An application of PCC model: risk measurement of extreme event in Chinese stock market

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Abstract. We focus on analyzing extreme event on stock market with the pair-copula constructions (PCC) based multivariate models with GARCH (p, q) margins. We utilize the PCC model to get the estimation of joint PDF parameters. Then, we use six indices construct the decomposition of the PCC copula. As for different tail dependence of these log-returns series, we build the estimating model with bivariate t-copulas. Finally, we apply Monte Carlo method to simulate the extreme loss with parameters estimated from decomposing steps.

Introduction

Extreme event happens in finance market and it could generate huge loss to the investors even though its frequency is low. In this paper, we focus on the stock market violation and we are intending to observe the high dimensional factors' decomposition so as to analysis the extreme risk measure and management of the stock market based on the pair-copula theory. We use the pair-copula constructions (PCC) to obtain the joint probability distribution based on multivariable data series. The appearance of pair-copula theory supplies a kind of way to solve the problem of decomposing the high dimensional copula. And Pair-copula well solved the problem of estimating the high dimensional copula parameters.

This paper intends to analysis of extreme event on Chinese stock market. We first introduce the relevant theory and methodology, and then give the empirical analysis. Section 2 gives the pair-copula construction model. Section 3 uses six indices to construct the decomposition of the PCC copula and build the estimating model with bivariate t-copulas. Finally, we apply Monte Carlo method to simulate the extreme loss with parameters. Section 4 concludes this paper.

Pair-copula Construction

1.1. Decomposition of pair-copula for multivariate density functions

Consider a n-dimensional random variables $X = (X_1, X_2, \ldots X_n)$ with a joint density function $f(x_1, x_2, \ldots x_n)$. The density can be factorized uniquely as

$$f(x_1, x_2, \ldots x_n) = f(x_1) \cdot f(x_2 \mid x_1) \cdot f(x_3 \mid x_1, x_2) \cdots f(x_n \mid x_1, \ldots x_{n-1}) \qquad (1)$$

We can get the pair-copula decomposition formula:

$$f(x_1, x_2, \ldots x_n) = \prod_{r=1}^{n} f(x_r) \times \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j, j+i \mid 1, \cdots, j-1} \quad (j = t - k, \ j + i = t) \qquad (2)$$

In the next we need to know the exact decomposition step of the copula function $c_{j, j+i \mid 1, \cdots, j-1}$.

1. Numerical analysis: an application of PCC to the stock market

We utilize the above model to the stock market. We consider six time series of daily log-returns

of six indices' prices as follows: industrial index (GY); public utility (GS); finance index (JR); Consumer Discretionary (KX); consumer staples index (RX); Information Technology Index (XJ). The data's resource is wind database, and the period is between December 31st, 2009 and December 31st, 2011.

1.1. Estimation of marginal distribution

We use log-return series of industrial index (GYR); public utility (GSR); finance index (JRR); Consumer Discretionary (KXR); consumer staples index (RXR); Information Technology Index (XJR), and we take the lowest daily price of each index as the sample of log-returns. Here we apply the GARCH (1, 1) –t model to the log-return series and obtain the standardized residuals to describe the marginal distribution of return series with conditional heteroscedasticity .

Table1 gives the parameter estimation of GARCH(1,1)-t for the six log-return series. We could see that each parameter pass the test on a high level of significance, and thus each log-return can be described by GARCH(1,1)-t. After the estimation of each parameter, we could get the daily log-return distribution of the six indices.

Table 1. Parameter estimation of GARCH model

	Parameter 0	Parameter 1	Parameter 2	Parameter 3	Degree of Free
GY	0.020	0.093	0.046	0.920	5.338
(P Value)	(0.017)	(0.021)	(0.029)	(0.000)	(0.000)
GS	0.032	0.120	0.057	0.884	4.813
(P Value)	(0.028)	(0.020)	(0.033)	(0.000)	(0.000)
JR	-0.062	0.070	0.035	0.928	3.451
(P Value)	(0.019)	(0.031)	(0.018)	(0.000)	(0.000)
KX	-0.004	0.074	0.048	0.926	6.116
(P Value)	(0.010)	(0.021)	(0.032)	(0.000)	(0.002)
RX	0.103	0.058	0.087	0.893	7.396
(P Value)	(0.001)	(0.010)	(0.005)	(0.000)	(0.027)
XJ	0.008	0.118	0.071	0.897	8.856
(P Value)	(0.009)	(0.020)	(0.030)	(0.000)	(0.035)

1.2. Pair-copula decomposition

We use these standardized residuals generated from GARCH model as the data series so as to choose the pair-copula function. Figure 1 is the scatter plots of each pair of marginal distribution series. From the figure we could see obvious upper and lower tail dependence.

Table 2 gives the estimation results and the shadowed cells are the number we choose to get the root node. From the table we could see that node JR has the most shadowed cells, hence node JR is the root node we seek.
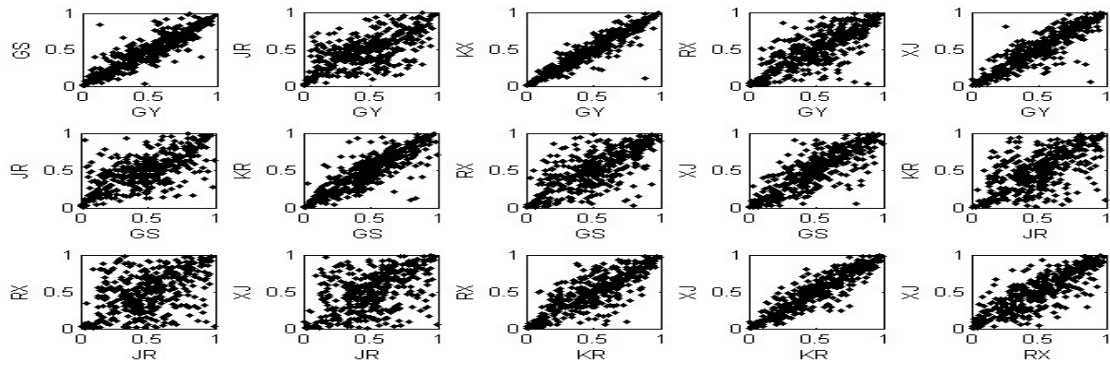
Fig. 1. Scatter plots of the unit interval transformed standardized residuals

Table 2. Estimated degree of free in a bivariate t-copula model

	GS	JR	KX	RX	XJ
GY	3.373	4.258	2.346	3.530	2.648
GS		2.316	3.226	3.064	4.395
JR			2.257	1.825	2.458
KX				3.905	4.581
RX					6.482

Table 3. Estimated start and final ML parameters

Parameters	Start		Final	
	Correlation	Degree of free	Correlation	Degree of free
14	0.972	2.352	0.972	2.346
23	0.800	2.320	0.800	2.316
34	0.753	2.261	0.752	2.257
35	0.561	1.825	0.560	1.825
36	0.616	2.458	0.616	2.458
24|3	0.803	5.343	0.796	5.372
25|3	0.674	9.103	0.675	9.103
26|3	0.790	8.643	0.795	8.640
13|4	-0.280	10.204	-0.288	10.194
12|34	-0.271	16.988	-0.269	17.324
45|23	0.598	8.637	0.597	8.625
56|23	0.582	16.324	0.581	16.323
15|234	0.004	24.877	0.004	25.947
46|235	-0.333	27.761	-0.332	27.760
16|2345	-0.319	6398385	-0.318	6398370
MLE	1957.319		1957.438	

Table 3 lists the result of pair-copula's parameters. We got the initial value and the final value during the whole estimation, and we would use the final value together with the parameters of marginal distribution to construct the joint probability distribution of the whole market.

1.3. Monte Carlo simulation result

Here we set change rate7% as the trigger of extreme event and utilize Monte Carlo method to simulate the distribution. According to the result, the probability of downward change rate over 7% in the next day is less than 5%, meanwhile, the probability of downward change rate over 8% is less than 1%. And we could see these probabilities as the small probability event which means there is little chance of breaking out financial crisis or fierce violation of stock market.

Conclusion

This paper presents an analysis of extreme event on stock market with the PCC copula based multivariate models with GARCH (p, q) margins.

The results show that, pair-copula could well describe the dependence structure of different risk factors and during the decomposing process, we could adjust the pair-copula function used in different tree so as to find the most appropriate structure, this is also the most important reason we choose pair-copula model to value the risk factors. PPC model has wider application beyond our article, such as insurance portfolio investment and futures market. In this context, we focus on the application of forecasting the probability of extreme event occurrence. PPC model provides us a more flexible way to measure the composite risk a market faces; moreover, it well presents the dependence structure of the risk factors, which is extremely practical because the composite index is too composite to observe the interaction of different factors.

Acknowledgements

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