A preemptive scheduling and due date assignment for single-machine in batch delivery system

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Abstract

In this paper we have proposed a new linear mathematical model for due date assignment on the single machine scheduling problem in a batch delivery system. Some assumptions are used as due dates are controllable, dynamic job arrivals; unforced machine idle and preemption are allowed. In this paper we propose a novel method of finding the sequence of jobs, batching of jobs for delivery and due date assignment of the jobs, so as to minimize the total of earliness, tardiness, holding, due date assignment, setup and delivery costs in just-in-time environment. Due to complexity of the problem it is NP hard too.

Keywords: batch delivery; due date assignment; earliness; tardiness; preemptive scheduling.

1. Introduction and Literature Review

Just-In-Time (JIT) scheduling was applied in a lot of studies about the problems which consist of earliness and tardiness penalty. A job is called tardy when it is delivered after its due date, and it is called early when it is delivered before its due date. Therefore, an ideal schedule is one in which all jobs are completed exactly on their due dates. In the most commonly scheduling models considered due dates are determined by customers [5]. But there are integrated systems that due dates can be determined according to the system's ability to meet the quoted requirement. For the existing ability many papers have investigated scheduling models with controllable due dates. It is shown that this ability can be a major factor in improving the performance of the systems. Promising short delivery dates may cause a company pay tardiness cost. On the other hand, quoting delivery dates too far in the future may not be acceptable to the customer or may force a company to offer price discounts in order to retain business clients [3]. Thus it can be an important duty to trade-off between short due dates, and due dates that are attainable. There are two main methods in due date assignment. The unrestricted method in which each job can have a different due date (this method is usually referred to as the DIF method) [2]. The constraint method referred to as CON method in which we have to assign a common due date for all jobs [11].

In all the above cases attempts have been made to deliver jobs exactly after completion, and delivery costs are ignored. However, in the real systems delivery costs are important and effective factors in production costs. In order to decrease these costs, completed jobs should be delivered within the same batches. In such situations, a job might have different completion and delivery/provide times (always, delivery time is greater than or equal to the completion time). Smith et al have a review on scheduling with batching [6]. Chen studied single machine batch delivery scheduling to minimize an earliness/tardiness related objective problem with a CON due date assignment method [7], Shabtay presented the JIT scheduling problem of single machine with batch delivery to minimize the cost of earliness, tardiness, due date assignment and holding cost [4].

Preemptive scheduling problems are those in which the processing of a job can be temporarily interrupted, and is restarted at a later time [1], although a lot of researchers studied preemptive scheduling, but a few of them considered it in the context of JIT-scheduling problems [9]. However, preemption seems to be disregarded in JIT-scheduling problems with due date assignment.

Many studies have dealt with the single machine scheduling problem with the due date assignment. Cheng et al [8] showed that the problem with common due date assignment and ready time is strongly NP-hard and proposed approximation algorithms for solving this problem. Cheng and Kovalyov[10] presented a Batch scheduling problem and common due-date assignment on a single machine that the objective is to minimize the sum of the due date assignment penalty and the weighted number of tardy jobs that several special cases of this problem were shown to be ordinary NP-hard. Shabtay [4] considered a single machine scheduling problem with due date assignment to minimize earliness, tardiness, holding, due date assignment and batch delivery costs with general cost coefficients and proved that the problem is NP-hard. Due to the complexity of these problems, a problem that is a developmental model of the Scheduling and due date assignment problem is NP-hard too.

This paper is organized as follows. In section 2 we describe the problem and give our assumptions, notations (indices, parameters and variables) parameters and decision variables. The mathematical model for this problem is discussed in section 3 and we use this model in section 4 for solving a numerical example and finally in section 5 the concluding remarks and future research are presented.

2. Problem Description

In a single machine scheduling problem a set of n independent jobs $\{J_1, J_2, \cdots, J_n\}$ has to be scheduled with preemptions on a single machine that can handle at most one job at a time. In this problem there are n unrelated jobs that may not be ready in zero time have to be scheduled in a single machine. In this study, a mathematical model is proposed for due date assignment on the single machine scheduling problem in a batch delivery system. The objective is to minimize the total of earliness, tardiness, holding, due date assignment, setup and delivery costs. Dynamic job arrivals, unforced machine idle and preemption are allowed. A new linear mathematical model is presented and due to it, the complexity of the problem is shown. The problem is formulated according to the following assumptions.

2.1. *Assumptions*

- The processing time, release time and the cost coefficients of each job are known.
- Each job has a lead time that due date assignment penalty occurs by assigning a due date to be greater than A_j .
- Each batch is delivered when all its jobs are completed
- All of the jobs can be preempted.
- Whenever a job is to be started, whether initially or after preemption, a setup is necessary and total setup cost is only related to the number of setups, and independent of the jobs.
- The capacity of the batches is unlimited.

- The delivery cost of all of the batches is the same and fixed and is independent of the number of the jobs, that be allocated to the batch.
- For the jobs which be allocated to a same batch must be assigned equal due date and jobs that are delivered in separated batches can have a different due date.
- The batches are delivered immediately after completion.
- The machine can handle at most one job at a time.
- The machine is assumed to be continuously available and breakdown is not occurring.

2.1.1 Indices

i index for jobs $i = 0,1,\ldots,n$

j index for jobs $j = 1,\ldots,n$

k index for parts $k = 1,\ldots,P(i)$

l index for parts $l = 1,\ldots,P(j)$

m index for batches $m = 1,\ldots,n$

2.1.2 Parameters

P_j processing time of job *j*

R_j release date of job *j*

A_j lead time of job *j*

α_j cost of earliness of job *j*

β_j cost of tardiness penalty of job *j*

γ_j cost of due date assignment of job *j*

θ_j holding cost

δ delivery cost of each batch

μ setup cost

M a big number

2.1.3 Variables

C_{jl} completion time of *l* th part of job *j*

D_j delivery time of job *j*

dj due date of job *j*

E_j earliness of job *j*

T_j tardiness of job *j*

H_j holding time of job *j* $(Dj - P_j - Rj)$

G_j lead time positive deviation of assigned due date of job *j*

ω number of setup

$x_{ik,jl}$ equal to 1 if lth part of job *j* is processed immediately after kth part of job i and 0 otherwise

y_{im} equal to 1 if job i is delivered with mth batch and 0 otherwise

z_m equal to 1 if at least a job is assigned to mth batch

3. The mathematical model

$$\min Z = \sum_{j=1}^{n} \alpha_j . E_j + \sum_{j=1}^{n} \beta_j . T_j + \sum_{j=1}^{n} \theta_j . H_j + \sum_{j=1}^{n} \gamma_j . G_j + \delta . \sum_{m=1}^{n} z_m + \mu . \omega \quad (1)$$

$$\sum_{i=0}^{n} \sum_{k=1}^{P_i} x_{ikjl} = 1 \quad \forall j,l \quad (2)$$

$$\sum_{j=1}^{n} \sum_{l=1}^{P_j} x_{ikjl} \le 1 \quad \forall i,k \quad (3)$$

$$C_{j1} \ge R_j + 1 \quad \forall j \quad (4)$$

$$C_{j1} \le R_j + 1 + F_j . M \quad \forall j \quad (5)$$

$$C_{j1} \ge C_{ik} + 1 - (1 - x_{ikj1}) . M \quad \forall j,i,k \quad (6)$$

$$C_{j1} \le C_{ik} + 1 + (2 - F_j - x_{ikj1}) . M \quad \forall j,i,k \quad (7)$$

$$C_{jl} < C_{jl+1} \quad \forall j,k,i,l \quad (8)$$

$$C_{jl} > C_{ik} - (1 - x_{ikjl}) . M \quad \forall j,i,l,k \quad (9)$$

$$\sum_{m=1}^{n} y_{jm} = 1 \quad \forall j \quad (10)$$

$$D_j \ge C_{iP_i} - (2 - y_{jm} - y_{im}) . M \quad \forall i,j,m \quad (11)$$

$$d_j \ge d_i - (2 - y_{jm} - y_{im}) . M \quad \forall i,j,m \quad (12)$$

$$d_j \le d_i + (2 - y_{jm} - y_{im}) . M \quad \forall i,j,m \quad (13)$$

$$D_j + E_j - T_j = d_j \quad \forall j \quad (14)$$

$$H_j = D_j - R_j - P_j \quad \forall j \quad (15)$$

$$z_m \ge y_{jm} \quad \forall j,m \quad (16)$$

$$z_{m-1} \ge z_m \quad m = 2,\ldots,n \quad (17)$$

$$G_j \ge 0 \quad \forall j \quad (18)$$

$$G_j \ge d_j - A_j \quad \forall j \quad (19)$$

$$\omega = \left(\sum_{j=1}^{n} P_j - \sum_{k=1}^{P_j - 1} x_{jkjk+1} \right) \quad (20)$$

$$x_{ikjl}\ , y_{jm}\ , z_m \text{ and } F_j \text{ are binary} \quad \forall i,j,k,l,m \quad (21)$$

$$d_j, C_{jl}, G_j \text{ and } D_j \text{ are integer} \quad \forall j,l \quad (22)$$

The objective function and constraints can be formulated as follows:

Equation (1) shows the objective function which minimizes all costs. Constraint (2) ensures that each part only can be processed after another part constraint (3) ensures that at most one part can be processed after each batch (there is not any part after the last part). Constraint (4) to (7) force first part of job *j* be stored on maximum of the release date and completion time of a previous job, constraint (8) forces parts of a job complete consecutively. Constraint (9) and (10) calculate the complete time of parts of jobs, constraint (11) ensures that each job is assigned a batch, constraint (12) shows jobs which are assigned to a same batch are delivered at a same time. Constraint (13) and (14) force jobs which are assigned to a same batch have a same due date, constraint (15) calculates earliness or tardiness of jobs, constraint (16) shows holding time of jobs. Constraint (17) and (18) force batches be used consecutively, constraint (19) calculate lead time positive deviation of assigned due date, constraint (20) calculate the number of setups, constraint (21) shows binary variables and constraint (22) shows integer variable.

4. Numerical Problem

For example we consider a three jobs on single machine with processing time of jobs, release date of jobs, lead time of jobs, holding, earliness, tardiness and due date assignment cost of jobs. The setup and delivery cost are 3 and 25 respectively for this problem. For example three jobs are represented in Table 1:

Table.1: data of the example

	P_j	R_j	A_j	θ	α	β	γ
J_1	6	0	6	1	2	4	3
J_2	4	5	8	2	1	2	4
J_3	2	3	3	19	3	1	2

λ =3 δ=25

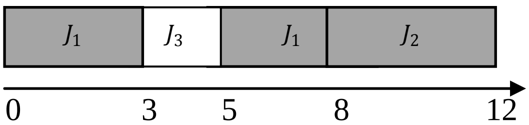

Fig.1: a feasible solution of the problem

In fig.1 two batches is delivered that a batch is contained j_3 and another batch is contained $j_1$and j_2. In the table.2 is showed value of the costs for feasible solution of fig.1:

Table.2: value of the costs

	d_j	$\max(0, d_j - A_j)$	H_j	T_j	E_j
J_1	8	2	12	4	0
J_2	8	0	7	4	0
J_3	3	0	2	0	0

m=2

According to table.2 value of the objective function is calculated as following:

Objective=0+(4*4+4*2+0*1)+(12*1+7*2+2*19)+(2*3+ 0*4+0*2)+(4*3)+(2*25)=156

5. Conclusion &future researches

In this paper, a mathematical model is proposed for the single machine scheduling problem under assumptions: due dates are controllable , dynamic job arrivals, preemption is allowed and the objective is to minimize total of the earliness, tardiness, due date assignment, holding, setup and delivery costs. We also provided a numerical example for verifying this model.

We have introduced one more parameter in comparison to [5]. This implies that our result extends the result in [5].

For the future research, researchers may extend the proposed model for the case of other shop scheduling systems e.g. flow shop, job shop etc. and also considering fuzzy data to solve this problem. It will be a worth for extending the existing model with multiple machines like parallel batch processing machines and also meta-heuristics algorithms to solve different types of problems.

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