Textbook design of Kalman Filter for undergraduates

Kedong Wang

School of Astronautics, Beihang University, Beijing 100191, China

Abstract

Kalman Filter is a specialized course for some majors in engineering. In our textbook, the theory deducing is condensed as much as possible. The relative figures are provided schematically to aid for understanding. No less than 5 examples are designed in each chapter to address the major points as emphasis. No less than 10 assignments are supplied in each chapter for the students to review the major points out of class. In the algorithm parts, the example codes of the algorithms are provided for reference. The textbook design method is good for the book design of other specialized courses.

Keywords: Use “keywords” style here

1. Introduction

Kalman filter, a kind of optimal estimation algorithm, has been widely applied in multi-sensor information fusion, finance, meteorology, and etc. Hence, Kalman filter is a basic specialized course in many undergraduate and graduate programs. The textbook is the cornerstone for teaching. The classic textbook of Kalman filter is Applied Optimal Estimation edited by Arthur Gelb and pressed in 1974 by MIT Press [1]. The advantage of the book is its brevity and simplicity in theory. There is no too much formula deduction, which is beneficial for the readers to focus on the principles of the algorithms and their

applications. However, the book is out of date to some extent since there are many breakthroughs in Kalman filter now. The new results in the study of Kalman filter have been edited in the recent books such as Kalman Filtering: Theory and Practice Using MATLAB edited by Mohinder S. Grewal and Angus P. Andrews [2]. The third edition of the book has been pressed in 2008 by Wiley. The newly proposed algorithms, such as unscented Kalman filter and particle filter, are included in the book. The other advantage of the book is that there are example codes of all of the algorithms programmed in MATLAB. However, the main drawback of the book is that there are too few examples and assignments, which is not beneficial for the readers to understand the major points thoroughly and review out of class.

Kalman Filter is a specialized course for some majors in engineering, which is very difficult to learn due to its complexity in mathematics and its depth in theory [3]. Moreover, the current textbooks are not easy for the students to follow the major points. The reasons include at least that: (1) the books focus too much on theory and mathematics, which is too boring for the students to learn; (2) there are too few examples in the books so that it is difficult for the students to understand and follow the algorithms and the major points; (3) there are too few assignments for the students to excise out of class; and (4) there are no example programs of the algorithms for the students to refer to. In this paper, a

new textbook of Kalman filter is designed for undergraduates. In the book, formulas are reduced as few as possible. What is more important is that the formulas are deduced logically. No less than 5 examples and 10 assignments are designed in each chapter to help the readers understand the major points easily. The example codes of all of the algorithms are also provided in MATLAB, but what we stress on is the way of programming the algorithms in codes.

2. The content design

In the book, there are 9 chapters except for the introduction. The 10 chapters can be divided into 4 parts.

In the first part, there are 3 chapters including introduction, the mathematical basis, and the basis of the linear system. The concept of estimation, the task of Kalman filter and the history of Kalman filter are described in the introduction. The trend of Kalman filter is also introduced in the first chapter. Then, the operation of matrix and the analysis method of random process are provided briefly. Finally, the basis of the linear system is summarized. The points include the linear system modeling, the discretization of the continuous system, the error analysis, and the observability of the linear system. The first 3 chapters are the basis of the book although the mathematical basis and the basis of the linear system are the contents of Linear Algebra and Modern Control.

In the second part, there are 4 chapters including the main optimal estimation algorithms, Kalman filter, the application methods of Kalman filter, and the nonlinear filter. The optimal estimation algorithms are divided into two kinds: the minimum variance criterion and the maximum probability criterion. The minimum variance estimation algorithms

include least squared method, minimum variance estimation, linear minimum variance estimation, Kalman filter, Wiener filter, and etc. the maximum probability estimation algorithms include maximum likelihood estimation and maximum posterior estimation. Bayes estimation algorithm can be turned into minimum variance estimation or maximum probability estimation if its penalty function is determined. All of these main optimal estimation algorithms will be summarized in the fourth chapter concisely. In the fifth chapter, the core content of the book, Kalman filter, is introduced intensively from its deduction. Its initialization, the model discretization and its error analysis are the necessary points of Kalman filter. In the sixth chapter, the application methods of Kalman filter are introduced. The three problems, including the colored noise, the computing instability or load, and the modeling error, are discussed and the candidate methods are also proposed. The last chapter in this part is the nonlinear filter. The three nonlinear filter algorithms, including extended Kalman filter, unscented Kalman filter, and particle filter, are introduced.

In the third part, there are 2 chapters including the optimal smoothing and the federal filtering. In the optimal smoothing, the fixed-point smoothing, the fixed-interval smoothing, and the fixed-delay smoothing are introduced. The smoothing performance is also compared with the filtering performance. In the ninth chapter, the federal filter is described in detail. The redundant and fault-tolerant design of the federal filter is focused on.

In the last part, the typical applications of Kalman filter are introduced. The method of designing Kalman filter in practice is concentrated on. The applications include the integrated navigation of GPS and inertial navigation

system, the target tracking, and the finance analysis.

In the content design, some chapters may be beyond the requirement for the undergraduate. For example, nonlinear filter and federal filter are too complex for the undergraduate to learn in a term. Hence, the most content of the book is suitable for the undergraduate, but some content is designed for the advanced study or the graduate. If it is possible, the last part of the book can be extended as required in the revision. For instance, more examples in practice can be added if necessary. That is, the structure of the book is convenient for its revision.

3. The example and the assignment design

The examples and the assignments are the necessary part of the books for undergraduate, but they are the main disadvantage of the existing books. Hence, in this book, the proper examples and the sufficient assignments are designed with the extra effort.

Except for the first chapter, no less than 5 examples are provided in each chapter. The examples are to remind the readers to stress on the key points. The performance of the key points can be described more clearly by the examples. For example, in the third chapter, the way to model a system and the way to discrete a linear model are the two key points. To emphasize the two points, two examples are designed in the book. The first example is to model a mass-spring-dashpot system while the second is to discrete a second-order linear model. In the chapters on the algorithms, the examples are designed to spot on the performance of the algorithms if necessary.

In each chapter, no less than 10 assignments are designed for the readers' self-study. The assignments are designed

in three layers. The first layer focuses on the fundamental concepts. The assignments in the first layer may be some questions to inspire the readers to think further. The second layer assignments stress on the problem solving. The assignments in this layer are the problems to be solved by deducing or using the learned skills comprehensively. The third layer assignments emphasize on the further analysis of the problems. The assignments in this layer are the problems with the questions for further thinking. The typical assignments of the three layers may be like: 'what are the three kinds of estimation?' (the first layer), 'Please deduce the weighted least squared method.' (the second layer), and 'Please analyze the effect of the gain matrix in Kalman filter.' (the third layer). In the book, most of the assignments are provided with their reference answers, which is convenient for the readers.

In this book, some comprehensive assignments are also designed for the final stage study. These assignments are the problems with many questions ranging from the system modeling to the filtering result analysis. The following tasks have to be done at least in these assignments: (1) the system modeling, (2) Kalman filter algorithm design, (3) algorithm coding, and (4) filtering result analysis. The assignments are helpful for the readers to understand Kalman filter better by trying to use it in practice.

In the book, the examples and the assignments may be edited in an independent part based on their length.

4. The example code design

Although some existing books have provided some example codes, there are no extra comments on the code design, which is not convenient for the readers to understand the codes. In the book design, all of the algorithms are coded in

MATLAB. What is more important is that the design process of the codes is described in detail, which is good for the readers to be familiar with the coding of the Kalman filter algorithms. The results of the codes are also analyzed to judge the correctness of the codes or the performance of the algorithms. To help the readers improve the ability of debugging the codes, some scenarios with typical errors are designed. To evaluate the performance of the algorithms, some typical scenarios are also designed with the different initial conditions so that the results can be compared. The example code design is the main part of the chapters focused on algorithms.

5. Concluding remarks

Since many breakthroughs have been achieved in Kalman filter, it is necessary to include the newly research outcomes in the textbook. The obvious drawbacks of the existing books, such as the few examples and assignments, are also not

good for the readers to learn by themselves. In this paper, the design of a new Kalman filter book for undergraduate is put forward. The structure of the book content is convenient for the future expansion. The examples and the assignments are targeted for the readers to learn by themselves. The code design processes are especially useful for the new learners to practice the algorithm tentatively. The book design method can also be referred as an example for other specialized books.

6. References

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