"JACOBI DOCET"
An issue to honour Jacobi's bicentennial
Foreword

2004 and 2005 are bicentennials for two famous mathematicians, Carl Gustav Jacob Jacobi (born December 10, 1804) and William Rowan Hamilton (born August 4, 1805). A celebration of their individual contributions to Mathematics is particularly apt because their names continue to be cited very frequently in the mathematical literature (in 2004, 502 and 313 hits in MathSciNet, respectively). Importantly the extraordinary work of these two great mathematicians, especially Jacobi, has been extremely valuable to and enabled past and recent advances in research in a variety of areas. While a celebration of Hamilton's bicentennial has been already organized thanks to The Hamilton Mathematics Institute at Trinity College in Dublin, Ireland (see www.hamilton.tcd.ie), nothing has been done so far to honour Jacobi's bicentennial, except for a special session which was organized by MC Nucci and P Winternitz at the 111th Annual meeting of the American Mathematical Society in January 5-8, 2005 (see www.ams.org/amsmtgs/2091_program_ss37.html#title). Some of the papers presented there are published in the present issue.

Jacobi worked in many mathematical areas, which are still actively researched, from special functions to number theory, from calculus of variations to astronomy, from differential equations to analytical mechanics, until his premature death on February 18, 1851. Also he was an excellent teacher. In his biography of Jacobi (published in 1904 on the occasion of the first Jacobi centennial) Koenigsberger wrote that Jacobi was always able to captivate the interest of his audience not only because he engaged his audience's attention by the fascination of his ideas, conveyed his great amount of knowledge and revealed new view points but also because of his happy habit of connecting the historical development of problems to their manifold solutions, problems which were taken from the most apparently disparate disciplines of mathematical science and critically enlightened. In the words of Dirichelet Jacobi's lectures are singled out because of their high level of clarity, not at all the same type of clearness that often accompanies penury of ideas. In the words of his student, Borchardt, Jacobi's characteristic was a combination of sharp acumen and sense of humor. In spite of his brother's warning that nobody was a freer intellectual than him and only after freedom was invented he became not free, Jacobi liked to speak freely without thinking of the consequence and, when he was forced to bow to the authority of the King, at the same time he wanted to show that "without any arrogance I am still standing straight".

This issue is a very humble homage to this great man and scientist. It focuses on differential equations and their applications to Physics and other sciences in the wake of Jacobi's accomplishments. In the introduction to his lectures on dynamics Jacobi stated that "any progress in the theory of differential equations must also bring about a progress in mechanics" and later in his lectures on analytical mechanics he warned his students by saying that "wherever Mathematics is mixed up with anything, which is outside its field, you will find attempts to demonstrate these merely propositions a priori, and it will be your task to find out the false deduction in each case". What Jacobi docet is of such depth and modernity that his advice remains valid in the year 2005 which quite coincidentally has been declared International Year of Physics by the General Assembly of the United Nations Organization.

This issue contains state of the art articles on the following topics: separation of variables for the Hamilton-Jacobi and Schrödinger equations in classical and quantum mechanics, respectively, with special attention to Lie symmetries and the connection to special functions, the last topic initiated by Jacobi in his search for geodesics on an ellipsoid; Jacobi's last multiplier and its applications to dynamical systems and classical and quantum mechanics.

Rosati and Nucci, after some historical considerations about elliptic integrals, apply Lie group analysis to a third-order ordinary differential equation which in 1829 was derived by Jacobi in order to link two different moduli of an elliptic integral. Apparently any remembrance of this equation has been lost in the scientific literature since the publication of Cayley's treatise on elliptic functions in 1895.

Chouikha makes use of a trigonometric expansion of Jacobi theta functions to establish a differential system derived from the heat equation. Similar expansions for Jacobi's elliptic and zeta functions are examined.

Dragovic presents an interesting connection between separability of partial differential equations in the sense of Hamilton-Jacobi and the theory of hypergeometric functions as a consequence of separable perturbations of integrable billiards systems and the Jacobi problem for geodesics on an ellipsoid.

Kalnins, Thomova and Winternitz examine the separation of variables for the Hamilton-Jacobi equation and the Schrödinger equation in complex and real four-dimensional flat spaces. Abelian subgroups are used to generate coordinate systems with ignorable coordinates. A coherent picture is presented in terms of graphic chains.

Kalnins, Kress and Miller discuss Jacobi's method to integrate the Hamilton-Jacobi equation, separable elliptic coordinates and the integrability of the Schrödinger operator with particular reference to superintegrable systems which have the attractive property of separability in several coordinate systems. They present a complete listing of all real and complex two-dimensional spaces exhibiting the general structure of superintegrable systems.

Cariñena, Rañada, Santander and Sanz-Gil discuss six coordinate systems allowing separation of variables in the Hamilton-Jacobi equation for Hamiltonians of 'natural' form from a characterisation of types of separable potentials in curved spaces of variable metric. As examples they show that the simple harmonic oscillator and the Kepler Problem are superintegrable in all the spaces considered. A strong argument is presented for going from the general structures of curved space to particular instances to avoid the multiple derivation of the same results in different spaces.

Persson and Rauch-Wojciechowski provide a complete list of superintegrable triangular newtonian systems in two dimensions through explicit construction based upon the posited existence of three autonomous integrals of the motion.

Naicker, Andriopoulos and Leach examine a development of the Hamilton-Jacobi equation, the Hamilton-Jacobi-Bellman equation, which has found application in the modern field of the Mathematics of Finance, from the approach of Lie point symmetries and provide solutions of several classes of mean-variance equations subject to a terminal condition.

Nucci, after reviewing the historical development and manifold applications of the Jacobi last multiplier, introduces a new method which exploits the Jacobi last multiplier to the purpose of finding Lie symmetries of first-order systems. Several illustrative examples are given.

Finally Nucci and Leach exploit Jacobi's last multiplier and some recent developments in the theory of complete symmetry groups to determine the complete symmetry group of the Ermakov-Pinney equation for which the difficulty of the task is quite incommensurate with the simplicity of the equation!

All the papers have been carefully reviewed by anonymous referees. We thank all of them for their efforts. Last, but not least, we are indebted to Norbert Euler for his help and support.

M C Nucci
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P G L Leach
Durban, April 8, 2005