

## Modified iterative sphere decoding algorithm in LTE system

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**Abstract:** In the long-term evolution (LTE) system, channel equalization makes compensation to restore the original signal, the paper puts forward iteration sphere decoding algorithm which combines traditional sphere decoding and the improved QR based on the conventional QR decomposition detection algorithm. It can effectively reduce the system complexity. At the same time, in QPSK and 16QAM, the simulation results show that the improved QR iterative sphere decoding algorithm performance is better with higher SNR in AWGN channel.

*Keywords:* QR sphere decoding, iteration algorithm

### 1 INTRODUCTION

In LTE (Long Term Evolution) system, in order to recover the transmitted signals, the receiver often uses signal detection and estimation methods to retrieve information from the received data. This process is referred to as channel equalization. Channel equalization is an anti-fading measures which taken in the communication system ,in order to improve the performance of transmission, the mechanism is to compensate for the channel in order to eliminate or weaken inter-symbol interference (ISI) which comes from the multi-path delay.

In recent years, a lot of the detection algorithms for channel equalization to recover the transmitted signal have been proposed. This paper expounds the principle of decomposition algorithm and improved QR decomposition iterative detection algorithm. An improved algorithm is presented by combining the QR algorithm with sphere decoding methodology. In addition, the improved algorithm will be compared with the Maximum Likelihood (ML) algorithm. This paper is organized as follows: the QR decomposition detection principles will be described in section 2. Section 3 will present the improved iterative QR decomposition detection method. The combination of sphere decoding methodology and improved QR decomposition detection method will be given in section 4. Section 5 shows the simulation results, whereas conclusions are drawn in section 6.

### 2 QR decomposition detection principle

Channel model:  $Y=HX+N$ .QR will decompose the channel impact response matrix H at (k, l). The result is  $nT*nT$  of the unitary matrix Q and  $nT*nT$  of the upper triangular matrix R, and then the matrix QH left multiplied by the received vector Y, after conversion.

The result is received vector  $\tilde{y}$ . In the equation right, QH\*Q transform identity matrix I, and after conversion expressions are as follows:

$$\tilde{y} = Q^H Y = RX + Q^H n = RX + \eta \quad (1)$$

Where R is upper triangular matrix  $nT*nT$ . Because n is a Gaussian random variable  $Q^H n = \eta$ , The equation is also a Gaussian random variable. The above formula converted to the form of a matrix can be obtained:

$$\begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_{nR} \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & \dots & r_{1,nT} \\ 0 & r_{22} & \dots & r_{2,nT} \\ & & \dots & \\ 0 & 0 & \dots & r_{nR,nT} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{nT} \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_{nR} \end{pmatrix} \quad (2)$$

After decomposing the above matrix, we can obtain the following equations:

$$\begin{cases} \tilde{y}_{nT} = r_{nT,nT}x_{nT} + \eta_{nT} \\ \tilde{y}_{nT-1} = r_{nT-1,nT-1}x_{nT-1} + r_{nT-1,nT}x_{nT} + \eta_{nT-1} \\ \dots\dots\dots \\ \tilde{y}_1 = r_{1,1}x_1 + \sum_{j=2}^{nT} r_{1,j}x_j + \eta_1 \end{cases} \quad (3)$$

For the first equation,  $\tilde{y}_{nT}$  does not contain the interference from the other transmission signal, can be

$$\hat{x}_{nT} = \frac{Q [ y_{nT} ]}{r_{nT,nT}} \quad \text{which processed in the first, get } \hat{x}_{nT} \text{ is the estimated value of } x_{nT}, \text{ and Q represents a}$$

judgment of the quantization process, mapping  $\frac{y_{nT}}{r_{nT,nT}}$  to the nearest constellation point of the Euclidean distance.

to eliminate the interference caused by  $x_{nT}$ , and get

$$\hat{x}_{nT-1} = Q \left[ \frac{\tilde{y}_{nT-1} - r_{nT-1, nT} x_{nT}}{r_{nT-1, nT-1}} \right]$$

estimated value of  $x_{nT-1}$  by bringing  $\hat{x}_{nT}$  to the next equation. repeat the above steps, you can get the estimated value of all complex-valued symbols from the sender  $\hat{x}_{nT}, \hat{x}_{nT-1} \dots \hat{x}_1$ . Whereby based on the received signal Y, we can get the detected transmission signal X.

### 3 Improved QR decomposition Iterative cycle detection algorithm

First of all, apply a QR decomposition algorithm for signal detection. All the judgment results only retain the estimated value of the last detection layer and abandon the other layers of the estimated values. Adjust the channel impulse response matrix of a column vector, so that the final estimation value of the last detection layer corresponds to lower layer of the preceding detection layer. Make signal detection for the channel impact response matrix H which is adjusted using QR, and retain the last detection layer and adjusted to the next previous layer which is first detected layer, repeat the process until all of the symbols to be detected are completed. The basic steps are as follows:

(1) Use the traditional QR decomposition detection algorithm for the received vector Y, to obtain layers of

the detected value  $\hat{x}_{nT}, \hat{x}_{nT-1} \dots \hat{x}_1$

(2) Assume  $i=1$ ;

(3) Adjust the order, and modulate the original

detection order to get  $[M, N]$ , where  $M=[\hat{x}_i \dots \hat{x}_1]$ ,

$N=[\hat{x}_{nT} \dots \hat{x}_{i+1}]$ , adjust the channel impulse response matrix H according to the adjusted detection order, get  $[h_{i+1} \dots h_{nT} \ h_1 \dots h_{i-1} \ h_i]$ ;

(4) Make QR decomposition impact on the adjusted channel response matrix, introduce  $\hat{x}_i \dots \hat{x}_1$  as the detected value to obtain the new  $\hat{x}_{nT} \dots \hat{x}_{i+1}$ ;

(5) Keep the last  $\hat{x}_{i+1}$  as detection value;

(6) Assume  $i=i+1$ , if  $i < nT$ , then, repeat step 3 circularly, if  $i = nT$ , representing all symbols are detection finished.

Here to complete a sub-carrier signal detection, the same operation is performed on all sub-carriers in the time-frequency location to recover the complex-valued symbols in the transmitting side all of the resources in the particles.

### 4 Sphere decoding algorithm based on the improved QR iteration of the loop

The basic idea of sphere decoding is search multidimensional grid point in sphere of radius d and treat a vector x as centered, thereby reducing the number of points of the search by limiting or reducing the search radius, so that reduce calculation time. The advantage of sphere decoding algorithm is unnecessary like the conventional ML decoding algorithm to search all the grid points in the entire grid, but only need to be searched in a previously set limited spherical area, the search time will be greatly reduced if the number of points contained in the region is quite smaller relative to the total number of points within the entire grid.

Sphere decoding is to solve the following problems:

$$\min_{s \in D \subset Z^m} \|r - Hs\|_2$$

Where  $Z^m$  is an integer lattice point set, r is the received signal, and s is the transmission signal. Now it is

needed to find an r to satisfy  $d^2 \geq \|r - Hs\|_2^2$ , where

d is the selected appropriate radius, and the parameter d can be set by channel estimation, and therefore do not

excessive repeat. In this article, a known parameter d is set to the initial time. Based on the improved QR decomposition algorithm derivation, concrete steps are as follows:

① As known is first input condition of receiver:  $d$ ,

$$Q = [Q_1, Q_2], \quad R, \quad x, \quad y = Q_1^* d$$

② QR decomposition on channel matrix H, assume  $N \geq M$ , we have

$$H = [Q_1, Q_2] \begin{bmatrix} R \\ 0_{(N-M) \times M} \end{bmatrix} \quad (4)$$

Where, Q is  $n \times n$  of orthogonal matrix,  $Q_1 \in R^{N \times M}$ ,  $Q_2 \in R^{N \times (N-M)}$  and R is  $M \times M$  dimension of the upper triangular matrix. The decomposition of H is

$$d_2^2 = d_s^2 - \|Q_2^H y\|_2^2, \quad y = Q_1^H r \quad (5)$$

So,  $d_2^2 = d_s^2 - \|Q_2^H y\|_2^2, y = Q_1^H r$  can be decomposed according to the following:

$$d^2 \geq \left\| x - [Q_1 \ Q_2] \begin{bmatrix} R \\ 0 \end{bmatrix} s \right\|^2 = \left\| \begin{bmatrix} Q_1^* \\ Q_2^* \end{bmatrix} x - \begin{bmatrix} R \\ 0 \end{bmatrix} s \right\|^2 = \|Q_1^* x - R s\|^2 + \|Q_2^* x\|^2 \quad (6)$$

The \* indicates the conjugate transpose, and the right shift to the left:

$$d^2 - \|\mathcal{Q}_2^* x\|^2 \geq \|\mathcal{Q}_1^* x - Rs\|^2 \quad (7)$$

Assume:  $y = \mathcal{Q}_1^* x$ ,  $d'^2 = d^2 - \|\mathcal{Q}_2^* x\|^2$ , bring this condition to the above formula will be:

$$d'^2 \geq \sum_{i=1}^m \left( y_i - \sum_{j=1}^m r_{ij} s_j \right)^2 \quad (8)$$

③ Its expansion inequalities by making use of R matrix of the upper triangular attributes:

$$d'^2 \geq (y_m - r_{m,m} s_m)^2 + (y_{m-1} - r_{m-1,m} s_m - r_{m-1,m-1} s_{m-1})^2 + \dots \quad (9)$$

The first one on the right relies only on  $s_m$ , the second one is only the relevant with  $s_m$ ,  $s_{m-1}$ , and so forth, so, it is a necessary condition that  $d'^2 \geq (y_m - r_{m,m} s_m)^2$ , after decomposition, then:

$$\frac{-d' + y_m}{r_{m,m}} \leq s_m \leq \frac{d' + y_m}{r_{m,m}} \quad (10)$$

Change to the form of interval:

$$\left\lceil \frac{-d' + y_m}{r_{m,m}} \right\rceil \leq s_m \leq \left\lfloor \frac{d' + y_m}{r_{m,m}} \right\rfloor \quad (11)$$

Where,  $\lceil \cdot \rceil$  represents rounding up, and  $\lfloor \cdot \rfloor$  represents rounding down, then it can get the range of  $s_m$ .

④ After getting  $s_m$ , make  $d_{m-1}'^2 = d'^2 - (y_m - r_{m,m} s_m)^2$  and  $y_{m-1|m} = y_{m-1} - r_{m-1,m} s_m$ , get

$$d'^2 \geq (y_m - r_{m,m} s_m)^2 + (y_{m-1} - r_{m-1,m} s_m - r_{m-1,m-1} s_{m-1})^2 + \dots \quad (12)$$

As known, the first and second of left formula is necessary condition, it can be launched the value space by the necessary condition:

$$\left\lceil \frac{-d_{m-1}' + y_{m-1|m}}{r_{m-1,m-1}} \right\rceil \leq s_{m-1} \leq \left\lfloor \frac{d_{m-1}' + y_{m-1|m}}{r_{m-1,m-1}} \right\rfloor \quad (13)$$

And so on, can pass out the value space of  $s_{m-2}, \dots, s_1$ .

⑤ Assume  $k = m$ ,  $d_m'^2 = d^2 - \|\mathcal{Q}_2^* x\|^2$

⑥ Set upper and lower boundary of  $s_k$ , where the

lower bound is  $\left\lceil \frac{d_k' + y_{k|k+1}}{r_{k,k}} \right\rceil - 1$  the upper bound is  $\left\lfloor \frac{d_k' + y_{k|k+1}}{r_{k,k}} \right\rfloor$

⑦ It begins traverse from the smallest integer in  $s_k$

where  $k=m$ , if  $s_k \leq UB(s_k)$ , then  $k = k-1$ ,

$$y_{k|k-1} = y_k + \sum_{j=k+1}^m r_{k,j} s_j$$

$$d_k'^2 = d_{k+1}'^2 - (y_{k+1} - r_{k+1,k+1} s_{k+1})^2$$

In the same way, according to the above method to set the interval  $s_{k-1}$  and begin to judge  $s_{k-1} \leq UB(s_{k-1})$  form the minimum one of the interval  $s_{k-1}$ , if meet to condition and then continue to decline  $k=1$  until  $k=1$ , cancel  $s_k, s_{k-1}, \dots, s_1$  and save  $s$  and the error  $d_m'^2 = d_1'^2 + (y_1 - r_{1,1} s_1)^2$ .

⑧ In step 7, judging  $s_k \leq UB(s_k)$  after decreasing  $k$ , if the conditions are not satisfied, then  $k = k+1$ , stop operation until  $k = m+1$ , otherwise,  $s_k = s_k + 1$ , continue to traverse.

⑨ The sphere decoding algorithm will traverse all value, sometimes, some can not meet the condition not until  $k=1$ , then cancel it, just save  $s_k, s_{k-1}, \dots, s_1$  and compare every kind of error  $d_m'^2 = d_1'^2 + (y_1 - r_{1,1} s_1)^2$ , save the value of which error is the least.

The improved QR improves the quality of the channel equalization greatly.

## 5 Simulation Results

Simulations have been done using MatLab, To compare the performance and efficiency of the above three algorithms. The simulation uses uniform conditions and simulation conditions and parameters are shown in table 1: the simulation based on the LTE system, the iterative number is 2000, symbol number of each transmitting antenna  $t=100$ . Where ML is the

maximum likelihood detection algorithm, rev-QR is improved QR decomposition iterative detection algorithm, SD is sphere decoding algorithm, rev-QR-SD is improved sphere decoding algorithm based on modified QR iterative algorithm.

TABLE 1 The condition and parameter of simulation

name	value
Modulation mode	QPSK/16QAM
number of Resource block	20
Doppler frequency shift	5Hz
The number of transmit antennas	2/4
The number of receive antennas	2/4
Path delay	[0 30 150 310 370 710 1090 1730 2510]
the power gain of route	[0 -1.5 -1.4 -3.6 -0.6 -9.1 -7.0 -12.0 -16.9]
Channel model	AWGN/EPA5
Frame structure	Common CP
Carrier frequency	2GHz
Carrier spacing	15kHz
Channel estimation algorithm	MMSE
Demodulation method	Soft demodulation

The following four plots show that the result of simulation which using Monte Carlo simulation in the QPSK and 16QAM in two-input two-output and four-input four-output of MIMO transmission mode, it is BER of comparison between the maximum likelihood algorithm ( ML ), sphere decoding algorithm ( SD ), improved QR detection algorithm and proposed iterative sphere decoding algorithm based on the improved QR the algorithm to show the different performance with different algorithm.

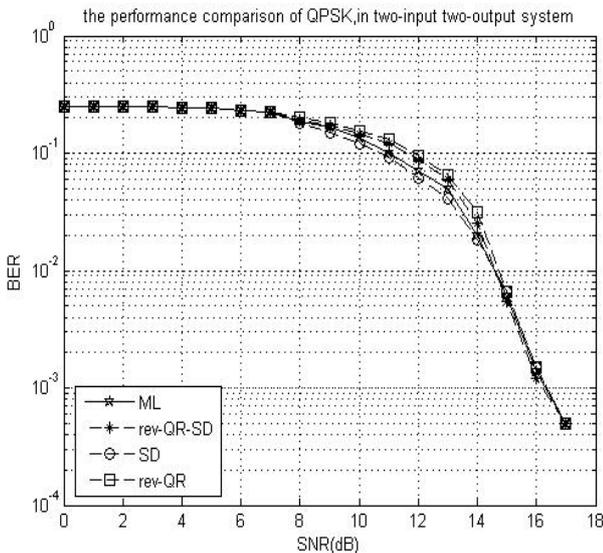


Figure 1 The performance comparison of QPSK, in 2\*2 system

By the graph 1, using QPSK, in the condition of low SNR, the performance of the four algorithms are very close; from the medium to high SNR, it can be seen that, improved sphere decoding algorithm capability has better performance than other algorithms. in the high SNR, algorithms have the unanimous performance.

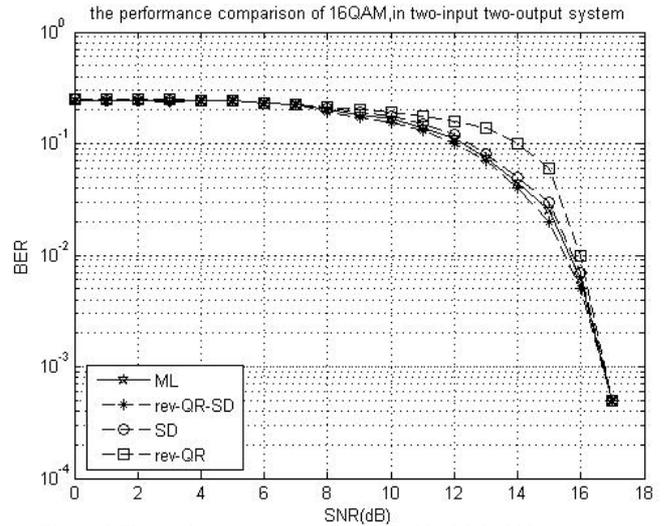


Figure 2 The performance comparison of 16QAM, in 2\*2 system

Figure 2 shows that, using 16QAM algorithm, from the medium to high SNR, sphere decoding algorithm performance are improved much more obviously than the other three algorithms.

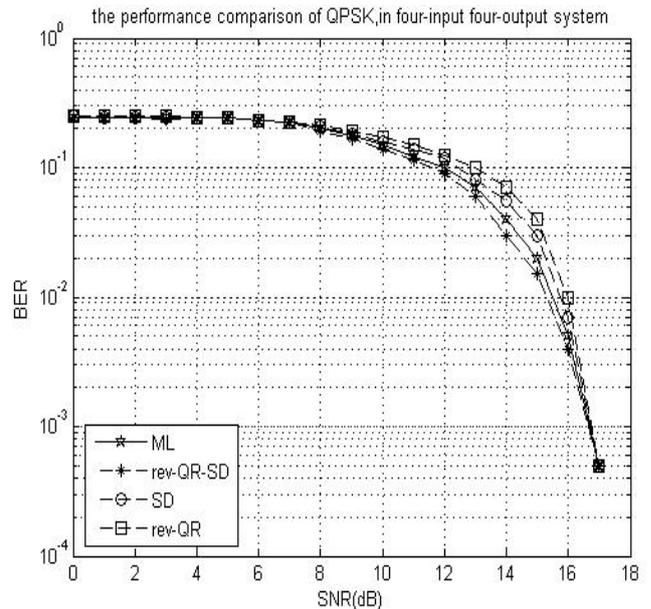


Figure 3 The performance comparison of QPSK, in 4\*4 system

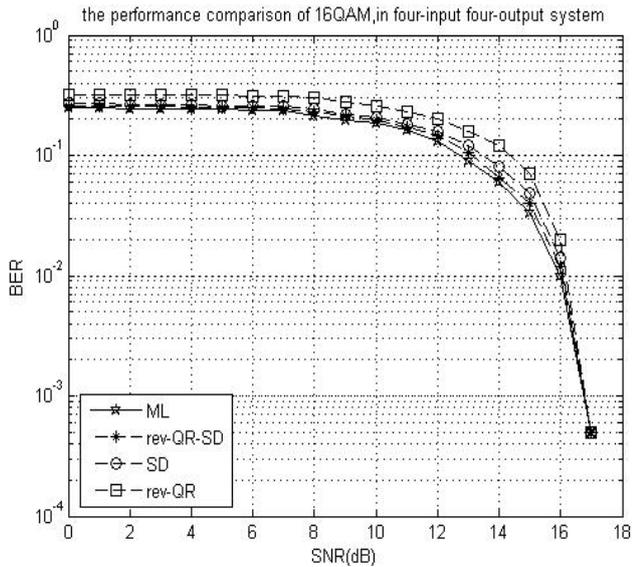


Figure 4 The performance comparison of 16QAM, in 4\*4 system

Based on the simulation results shown in the figures, we can conclude that: comparison of the figure 1 and figure 2, figure 3 and Figure 4, in the low SNR conditions, the above algorithms have the consistent performance, no matter which is in  $2 \times 2$  or  $4 \times 4$  MIMO cases, using 16QAM modulation algorithm, in the high SNR conditions, improved sphere decoding have the better performance than other algorithms. Longitudinal comparison of the figure 1 and figure 3, figure 2 and Figure 4, with the antenna number increases, the performance of the four algorithms have the growing gap. In high SNR, with the antenna number increases, NR, improved sphere decoding have more obvious advantages.

## 6 CONCLUSIONS

Based on the traditional QR detection algorithm, this paper proposed an improved iterative QR decomposition detection method. The channel can make full use of the diversity gain of the last detection layer, which will result an improvement of the detection performance. In addition, the improved method was combined with the sphere decoding methodology to further enhance the system performance. Simulation results showed that the proposed QR iterative sphere decoding algorithm can not only significantly reduce the computational complexity, but also improve the system achievability.

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