$$\sum_{i=0}^{p-1} f_i(i+1) \le \tau + 1 < \sum_{i=0}^{p} f_i(i+1). \tag{17}$$

Let $g_p = \lfloor (\tau+1-\sum_{i=0}^{p-1} f_i(i+1))/(p+1) \rfloor$ and $i_0 = \tau+1-\sum_{i=0}^{p-1} f_i(i+1) - g_p(p+1)$. For $0 \le i < p$, let

$$g_i = \begin{cases} f_i + 1, & \text{if } i = i_0 - 1, \\ f_i, & \text{otherwise}. \end{cases} \tag{18}$$

Then $h_0, g_0, h_1, g_1, ..., h_p, g_p$ are positive integers satisfying (11) and (12). From the left inequality of (17), we see

$$b^{-1}(\tau+1)^{1-a} \ge \sum_{i=0}^{p-1} (i+1)^{1-2a} \ge \int_0^p x^{1-2a} dx = p^{2-2a}/(2-2a),$$

and thus

$$p \le (\tau+1)^{1/2}((2-2a)/b)^{1/(2-2a)}. \tag{19}$$

Hence,

$$\begin{aligned} &\sum_{i=0}^{p-1} \frac{(h_{i+1}-h_i)^2}{2g_i h_i + h_{i+1} - h_i} = \sum_{i=0}^{p-1} \frac{1}{2g_i(i+1)+1} \\ &\le \frac{1}{2b(\tau+1)^a} \sum_{i=0}^{p-1} (i+1)^{2a-1} \le \frac{1}{2b(\tau+1)^a}(1 + \int_1^p x^{2a-1} dx) \\ &= \frac{1}{2b(\tau+1)^a}(1 + \frac{p^{2a-1}}{2a}) \le \frac{p^{2a}}{4ab(\tau+1)^a} \le \frac{((2-2a)b^{-1})^{a/1-a}}{4ab}. \end{aligned}$$

If we choose b further as

$$b = 3^{1-a}(20a)^{a-1}(2-2a)^a, \tag{20}$$

Then we have

$$\sum_{i=0}^{p-1} \frac{(h_{i+1}-h_i)^2}{2g_i h_i + h_{i+1} - h_i} \le 5/3,$$

and thus according to Theorem 1, the Chase-like algorithm $C(U_J)$ achieves BD decoding.

From $g_p \le f_p - 1$, (19) and (20), we have

$$\begin{aligned} &\sum_{i=0}^{p} g_i \le p + 1 + \sum_{i=0}^{p} b(\tau+1)^a (i+1)^{-2a} \\ &\le p + 1 + b(\tau+1)^a \left(1 + (p+1)^{-2a} + \int_1^p x^{-2a} dx\right) \\ &= p + 1 + b(\tau+1)^a \left(1 + (p+1)^{-2a} + \frac{p^{1-2a}-1}{1-2a}\right) \\ &\le 1 + 2b(\tau+1)^a \\ &\quad + \left((2-a)^{\frac{1}{2-2a}} b^{\frac{-1}{2-2a}} + \frac{1}{1-2a}(2-2a)^{\frac{1-2a}{2-2a}} b^{\frac{1}{2-2a}}\right)(\tau+1)^{1/2} \\ &= 1 + 2a(\tau+1)^a + \sqrt{\frac{40a(1-a)}{3}}\left(1 + \frac{3}{20a(1-2a)}\right)(\tau+1)^{1/2}. \end{aligned}$$

Hence, we have proved the following theorem.

Theorem 2 When the Hamming distance d of the code approaches infinity, the Chase-like algorithms can achieve BD decoding with $(\psi + o(1))d^{1/2}$ input vectors, where

$$\psi = \min_{0<a<1/2}\left(1 + \frac{3}{20a(1-2a)}\right)\sqrt{\frac{20a(1-a)}{3}} \approx 2.218.$$

V . Conclusions

In literature, there are many works to estimate the smallest size, denoted by $\Delta(d)$ for binary block code of Hamming distance d , of input vector sets of Chase-like algorithms which achieve BD decoding. Unlike most of these works, we deal with in this paper some Chase-like algorithms with an additional input vector whose nonzero entries are not confined in the most unreliable positions. With a similar method used in [7], we show that such a Chase-like algorithm has also a unique minimal vector in its unchecked region and then improve the best known upper bound on $\Delta(d)$ to: $\Delta(d) \le (\psi + o(1))d^{1/2}$, where $\psi \approx 2.218$.

References

[1] D. Chase, "A class of algorithms for decoding block codes with channel measurement information," IEEE Trans. Inform. Theory, vol.IT-18, no.1, pp.170--182, Jan. 1972.

[2] G. Aricò and J. H. Weber, "Limited-trial Chase decoding," IEEE Trans. Inform. Theory, vol.49, no.11, pp.2972--2975, Nov. 2003.

[3] J. H. Weber,"Low-complexity Chase-like bounded-distance decoding algorithms,"in IEEE GLOBECOM, San Francisco, CA, Dec. 1-5, 2003, pp.1608--1612.

[4] J. H. Weber and M. P. C. Fossorier, "Limited-trial Chase-like algorithms achieving bounded-distance decoding," IEEE Trans. Inform. Theory, vol.50, no.12, pp.3318--3323, Dec. 2004.

[5] Y. Tang and X. Huang, "A note on limited-trial Chase-like algorithms achieving bounded-distance decoding," IEEE Trans. Inform. Theory, vol.55, no.3, pp.1047--1050, Mar. 2009.

[6] Y. Tang, X. Huang and T.Yan, "On the number of search centers of Chase-like bounded-distance decoding," ISITA2008, Auckland, New Zealand, pp.1073-1076, Dec. 2008

[7] T. Yan, Y. Tang and M. Chen, "Chase-3-like algorithms achieving bounded-distance decoding," Proceedings of IWSDA'09, Fukuoka, Japan, pp.80-83, Sept. 2009.

[8] M. P. C. Fossorier and S. Lin, "A unified method for evaluating the error-correction radius of reliability-based soft-decision algorithms for linear block codes," IEEE Trans. Inform. Theory, vol.44, no.2, pp.691--700, Mar. 1998.

[9] M. P. C. Fossorier and S. Lin, "Chase-type and GMD coset decodings," IEEE Trans. Commun., vol.48, pp.345--350, Mar.

[10] Y. Tang, T. Fujiwara and T. Kasami, "Asymptotic optimality of the GMD and Chase decoding algorithms," IEEE Trans. Inform. Theory, vol.48, no.8, pp.2401--2405, Aug. 2002.

[11] Y. Tang, S. Ling and F. W. Fu, "On the reliability-order-based decoding algorithms for binary linear block codes," IEEE Trans. Inform. Theory, vol.52, no.1, Jan. 2006.