New Multi Level Spreading Codes for DS CDMA Communication

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Abstract

Spreading codes are used to distinguish users and spread the signal to occupy much wider band width than the minimum required band width. Spreading codes are also called as user codes. Walsh, Gold and Kasami codes are the popularly used Binary (2-level) user codes. This paper proposes new multi level spreading codes constructed using ternary and quaternary Gray and Inverse Gray codes for multiuser Direct Sequence Code Division Multiple Access systems. Multi level spreading codes discussed in this paper are non zero mean, varying power codes. An attempt is made to analyze these multi level user codes through correlation properties. The methodology explained in this paper allows to generate 3-level user codes of length-6 and 4-level user codes of length-8. It also allows generation of codes whose lengths are even multiples of the codes of length 6 and length 8. The auto and cross correlation properties and bit error rate performance of these codes and their comparison with those of Walsh and Gold codes are presented in this paper.

Keywords: Generalized Gray Codes, Inverse Gray Codes, Multi level Spreading Codes, Auto and Cross Correlation metrics, DS CDMA.

1. Introduction

The field of spread spectrum communications covers the art of secure digital communication that is now being exploited for both civilian and military purposes. The Code Division Multiple Access (CDMA) technique is widely used in the existing wireless communication systems such as W-CDMA and CDMA2000[1, 2]. In multiuser CDMA systems, spreading that are used to distinguish users and spread the signal, play an important role. Spreading codes are also called as user codes. Binary valued Walsh and Gold codes are widely used as spreading codes in wireless CDMA systems. Walsh codes are perfectly orthogonal codes and are ideal for synchronous CDMA communications[1]. Availability of these code sets is limited and these sequences are not useful for asynchronous communication with random delays. To reduce Multiple Access Interference (MAI) in asynchronous CDMA systems, where relative delays between transmitter code sequences are arbitrary, it is ideal to have spreading sequences having both impulsive auto-correlation and zero cross-correlation. However, mathematically, it is not possible to design such ideal spreading sequences with finite lengths for all possible delays. Hence, spreading sequences with high auto and low cross-correlation values such as m-sequences, Gold sequences and Kasami sequences are used in asynchronous reverse links or in uplink communications. Different types of spreading sequences exhibit different properties, such as code length, code set size and auto and cross correlation values. Most of the available code sets yield degraded performance in multiuser conditions, especially when the system is heavily loaded and the channel is dispersive. Binary code sets have the chip levels (+1, -1) and generate constant envelope or fixed power modulating signal, which enables efficient use of the available RF power in CDMA. While this is true in the case of single user scenario, in a multiuser case, this is not true when multiple binary signals are added together. Amplifiers usually operate as a linear device under small signal conditions and become more non-linear and distorted with increase in input drive level. Increasing the input drive signal in turn increases the efficiency of the RF amplifier and thus increases the total transmitted power. Current research on RF amplifier design is aimed at increasing the linear range of RF amplifiers with lower distortion levels and higher efficiency. With such technological advances, implementation of varying power codes is becoming feasible for wireless and radio communications. The limitations of binary code sets discussed above lead to the design of new multi level (varying power) user codes. Short length codes are more useful for applications like wireless sensor networks or mesh networks where communication nodes are placed at close proximity and system complexity is of great concern. With increasing levels of hardware implementation, multi level sequences make the design method more flexible and also further improve the system performance. In Ref. 3 a method for the construction of multi level Hadamard matrices is proposed. Usage of multi level integer valued orthogonal codes for CDMA communications is initially proposed in Ref. 4, wherein an exhaustive search scheme to construct N-length multi level user codes is discussed. In this method, the multi level representations of integers from 1to 2^{N-1} are checked for orthogonality to obtain a code set. This method is proved to be tedious as the length of the code increases. There are few works addressing the design methods for multiple level user codes and their application to MC-CDMA systems [5, 6].

Gray code, a unit distance code of n-bit, is defined as a list of all 2^n bit strings such that successive code words differ in only one bit position [7, 8]. An 'n' bit Inverse Gray Code , is defined exactly opposite to Gray code, it is a list of all 2^n bit strings of length 'n' each, such that successive code words differ in (n-1) bit positions [9, 10]. By concatenating binary Gray and Inverse Gray codes, binary user codes are constructed. The procedure for constructing the codes and the performance of these codes has been reported earlier in Ref. 11. A similar technique for the construction of multilevel user codes using ternary and quaternary Gray and Inverse Gray codes is proposed in the present work. Rest of the paper is organized in the following manner: Section II discusses the proposed technique for the construction of 3-level and 4-level user code sets from n-git ternary and quaternary Gray and Inverse Gray codes respectively. In Section III, the auto and cross correlation properties of the proposed user codes are analyzed. And finally, Section-IV gives the conclusions.

2. Multi level Spreading Codes

In multi user CDMA systems, user codes or spreading sequences are used to distinguish users and spread signals. In multi level user codes, discrete amplitude levels, similar to pulse amplitude modulation (PAM) levels, are used as chip signals for spread spectrum codes. Chip amplitudes are chosen such that they have zero mean.

In order to minimize the average transmitted energy and to obtain M signal amplitude levels symmetric about zero and equally spaced, the following formula is used.

$$A_m = (2m-1-M),\ m=1, 2, \ldots, M.$$

The algorithm used to obtain generalized Gray codes[8] is as follows:

(n, r) Gray Code (radix 'r')

In this algorithm, as a generalization, a digit is referred as a git (generalized unit). Let an n-git radix 'r' Gray code be needed. Let $(P_1, P_2 \ldots\ldots P_n)$ be a permutation of (1, 2, 3, …..n). The r^n integers $(0,1,2,\ldots\ldots(r^n-1))$ can be arranged in the following doubly indexed indicial sets.

$$Q_{j,k} = r^j\{k, k+r, \ldots\ldots\ldots k+mr\}$$

$$j = 0, 1, 2, \ldots\ldots n$$

$$k = 1, 2, \ldots\ldots r-1$$

Where 'm' is the largest positive integer such that $m \leq r^{n-j-1}-k/r$.

Over the integers (0, 1, ….. ,(r-1), 0) a new succession order $(0, s_1, s_2, \ldots s_{r-1}, 0)$ is defined where $s_1, s_2, \ldots, s_{r-1}$ is a permutation of (1, 2, 3, ……(r-1)). Then, starting with the row of all zeros as a zeroeth row, the i^{th} row is obtained from the $(i-1)^{th}$ row by replacing the P_j^{th} git by its successor, if it is in $Q_{j-1,k}$.

2.1. *3-level, 6-length and 4-level, 8-length Spreading Codes*

The following are the steps required for the construction of multi level, 2n-length user codes:

Step 1: Generate n-git Gray code using the algorithm discussed in Ref. 3 with any permutation.

Step 2: Using the same permutation generate n-git Inverse Gray code as in Ref. 10.

Step 3: Append Inverse Gray Code to the Gray Code to result in 2n-length GIG code (Gray Inverse Gray).

Step 4: Each row of this 2n-length GIG code is mapped to the 'm' chip amplitude levels to construct multi level code set.

Binary user codes (2-level) are designed by mapping radix-2 elements {0,1} to chip amplitude levels {-1,1}. Similar procedure is adopted to construct multi level orthogonal codes. For example, for a 3-level coding radix-3 (ternary) coding elements {0,1,2} are mapped to chip amplitudes levels {-1,0,1} and for a 4-level coding, chip amplitudes levels {-3,-1,1,3}are obtained by mapping the coding elements of quaternary (radix-4) number system {0,1,2,3}.

Step 5: Select any one codeword as the first code word and check the orthogonality of remaining code words with the first code word to obtain m-level, 2n length user code set.

This five step procedure is repeated by selecting a different codeword as the first basis function to obtain another user code set. Table.1 gives the construction of 3-level, 6-length GIG code set using the permutation {3,2,1}.Weights of the coding elements in radix-3 for n-length code are $\{3^{n-1}, 3^{n-2},\ldots\ldots, 3^1, 3^0\}$ and similarly in radix -4, element weights are $\{4^{n-1}, 4^{n-2},\ldots\ldots, 4^1, 4^0\}$. For example, a 3-level, 6-length ternary code {1, 2, 0, 2, 1, 0} is equivalent to $\{3^5.1+3^4.2+3^3.0+3^2.2+3^1.1+3^0.0\}$ = 426 in decimal notation. Similarly, 4-level 8-length user code sets can be obtained using Quaternary GIG codes. Table. 2 displays one such 3-level, 6-length user code set and 4-level 8-length user code sets along with their decimal representations.

Considering the 6 & 8 –length code sets as basic sets, code sets of greater lengths which are even multiples of lengths 6 & 8 can be constructed recursively using the following relationship (M=6 or 8)

$$C_{2M} = \begin{bmatrix} C_M & C_M \\ C_M & \overline{C_M} \end{bmatrix}$$

Table 1. Construction of 3-level, 6-length GIG codes

3-git Ternary Gray Code	3-git Ternary Inverse Gray code	3-git Ternary GIG Code	{0,1,2} mapped to {-1,0,1}
0 0 0	0 0 0	0 0 0 0 0 0	-1 -1 -1 -1 -1 -1
1 0 0	0 1 1	1 0 0 0 1 1	0 -1 -1 -1 0 0
2 0 0	0 2 2	2 0 0 0 2 2	1 -1 -1 -1 1 1
2 1 0	1 2 0	2 1 0 1 2 0	1 0 -1 0 1 -1
0 1 0	1 0 1	0 1 0 1 0 1	-1 0 -1 0 -1 0
1 1 0	1 1 2	1 1 0 1 1 2	0 0 -1 0 0 1
1 2 0	2 1 0	1 2 0 2 1 0	0 1 -1 1 0 -1
2 2 0	2 2 1	2 2 0 2 2 1	1 1 -1 1 1 0
0 2 0	2 0 2	0 2 0 2 0 2	-1 1 -1 1 -1 1
0 2 1	0 1 2	0 2 1 0 1 2	-1 1 0 -1 0 1
1 2 1	0 2 0	1 2 1 0 2 0	0 1 0 -1 1 -1
2 2 1	0 0 1	2 2 1 0 0 1	1 1 0 -1 -1 0
2 0 1	1 0 2	2 0 1 1 0 2	1 -1 0 0 -1 1
0 0 1	1 1 0	0 0 1 1 1 0	-1 -1 0 0 0 -1
1 0 1	1 2 1	1 0 1 1 2 1	0 -1 0 0 1 0
1 1 1	2 2 2	1 1 1 2 2 2	0 0 0 1 1 1
2 1 1	2 0 0	2 1 1 2 0 0	1 0 0 1 -1 -1
0 1 1	2 1 1	0 1 1 2 1 1	-1 0 0 1 0 0
0 1 2	0 2 1	0 1 2 0 2 1	-1 0 1 -1 1 0
1 1 2	0 0 2	1 1 2 0 0 2	0 0 1 -1 -1 1
2 1 2	0 1 0	2 1 2 0 1 0	1 0 1 -1 0 -1
2 2 2	1 1 1	2 2 2 1 1 1	1 1 1 0 0 0
0 2 2	1 2 2	0 2 2 1 2 2	-1 1 1 0 1 1
1 2 2	1 0 0	1 2 2 1 0 0	0 1 1 0 -1 -1
1 0 2	2 0 1	1 0 2 2 0 1	0 -1 1 1 -1 0
2 0 2	2 1 2	2 0 2 2 1 2	1 -1 1 1 0 1
0 0 2	2 2 0	0 0 2 2 2 0	-1 -1 1 1 1 -1

Table 2. Proposed multi level orthogonal user code sets

	Ternary GIG code	Decimal equivalent	3-level, 6-length user codes Basis elements {-1,0,1} $Norm^2 = 4$
Set I	0 0 0 0 0 0	0	-1 -1 -1 -1 -1 -1
	2 0 0 0 2 2	494	1 -1 -1 -1 1 1
	1 2 0 2 1 0	426	0 1 -1 1 0 -1
	0 2 1 0 1 2	194	-1 1 0 -1 0 1
	2 1 2 0 1 0	624	1 0 1 -1 0 -1
	1 0 2 2 0 1	316	0 -1 1 1 -1 0
	Quaternary GIG code	Decimal equivalent	4-level, 8-length user codes Basis elements {-3,-1,1,3} $Norm^2 = 20$
Set I	0 0 0 0 0 0 0 0	0	-3 -3 -3 -3 -3 -3 -3 -3
	3 0 0 0 0 3 3 3	49215	3 -3 -3 -3 -3 3 3 3
	2 3 0 0 3 2 1 1	45285	1 3 -3 -3 3 1 -1 -1
	0 2 3 0 1 3 2 1	11385	-3 1 3 -3 -1 3 1 -1
	2 2 2 1 1 1 1 2	43350	1 1 1 -1 -1 -1 -1 1
	1 0 2 1 3 0 2 3	18891	-1 -3 1 -1 3 -3 1 3
	2 1 2 2 1 2 1 1	39525	1 -1 1 1 -1 1 -1 -1
	2 2 2 2 2 2 2 2	43690	1 1 1 1 1 1 1 1
	1 2 2 2 2 1 1 1	27285	-1 1 1 1 1 -1 -1 -1
	2 2 1 2 1 1 2 1	42585	1 1 -1 1 -1 -1 1 -1
	0 2 0 3 1 3 1 2	9078	-3 1 -3 3 -1 3 -1 1

3. Time, Frequency and Correlation properties

The performance of spreading codes is evaluated by various auto and cross correlation metrics. The ability of a DS CDMA receiver to detect the desired signal relies to a great extent on the auto correlation properties of the spreading codes and on the other hand multi-user interference rejection depends on cross correlation properties of the spreading sequences. In synchronous DS CDMA system the code sequence in the receiver is exactly same with that in the transmitter. Orthogonal codes are most suitable for synchronous communication. Two sequences are said to be Orthogonal when the Cross-correlation (inter-code correlation) between them is zero. Codes with high auto correlation and low Cross correlation are preferred in synchronous communication. Walsh Codes are perfectly orthogonal, fixed power, binary user codes. Walsh codes of length 64 are used to develop individual channels in IS-95. Gold codes are binary, nearly orthogonal and possess poor auto correlation properties. Hence are preferred in asynchronous communications where random delays are observed in the received signals. The code properties, especially the auto correlation and cross correlation characteristics have a great effect on the system performance, especially since many communication systems do not operate in perfect synchronized conditions. The study of synchronized systems provides a good sense of how well a system can work, and thus is still of great importance. A typical 3-level, 6-length and 4-level, 8-length code are displayed in Fig. 1. Magnitude and Phase functions of these codes are also displayed in Fig. 2(a) and (b) respectively. Auto and Cross correlation sequences between an arbitrary pair of codes are displayed in Fig. 3(a) and (b). The performance of different spreading sequences is evaluated by Mean Square Aperiodic Auto Correlation (MSAAC), Mean Square Aperiodic Cross Correlation (MSACC), and Figure Of Merit (FOM) [1, 5, 12-16]. If $x(k)$ and $x(k+m)$ represent the non-delayed and delayed versions of a code word then 'm' is the number of units by which a code word is delayed. 'N' is the length of a code word x.

Correlation metrics and merit factor are defined as follows:

The discrete aperiodic Correlation is defined as

$$C_{x,y}(m) = \frac{1}{N}\sum_{m=1-N}^{N-1} x(n)y(n+m) \qquad (1)$$

The Mean Square Aperiodic Auto Correlation (MSAAC) for a Code set containing M sequences is given by

$$R_{AC} = \frac{1}{M}\sum_{x=1}^{M}\sum_{m=1-N,\, m\neq 0}^{N-1}\left|C_{x,x}(m)\right|^2 \quad (2)$$

Mean Square Aperiodic Cross Correlation (MSACC)

$$R_{CC} = \frac{1}{M(M-1)}\sum_{x=1}^{M}\sum_{y=1,\, y\neq x}^{M}\sum_{m=1-N}^{N-1}\left|C_{x,y}(m)\right|^2 \quad (3)$$

Figure Of Merit (FOM)

$$FOM = \frac{C^2_{x,x}(0)}{\sum_{m\neq 0}\left|C_{x,x}(m)\right|^2} = \frac{N^2}{2.\sum_{m=1}^{N-1}\left|C_{x,x}(m)\right|^2} \quad (4)$$

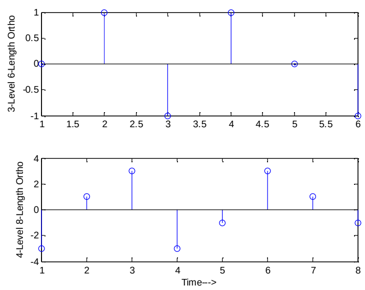
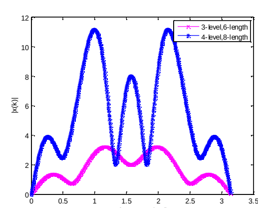
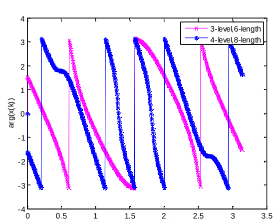


Fig. 1. Time domain representation of typical 3-level, 6-length and 4-level, 8-length proposed codes



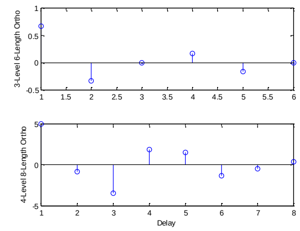
(a)



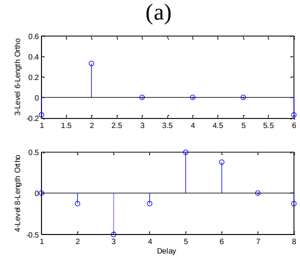
(b)

Fig. 2 (a) Magnitude and (b) Phase Functions of the proposed MLO codes

The auto and cross correlation properties of the proposed 3-level, 6-length and 4-level, 8-length user code sets and nearer length Walsh (length 8) and Gold (length 7) codes are displayed in Table 3.



(a)



(b)

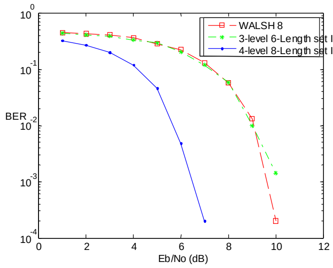
Fig. 3 (a) Auto correlation (b) Cross correlation sequences of typical ML codes

Codes with high auto correlation and low cross correlation are preferable in multi user communications. From Table 3 it can be observed that the proposed codes have correlation properties better than Gold and very close in their values to nearer length Walsh codes. Bit Error Rate (BER) performance is carried out for synchronous and asynchronous communication over

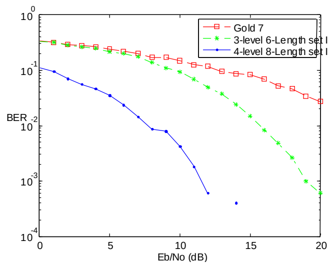
Table 3. Correlation measures of Walsh, Gold and proposed codes

Parameter	Proposed codes		W_8	G_7
	3-level, 6-length	4-level, 8-length		
MSAAC	1.2731	1.8641	2.375	0.9388
MSACC	0.7648	0.8572	0.6607	0.8503
FOM	0.7855	0.5365	0.4211	1.0652

Additive White Gaussian Noise (AWGN) channel for 2-user DS CDMA system. As Walsh codes are popular for synchronous communication, BER performance comparison of the proposed codes is done with the same. For asynchronous systems Gold codes are considered for comparison. Figure 4 (a) and (b) show the BER performance of the proposed codes over synchronous and asynchronous communications respectively. Form the figure it can be noted that the proposed codes are suitable for both synchronous and asynchronous systems.



(a)



(b)

Fig. 4 BER performance of (a) Walsh and proposed codes for synchronous communication (b) Gold and proposed codes for asynchronous communication over AWGN channel.

4. Conclusion

A new set of multi level user codes constructed using ternary and quaternary GIG codes for direct sequence CDMA communication is proposed in this work. The design methodology proposed allows to construct m-level, 2n- length user code sets. Using ternary (m=3) Gray and Inverse Gray codes 3-level, 6-length user code sets are obtained. Quaternary (m=4) GIG codes result in 4-level, 8-length code sets. In this paper construction of ML spreading codes is limited to 3 & 4 levels only. The higher length code sets (12,16,24,32 ..etc) can be constructed recursively using these basic sets. Spreading code sets of higher levels (m=5, 6,…) also can be obtained using the discussed construction procedure. Auto and Cross correlation properties reveal that the proposed codes are superior to Gold codes and are competent with Walsh codes. Performance analysis of the Walsh, Gold and the proposed codes for 2-user DS CDMA system over AWGN channel for synchronous and asynchronous communication is carried out. From the Bit Error Rate performance it can be observed that the proposed codes are suitable for both synchronous and asynchronous communication scenario.

References

1. Walsh J.L. A Closed set of Normal Orthogonal Functions, American Journal of Mathematics, 45 (1923).
2. Website for CDMA2000, WCDMA standards: http://www.3gpp.org
3. Trinh Q.K, Fan P, Gabidulin E.M. Multi level Hadamard matrices and Zero Correlation Zone Sequences, IEE Electronics Letters, 42 (13), (2006) p. 748.
4. Radharani Poluri, Design and Performance of CDMA Codes for Multiuser Communications, Ph.D Thesis, New Jersey's Science and Technology University, 2007.
5. Jingtao Zhang, David W. Matolak. Multiple level orthogonal codes and their application in MC-CDMA systems, Journal on Computer Communications 32, (2009), pp 492-500.
6. Xin Qiu, Xue-jun Sha, Xiao-yan Ning. A Multi-level Orthogonal Sequences Design and Implementation Method, *Proc. Of IEEE ICSP 2010* (2010), pp 1793-1796.
7. Gray F. Pulse Code Communication, US patent #2632058, 17th March (1953).
8. Sankar K. J., Pandharipande V.M., Moharir P.S. Generalized Gray Codes, *Intelligent Signal Processing and Communication Systems (ISPACS'04*, 2004) pp. 654-659.
9. Duc-Minh Pham, Premkumar A.B., Madhukumar A.S. Error Detection and Correction in Communication Channels Using Inverse Gray RSNS codes, *IEEE Transactions on communications*, 59(4), pp. 975-986, 2011.
10. Usha, K., Jaya sankar K. A Technique for the construction of Inverse Gray codes, *International Journal of Emerging Trends in Engineering and Development*, 2(2), (2012) pp 337-341.
11. Usha, K., Jaya sankar K. Binary Orthogonal code generation for Multi user communication using n-bit Gray and Inverse Gray Codes, *International Journal of Electronics and Communication Engineering*, 5(2), (2012) pp 165-174.
12. Sarwate D.V., Pursley M.B. Correlation properties of pseudo random and related sequences, *Proc. Of IEEE*, 68(5), (1980) pp 593-619.
13. Fazel K., Kaiser S. *Multi carrier and Spread Spectrum Systems*. (John Wiley and Sons, 2003).
14. Don Torreri, *Principles of Spread Spectrum Communication Systems* (Springer Publishers, 2005).
15. Mosa Ali Abu – Rgheff. *Introduction to CDMA Wireless Communications* (Elsevier Publishers, 2005).
16. Anil Kumar V., Abhijith Mitra, Mahadeva Prasanna S.R. On the Effectivity of different Pseudo-Noise and Orthogonal Sequences for Speech Encryption from Correlation Properties, *World Academy of Science, Engineering and Technology* 48, (2008) p 684-691.