex.*” The authors trace their interest in these topics to their attempt to construct a general theory of hypergeometric functions, indeed they claim that “*the place of discriminants in the general theory of hypergeometric functions is similar to the place of quasi-classical approximation (sic) in quantum mechanics*”. The justification for this remarkable statement is that multiple roots of a polynomial, or combined roots of several polynomials - which are the main theme of this book - identify the singularities of hypergeometric functions, through the vanishing of the highest symbols of the differential equations they satisfy. Hence, according to the authors, “*The relation between differential operators and their highest symbols is the mathematical counterpart of the relation between quantum and classical mechanics; so we can say that hypergeometric functions provide a <<quantization>> of discriminants.*”

The book, however, does not deal with differential equations; it provides instead a fascinating exposition of classical and novel results on discriminants (the objects that characterize multiple roots of polynomials), resultants (the objects that characterize simultaneous roots of several polynomials), and “*hyperdeterminants (analogs of determinants for multidimensional <<matrices>>)*”.

The book is clearly written, but cannot be fully appreciated without a considerable background in algebraic geometry. It is divided in three parts, entitled respectively *General Discriminants and Resultants* (4 chapters, approx. 130 pages), *A-Discriminants and A-Resultants* (7 chapters, approx. 210 pages), *Classical Discriminants and Resultants* (3 chapters, approx. 75 pages). Readers less expert in algebraic geometry may find it convenient to start from the third part, namely to read first chapters 12-14 (*Discriminants and Resultants for Polynomials in One Variable, Discriminants and Resultants for Forms in Several Variables, Hyperdeterminants*).

The *Bibliography* extends over 10 pages; it is ordered alphabetically by the name of the first author, not by the acronym used to identify the reference in the text. It is followed by 3 pages containing “*some additional bibliographic comments and references.*”

We guess this book will serve as standard reference tool for quite some time, and we expect it will cause a surge of research on these topics.

Bert Fristedt and **Lawrence Gray**: *A Modern Approach to Probability Theory*. Birkhäuser, Boston Basel Berlin, 1997. 776 pages. Hardcover (DM 118,00).

This well written textbook on probability requires as preliminary knowledge only elementary calculus and some linear algebra. It is meant for a basic 3-semester graduate course in probability theory; various paths suitable for a 2-semester treatment are also indicated. The clear exposition and pedagogical care make it also suitable for self-instruction: there are indeed many examples and well over a thousand exercises, about a quarter of which have their solutions available on INTERNET (a clear demonstration of how up-to-date this presentation is; another telling indication is the careful avoidance of any preferential use of the masculine pronoun “he” to identify, for instance, the reader; it is either “she or he”, or most often just “she”).

The scope of the book is best described by quoting from its cover blurb: “*The authors introduce the basic objects of probability theory (random variables, distributions and distribution functions, expectations, independence), at the same time developing concepts from measure theory as required. They then proceed through the standard topics in the subject, including laws of large numbers, characteristic functions, central limit theorems, conditioning, and random walks. The latter part of the book concern stochastic processes in both discrete and continuous time, with individual chapters being devoted to martingales, renewal sequences, Markov processes, exchangeable sequences, stationary sequences, point processes, Levy processes, interacting particle systems, and diffusions. The treatment of these topics is sufficiently advanced to bridge the gap between standard material and specialized research monographs.*”

The text has no Footnotes nor calls to References, but there is a 13-page Appendix with *Comments and Credits* concerning each Chapter and including References to original papers, as well as an extensive Bibliography (8 pages) detailing, firstly, *General probability books*, secondly, *General books on stochastic processes*, and then several sets of books appropriate to various sets of chapters. There is also a 6-page Appendix devoted to *Notation and Usage of Terms*, as well as a 20-page Index, alphabetically ordered (except for a slightly confusing beginning with some mathematical symbols, Greek letters and the like: the first and last items of this first part both read “0-1 trivial σ -field” but provide different page numbers – both, as it happens, appropriate).

The price appears reasonable: if this book will have the success it seems to deserve a much less expensive paperback edition will of course be essential to make it affordable by students.

Victor Ya Frenkel: *Yakov Ilich Frenkel - His work, life and letters*. Birkhäuser, Basel Boston Berlin, 1996. 332 pages. Hardcover (DM 198,00).

This book is dedicated to the Soviet theoretical physicist Yakov (“James”) Ilich Frenkel (1894-1952), well known for his many contributions in most fields of modern physics (especially in solid state physics: for instance, the defects in solids called Frenkel pairs, the theory of mobile dislocations called Frenkel-Kontorova solitons). Ya I Frenkel was also a prolific writer of textbooks, which educated many generations of Soviet physicists, indeed physicists everywhere, since some of his textbooks were also published in Germany in German and in England in English.

This book has been written by his son Victor Frenkel, who was himself a world-known authority in the history of modern physics (unfortunately, he also passed away recently – in fact, after this review was first drafted). This is an advantage, to the extent that the author draws on an intimate knowledge of the subject and brings to bear on this material his scholarship as an expert historian, as well as a considerable amount of archival research (entailing in several instances digging into archives which were opened in Russia only recently, after the demise of the Soviet regime - indeed the book under review here is a much enlarged version of a previous book by the same title published by the same author in the Soviet Union in 1966). The close (son-father) relationship between biographer and biographee is also an advantage, because it provides deep inside information, but it is also a liability, inasmuch as the book has inevitably an agiographic character.

The book contains a narrative part, but also reports many letters written by Yakov Ilich (to his parents, his wife, other relatives and colleagues) and by others to him. Since he entertained friendly personal relations with many of the outstanding physicists of this century, and indeed often collaborated with them (including Einstein, Born, Pauli, Bohr - and so on), this correspondence is of great interest. It also provides a vivid background description of the various developments of Soviet science and Soviet society.

Yakov Ilich, as most of his friends and colleagues in Russia, was a convinced communist - but he also suffered from the persecutions and fears which characterized the Soviet regime. For instance, he was harassed because of his defence of quantum mechanics, which he treated in detail in some of his textbooks, often using an imaginific language which he considered pedagogically useful but led to accusations of contradicting “Marxism-Leninism” and of leaning toward “idealistic” philosophical views. Although put forward by incompetent scientists, this kind of criticism entailed great dangers in the Soviet Union - indeed, analogous debates masterminded by Lysenko caused the destruction of Soviet biology, including the deportation and the death of many of its more competent practitioners. Probably the main reason why physicists - in particular, theoretical physicists - in the Soviet Union were spared a similar fate after the Second World War was the influence of Igor Kurchatov, who was a highly competent physicist and who carried much weight with the Soviet leadership - all the way up to Beria and Stalin - as head of the Soviet nuclear weapon program.

This is a well-written, entertaining and highly informative book, especially for readers interested in the history of modern physics. It also contains several interesting photographs. This hardly justifies its price, which seems rather exorbitant.

Richard H. Cushman and **Larry M. Bates**: *Global Aspects of Classical Integrable Systems*. Birkhäuser, Basel Boston Berlin, 1997. 452 pages. Hardcover (DM 78,00).

The best way to explain the motivation and scope of this bulky - and, given current prices, not-too-expensive - book is to quote selectively from its Forward: “*This book gives a complete global geometric description of the motion of the two dimensional harmonic oscillator, the Kepler problem, the Euler top, the spherical pendulum and the Lagrange top.*” “*We have not written a book on mechanics or Hamiltonian particle dynamics. We only discuss five special integrable systems, which is a very small sample of the rich variety of general Hamiltonian systems.*”. “*This book is written from a bottom up approach with examples being given prominence over theory. The examples are treated in a uniform way.*”

First the mathematical model is described and the equations of motion are derived. Next the symmetries and corresponding integrals are obtained and it is shown that the given problem is Liouville integrable. Finally, the geometry of the level sets of the energy momentum map, which gives a completely geometric description of the motion, are obtained by first using reduction to remove the symmetries and then reconstructing the geometry from the geometry of the reduced systems.” “The theory given in the appendices is what the authors feel is the minimum necessary to justify all the unproven assertions in the examples.” “This book was not written to be read in a sequential fashion. We strongly encourage the reader to browse.”

The book is organized in 5 Chapters and 5 Appendices (see above), each including a final Section devoted to Exercises. There are no footnotes, but 11 pages at the end are devoted to Notes with comments on specific paragraphs of the text. There is a Bibliography with 227 entries ordered alphabetically (14 pages - I was surprised not to find there A. M. Perelomov's name), and a 9-page Index including the puzzling item “*champagne bottle*”, that refers to the *champagne bottle potential* (on which one of the two authors has written a paper: L.Bates, *Monodromy in the champagne bottle*), namely the two-dimensional spherically-symmetrical “double-well” quadratic-plus-quartic potential (a bottle containing indeed a lot of champagne!)

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