The application of the theory of real estate investment option game in China

LIU Yachen
School of Management
Shenyang Jianzhu University
Shenyan China
lyc@sjzu.edu.cn

Ma Jian
School of Management
Shenyang Jianzhu University
Shenyang China
2430418194@qq.com

Niu Siqi
School of Management
Shenyang Jianzhu University
Shenyang China
nsq0920@126.com

***Abstract*—Real estate developers is the real estate investment main body,however facing the high risk of the real estate market,the investment of science and rationality has become the key factors for the success of business investment.The investment process of real estate item contains a real option,but the traditional evaluation method has a serious defect.This paper uses the options game theory,to the real estate investment made scientific analysis,guideline developers to rational investment,the real estate market healthy development and make due contributions to.**

Keywords-real estate;investment;dual strike option; preemption

I.THE IMPORTANCE OF THE BEACHHEAD GAME MODEL FOR REAL ESTATE INVESTMENT OPTIONS

With the rapid development of China's real estate industry, real estate investment option is especially important. However, the real estate investment options have characteristics of high-yield and high-risk, so the development of scientific investment decision-making is very important. Real estate investment option contains a real option. The fierce competition between projects solely to the net present value method represented by the traditional investment appraisal methods there is a huge defect. It ignores the irreversibility of the investment of the project, rigid investment mechanism. However beachhead game for real estate investment options can play a scientific and rational guiding role. It greatly improved the accuracy of the investment options and reduced the risk of investment. Optimal execution strategy using a beachhead game theory can be formulated as an investment option.This paper established a beachhead game model and combined with the actual case. This article describes the significance of the beach game in the real estate investment options.

II.REAL ESTATE INVESTMENT OPTIONS BEACHHEAD GAME MODEL

Investment in real estate projects with a lot of risk and uncertainty, accurate and scientific investment is particularly important. This paper discuss double option game of oligarchies monopolization under market conditions. In this paper, the competition between the two development projects with a strong alternative make a reasonable analysis and decision-making. Followers exercise strategy and an analysis of the case for the next chapter under beachhead game case the option.The paper analysis bullish investment options held by investors. Based on past experience, the first requirement is proposed value function of the follower. (t, t + dt) for the time interval, r for the risk-free rate,assuming such a portfolio, an option, the number of units of output short m capital of output short number of units n.. According to Ito's lemma can be obtained:

$$d(V - mR - nC) = \left(\frac{\partial V}{\partial R} - m\right) dR + \left(\frac{\partial V}{\partial C} - n\right) dC + \frac{1}{2}\left(\sigma^2 R^2 \frac{\partial^2 V}{\partial R^2} + 2\rho\sigma\alpha RC \frac{\partial^2 V}{\partial R \partial C} + \alpha^2 C^2 \frac{\partial^2 V}{\partial C^2}\right) dt \quad (2\text{-}1)$$

Due to the risk-free portfolio,Therefore equation (2-1) in the right of the dR and dC is a random variable, the coefficient is zero, i.e., $m = \frac{\partial V}{\partial R}$, $n =. \frac{\partial V}{\partial C}$. It can be concluded that the (t, t + dt), Capital gains:

$$\frac{1}{2}\left(\sigma^2 R^2 \frac{\partial^2 V}{\partial R^2} + 2\rho\sigma\alpha rC \frac{\partial^2 V}{\partial R \partial C} + \alpha^2 C^2 \frac{\partial^2 V}{\partial C^2}\right) dt \quad (2\text{-}2)$$

For short, to make the payment of the convenience yield of the corresponding capital and output is (m R + n C) dt. Have to make a payment equal to the risk-free return r (V-MR-nC) dt,.Finishing:

$$\frac{1}{2}\left(\sigma^2 R^2 \frac{\partial^2 V}{\partial R^2} + 2\rho\sigma\alpha rC \frac{\partial^2 V}{\partial R \partial C} + \alpha^2 C^2 \frac{\partial^2 V}{\partial C^2}\right) dt + (r - \mu) R \frac{\partial V}{\partial R} + (r - \omega) C \frac{\partial V}{\partial C} - rV = 0 \quad (2\text{-}3)$$

Equation (2-3) R and C as two independent variables,make partial differential equations,give followers` rational option value function, is applicable in the region of optimal options held in the space (R, C) without performing, meet the two boundary conditions. First, the value matching condition that the need for immediate implementation of the optimal regional boundary, the value reached the value of the investments of the followers. In this boundary, the option value is equal to the difference between the present value of the cash flow of future income and the construction cost:

$$V(R,C)=\frac{R}{\mu}-C \qquad (2\text{-}4)$$

(2-4) on both sides of the value function requirements must be tangent to the boundary, which generates two smooth pasting conditions:

$$\frac{\partial V}{\partial R}=\frac{1}{\mu} \qquad (2\text{-}5)$$

$$\frac{\partial V}{\partial C}=-1 \qquad (2\text{-}6)$$

If only the construction costs and the value of the project doubled,the optimal decision depends only on p $=\frac{R}{C}$. Then, in the (R, C) within the option value will be the number of a homogeneous type. (2-4) can be written as:

$$V(R,C)=Cf\left(\frac{R}{C}\right)=Cf(p) \qquad (2\text{-}7)$$

Where f is an unknown function, differential (2-7) into (2-3) finishing:

$$\frac{1}{2}p^2 f''(p)(\alpha^2+\sigma^2-2\alpha\sigma\rho)+pf'(p)(\omega-\mu)+f(p)\omega=0 \qquad (2\text{-}8)$$

The formula value matching conditions: $f(p)=\frac{p}{\mu}-1$ (2-9)

The two smooth paste conditions for: $f'(p)=\frac{1}{\mu}$ (2-10)

$$f(p)-pf'(p)=-1 \qquad (2\text{-}11)$$

According(2-9) and (2-10) (2-8) to solve available:

$$f(p)=Ip^{\beta_1}+Jp^{\beta_2} \qquad (2\text{-}12)$$

Of which: $\beta_{1,2}=\frac{1}{2}-\frac{(\mu-\omega)}{\gamma^2}\pm\sqrt{\left(\frac{\omega-\mu}{\gamma^2}-\frac{1}{2}\right)^2+\frac{2\omega}{\gamma^2}}$ (2-13)

$$\gamma^2=\alpha^2+\sigma^2-2\alpha\sigma\rho \qquad (2\text{-}14)$$

Future earnings is zero for p = $\frac{R}{C}$, p also approaches zero. That is, (2-12) and the J = 0 according to the boundary conditions, was calculated:

$$\frac{C}{\beta_1-1}\left(\frac{p}{p_F}\right)^{\beta_1} \qquad p<p_F$$

$$\frac{R}{r-\mu}-C \qquad p\geq p_F$$

Calculate the critical value: $p_F=\frac{(r-\mu)\beta_1}{\beta_1-1}$

From the above analysis ,it can be concluded: When p reaches a critical value, the follower is most suitable for project development.

That is, in the non-vied for the case of the game, when p reaches a critical value, the follower is most suitable for project development.

III. APPLICATION OF BEACHHEAD GAME MODEL IN THE REAL ESTATE INVESTMENT OPTIONS

The real estate project investment is very risky investment,the process time are faced with difficulties. Therefore real estate options theory is applied in many real estate investment.This paper taked two real estate developers in Tianjin for example. Using beachhead game analysised application for the investment in real estate options.

A.Case background

The case is in to Tianjin A, B two real estate developers. Company A is a project covering the whole country company operating in the real estate business longer, and there is certain influence. Company B enters the real estate industry a short time, the regional development strategy has just started. But with strong financial strength, it has a wealth of experience in the high-end market. A company in Tianjin in 2006, won the bid to develop a high-quality housing land development, and started in the same year. At the same time, B company won the bid a high-end real estate land near A company land. Since Company A has developed two projects. For Company B, the most important thing is to identify the timing of the beginning of development projects in order to get the project to maximize the output. Game commence in the second phase of the project of the A and Company B of the new projects.

B.The option design parameters estimated

Company B of the project data is confidential, so some of the data in the case is derived according to the same strength of the corporation's projections.And individual data is conducted technical processing.

Ⅰ)Risk-free rate r

The project is risk-free interest rate r. By convention, take a five-year bond interest rate of 5%.

Ⅱ)Future earnings volatility μ

For Company B having a more accurate value of the project, μ take the Shanghai Stock Exchange trading stock price volatility of the real estate industry listed companies.

So: $\pi_i=\ln\left(\frac{S_i}{S_i-1}\right)$

Where π_i is the i-th time interval continuous compounding income; S_i is stock price at the end of the i-th time interval; i = 1, 2, • • • • • • n (n is the number of observations).

General estimate of the standard deviation S values are:

$$S=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\pi_i-\bar{\pi})^2}=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\pi_i^2-\frac{1}{n(n-1)}\left(\sum_{i=1}^{n}\pi_i\right)^2}$$

of which, $\overline{\pi}$ is the average value of π_i.

According to the geometric Brownian motion concluded: π_i standard differential is $\mu\sqrt{t}$ .Therefore, the variable S is $\mu\sqrt{t}$ valuation. μ itself can be estimated as S^*. Can be obtained: $S^* = \frac{S}{\sqrt{t}}$

of which, t is expressed in units of length of the interval in years.

According to the relevant data of the Shanghai Stock Exchange, after calculating and appropriate processing, the derived estimates of future income fluctuations in the value of μ take 20.68%.

III)The volatility of the investment cost ω

Company B foreign investment cost volatility is confidential. We can only extrapolate according to the listed shares of the construction company signed a construction contract with Company B. Take 22% as the average volatility of the construction industry sigma,0.856 as the cost of real estate investment volatility with the volatility of the market value of the construction sector correlation coefficient. So the fluctuations of Company B in the cost of investment rate is 21.5%. The correlation coefficient between the investment cost volatility and future earnings volatility γ = 0.017.

IV)Expected investment costs and future income

According to the data disclosed in the financial sector and the forecast of the securities company, the specific circumstances of the project investment costs and future earnings of Company B reflected in the table in the next 3 to 5 years.

2007 as the starting year, the predictive value of the investment costs and future earnings for the year are shown in Table 5-1 and 5-2:

Table 5-1 Investment costs the annual predictive value (unit: thousand)

Years	2007	2008	2009
Investment cost	143872	210819	89076

Table 5-2 Sales revenue the annual predictive value (unit: thousand)

Years	2008	2009	2010	2011
Sales revenue	179672	306783	92265	31273

At a discount rate equal to the risk-free rate, take r = 5%. Investment cost C_{2006} = 427113.1 thousand, R_{2006} = 556528.4 thousand. Can be calculated:

$$p_{2006} = \frac{R_{2006}}{C_{2006}} = 1.293$$

2008 as the starting year, the annual investment costs and future earnings predictive value as shown in Table 5-3 and 5-4:

Table 5-3 Investment costs the annual predictive value (unit: thousand)

Years	2008	2009	2010
Investment cost	184521	259863	112471

Table 5-4 Sales revenue the annual predictive value (unit: thousand)

Years	2009	2010	2011	2012
Sales revenue	246823	437576	136143	43195

Same as above,take r = 5%. Investment cost C_{2007} =436824.7 thousand,R_{2007} =786726.2 thousand. Can be calculated:

$$p_{2007} = \frac{R_{2007}}{C_{2007}} = 1.593$$

C.Conclusion of the study

According to data calculated of the previous section, take them into Company B option value function can be obtained P_F =1.567. It can be seen that $P_{2006} < P_F$. It is descripted that Company B is not suitable for immediate investment, it should continue waiting for an opportunity. $P_{2007} > P_F$,this shows that it's the most appropriate investment that Company B arrive before that time. Company B is successful land in early 2007. It did not blindly develop projects in order to catch up with the progress of the Company A,while spotting the opportunity to select the best development period to gain maximum benefit.

IV.CONCLUSION

Real estate investment options face a lot of uncertainty. More and more investors introduced the concept of the financial-market option to the field of real options investment.Therefore it results real options. However, most real options different from financial options. Real option is often not an investment unique,while there is two or more competitors in the market. When making Investment decisions,we must take competitors factors into account that is from the perspective of the game to think about the implementation strategy of the options,we should find the best investment timing and scale of investment. Beachhead game theory is well positioned to meet the needs of investors. From pre-investment until the end of the investment,beachhead game theory points clearly the direction for investors, provides scientific and effective theoretical basis, greatly improves the accuracy of the real estate investment options.

REFERENCES

[1] Chu YongQiang, Sing Tien Foo·OptimalTiming ofReal Estate Investment under Asymmetric Duopoly [J]·Singapore - HongKongRealEstate Research Symposium, 2005, (7): 14-15.

[2]Xu Dajiang. Investment Decision multi-objective linear programming method [J]. Systems Engineering Theory and Practice 2007,(12):46～52

[3]Wang Chunfeng,Tu Xinsu . The research of portfolio under the significance of the utility function [J].China Management Science,2002,10(2):56～59

[4]Zhang Zengfang,Zhang Longping. Run and Her Change: the city economy mechanisms New Theory[M].Southeast University Press ,2006.1

[5]Li Hua. A new measure of portfolio risk [J].Mathematics in Practice and Theory,2006,33(6):16～21

[6]Li Zhongfei.Fuzzy Optimization Based on VaR indicators portfolio [J].Mathematics in Practice and Theory,2008,33(3):35～40

[7] Martha Amram,Kulatilaka Panarin. Real options: Uncertain Environment Strategic Investment Management[M].Mechanical Industry Press ,2009.5

[8]Xie Fanrong.Transaction costs Securities Investment and an algorithm for optimal investment program [J].Mathematics in Practice and Theory,2009,33(1):30～33

[9] W illiams Joseph T·Equilibrium andOptions on RealAssets[J]·Review ofFinancialStudies, 2004, 6 (4): 826-850.

[10] Grenadier Steven R·The Strategic Exercise ofOptions: Develop-ment Cascades and Overbuilding in Real Estate Markets[J]·Journal ofFinance, 2006, 51 (5): 1653-1679.