Bayesian Networks Application in Multi-State System Reliability Analysis

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***Abstract*—Aiming at the limitations of traditional reliability analysis theory in multi-state system, a method for reliability modeling and assessment of a multi-state system based on Bayesian Network (BN) is proposed with the advantages of uncertain reasoning and describing multi-state of event. Through the case of cell production line system, in this paper we will discuss how to establish and construct a multi-state system model based on Bayesian network, and how to apply the prior probability and posterior probability to do the bidirectional inference analysis, and directly calculate the reliability indices of the system by means of prior probability and Conditional Probability Table (CPT) . Thereby we can do the qualitative and quantitative analysis of the multi-state system reliability, identify the weak links of the system, and achieve assessment of system reliability.**

Keywords-multi-state; Bayesian network; system reliability; reasoning; modeling

I. Introduction

In traditional reliability theory components and systems are in only two states: the working state and the failure state, while in the multi-state system, components and systems are in multiple states. In the traditional reliability theory, systems and components in the fault tree are in only two states, multi-state system cannot be analyzed. To this end, Huang [1] introduced fault tree into the multi-state system, and proposed multi-state fault tree theory which applies for multi-state association system. Subsequently, Huang [2] and Bossche [3] offered a multi-state fault tree theory which is applicable to multi-state non-association system. Yi-Kuei Lin and John Yuan [4] analyzed the multi-state network system reliability with minimal path sets. Ramirez-Marquez, Coit and the Tortorella [5] analyzed multi-state two-terminal network reliability with multi-state minimal cut sets. In order to avoid the complex calculation of minimal path sets and cut sets, Zaitseva Levashenko [6-7] used the theory of multi-valued logic to analyze the reliability of multi-state series system, parallel system, series-parallel system and k/n system, and the effect of components' state changes on system reliability. Using the universal generating function method, Levitin[8]analyzed that multi-state system reliability has two failure modes.

Due to much concern about multi-state system reliability, the research of multi-state system reliability has made some progress, but there are some deficiencies of these methods. In recent years, as an advanced technology for reasoning and describing uncertain knowledge, Bayesian network (BN) has been widely applied to the field of reliability analysis, risk analysis and maintenance of complex systems [9-14]. Bayesian network model can express multi-state of the event, and uncertainty of logical causality and information. The system can be predicted, analyzed, and diagnosed by calculating probability. Therefore, a Bayesian network can better suit the requirements of multi-state system reliability analysis. This paper is organized as follows: we start by briefly introducing multi-state system, Bayesian network and its construction method lie in Section II, and we specifically introduce how to build a multi-state system model based on Bayesian network in Section III. Finally, we give a real-life case to analyze the system reliability in details in Section IV, and offer some conclusions in Section V.

II. Brief Introduction of Multi-State System and Bayesian Network

A. Multi-State System

In addition to the "working" state and "complete failure" state, system can also be in a variety of working (or failure) states, or the system can be run in a plurality of performance levels, such system is called a multi-state system (MSS) [15-16].

The multi-state system is divided into two types: multi-work (or failure) state system and the multi-performance level system.

1) Multiple work (or failure) state system

Multiple work (or failure) state system is that in addition to the two states of the working and complete failure, the system can be in a variety of work (or failure) states. Such as k/n (G) system in the traditional reliability is a typical multi-work (or failure) state systems.

2) Multiple performance level system

Multi-performance level system is that the system is able to run in a variety of performance levels. Such as for a 300 MW generator set, when it is completely normal, their power level is 300 MW, when the ventilator or pulverizer fails, the electricity generation levels will decrease to 150 MW, 200 MW, 225 MW and so on.

Assuming that multi-state system has M states, where $M \in Z^{+}$ and $M \geqslant 2$, when M=2. That is the traditional two-state system. System is defined as the state 1, when it is complete failure; the system is defined as the state M, when it is completely normal.

Traditional reliability analysis methods have the following deficiencies:

a) The traditional reliability theory divides all components and systems into being in "working" or "complete failure" state, ignoring the intermediate state, so the theory describes the multi-state of the components and systems inaccurately.

b) Analysis does not reflect the relationship between component performance and system performance, and between system reliability and system performance. It is precisely because of the above shortcomings that traditional reliability analysis methods possess. When making the analysis of multi-state system reliability, performance departs from the reliability. The traditional reliability analysis cannot reflect the underlying causes of system performance degradation and failure, which may lead to the inaccurate analysis results of system reliability.

B. Bayesian Network

Bayesian network (BN), also known as Belief Network, is a directed acyclic graph (DAG) with probability annotated [17]. Considering a finite discrete random variable sets $U = \{V_1, V_2, ..., V_k\}$, wherein, each variable V_i can take a finite number of values. Formally, a Bayesian network is a two-tuple $S = <G, P>$. Of these, the first part G is a directed acyclic graph where the nodes mirror the random variables V_1, V_2, ..., V_k, and directed arch shows conditional dependencies between variables. It contains the following conditional independence assumption: given set of parent nodes, each variable is independent on its non-descendant nodes. The second part P is a set of parameters that describe the network distribution of the conditional probability and quantitatively represents dependence of each node with its parent nodes. Node without the parent node is called the root node; node without descendants is called a leaf node. [18]

Based on the probability multiplication formula:

$$p(V_1,V_2,...,V_k)=\prod_{i=1}^{k} p(V_i|V_1,V_2,...,V_{i-1}) \qquad (1)$$

If $A(V_i)$ represents any subset of nodes which non-V_i descendant nodes constitute, $Pa(V_i)$ represents direct parent nodes, then the assumption based on the conditional independence is:

$$p(V_i|A(V_i), P_a(V_i))=p(V_i|P_a(V_i)) \qquad (2)$$

Variable U joint probability distribution Bayesian network described can eventually be determined by the following formula only:

$$p(U)=p(V_1,V_2,...,V_k)=\prod_{i=1}^{k} p(V_i|P_a(V_i)) \qquad (3)$$

Bayesian network construction steps are as follows:

a) Determine the corresponding relationship between variables with each node of the network.

b) Constructing a directed acyclic graph indicating conditional independence.

c) Determine conditional probability distribution parameters of nodes.

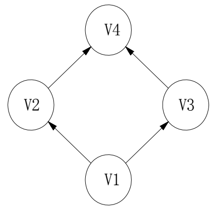


Figure 1. Bayesian network model (omitting the CPT)

Figure 1 is a simple yet typical Bayesian network model, the joint probability of the variables in the model can be deduced by equation (3):

$$\begin{aligned} & p(V_1,V_2,V_3,V_4) \\ & = \prod_{i=1}^{4} p(V_i|P_a(V_i)) \\ & = p(V_4|V_2,V_3)\,p(V_3|V_1)\,p(V_2|V_1)\,p(V_1) \qquad (4) \end{aligned}$$

Bayesian network inference is realized by computing the probability of certain conditions, including the joint probability, the marginal probability as well as the calculation of each conditional probability.

Let the Evidence set of variables be E, Query be Q, under the condition of given evidence variable values $E = e$, Bayesian network inference calculates conditional probability distribution of the query variable Q. It can be described as follows:

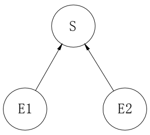
$$\begin{aligned} & p(Q \mid E = e) \\ & = \sum_{U-E} p(V_1, V_2, \ldots, V_k) \\ & = \sum_{U-E} \prod_{i=1}^{k} p(V_i \mid Pa(V_i)) \qquad (5) \end{aligned}$$

III. MULTI-STATE SYSTEM MODEL BASED ON BAYESIAN NETWORK

Here we use two simple examples to illustrate the constructing process of the multi-state system model of Bayesian network.

A. Series system

The system consists of two three-state components in series. In terms of system and component states as defined above, there are three states 0, 1, 2. P represents the probability of the system or component. The E_1, E_2 represent the states of the two components, S represents state of the system. In the BN a priori probability of the three states is given to the components E_1, E_2 respectively, with the conditional probability distribution table we analyze states of system nodes (under the conditions of different states of the two components E_1, E_2), as shown in Figure 2. Thus, multi-state components relationship, once being difficult to express, will be able to be clearly and easily described.



E1	E2	S	P(S=1|E1, E2)	P(S=2|E1, E2)
0	0	0	0	0
0	1	1	1	0
0	2	2	0	1
1	0	1	1	0
1	1	1	1	0
1	2	1	1	0
2	0	2	0	1
2	1	1	1	0
2	2	2	0	1

Figure 2. series system BN model of two three-state components

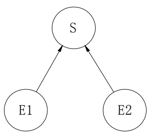
We use Bucket elimination algorithm to calculate the probability:

$$P(S)=\sum_{E_1,E_2}P(E_1,E_2,S)$$
$$=\sum_{E_1,E_2}P(S\mid E_1,E_2)P(E_1)P(E_2)$$
$$=\sum_{E_1}P(E_1)\sum_{E_2}P(S\mid E_1,E_2)P(E_2) \qquad (6)$$

According to the prior probabilities and the conditional probability table, we can easily calculate the probability of the series system.

B. parallel system

Similarly, when the system consists of two three-state components in parallel, it is the BN model shown in Figure 3.



E1	E2	S	P(S=1|E1, E2)	P(S=2|E1, E2)
0	0	0	0	0
0	1	0	0	0
0	2	0	0	0
1	0	0	0	0
1	1	1	1	0
1	2	2	0	1
2	0	0	0	0
2	1	2	0	1
2	2	2	0	1

Figure 3. two three-state components in parallel systems BN model

Also available:

$$P(S)=\sum_{E_1,E_2}P(E_1,E_2,S)$$
$$=\sum_{E_1,E_2}P(S\mid E_1,E_2)P(E_1)P(E_2)$$
$$=\sum_{E_1}P(E_1)\sum_{E_2}P(S\mid E_1,E_2)P(E_2) \qquad (7)$$

According to the prior probabilities and the conditional probability table, we can easily calculate the probability of the parallel system $P(S)$.

This shows that the form that series and parallel systems BN model calculates the probability formula is the same, the difference lies in the conditional probability tables.

If we want to know the states of other nodes (query Q), in the case of the known information (evidence e) for example, $P(S=1|E_1=1)$ or $P(E_2=1|\ S=1)$ can calculate the probability through equation (6). $P(S=1|E_1=1)$ calculation is causal reasoning, $P(E_2=1|S=1)$ calculation is diagnostic reasoning.

Of course, we can also calculate $P(S=1|E_1=2)$ or $P(E_2=2|S=1)$. So by the BN model, we are able to figure out how the components in different states affect the system under different states, and in different states of the system which components impact greater on the system. Through this analysis, we can predict and diagnose system state to conduct a comprehensive assessment of the reliability of the system. This is what traditional reliability theory cannot provide.

IV. Case Study

Li/MnO_2 cell production line is a high-speed automated production line which assembles negative shell, positive shell, lithium-chip and manganese chip steeply through PLC controlling cylinder, hydraulic cylinder and motors and other actuators. The main task of the production line is: successively add lithium chip, separator paper, and positive manganese chip soaked in the electrolyte into negative shell, and supplement positive chip with electrolyte. In the premise of ensuring electrolyte completely infiltrated, we stamp with the positive casing, sealing processes.

According to the characteristics of the system, the system is divided into five subsystems: they are add lithium chip into negative shell subsystem, separator paper into the shell subsystem, add manganese chip subsystem, the positive steel shell assembly subsystem, sealing subsystem, etc. We selected the add manganese chip subsystem to be analyzed.

To facilitate analysis, we build the fault tree model to represent the relationship between the events. Add manganese chip subsystem fault tree model as shown in Figure 4, *TE* is add manganese chip subsystem failure, *G1* is cylinder failure, *G2* is manganese chip positioning failure, *X1* is solenoid valve failure, *X2* is negative shell stuck cylinder, *X3* is insufficient air pressure, *X4* is the feeding device positioning error, *X5* is manganese chip defects, *X6* is negative shell deformation, *X7* is sensor failure.

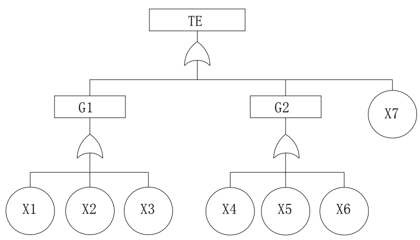


Figure 4. add manganese chip subsystem fault tree model

Then, we follow the steps of constructing the Bayesian network to construct multi-state add manganese chip subsystem BN model.

Production equipments at run-time will often encounter with the following problems: the component failure rate is higher, resulting in short equipment downtime, but it did not cause any impact on the performance of production equipment or the quality of product; component failure rate is very low, caused a great loss, resulting in the system shutdown for a long time, also taking a greater impact on the performance of production equipment or the quality of product. Therefore, the effective modeling will play an important role in a thorough analysis of the reliability of the system. So we have built a multi-state system model based on BN.

We define the following: The system has three states 0, 1, 2. 0 state is the normal working condition; 1 state is Class *A* failure (major fault) state, which leads to the prolonged failure of its system (physical damage to the device); 2 state is Class *B* failure (minor fault) state, i.e., causing system a short time lapse (external interference, poor contact and other reasons).

Provided the system is in the normal working state of probability P_0, the class *A* fault state probability is P_1, Class *B* fault state probability is P_2.

$$P_0 + P_1 + P_2 = 1 \tag{8}$$

In the add manganese chip subsystem, WORK is normal work state, FAUTL1 is fault state of Class *A*, FAULT2 is Class *B* fault state. According to Bayesian network constructing steps we construct a relative Bayesian network model (omitting CPT) in Figure 5.

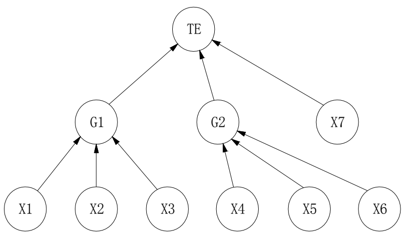


Figure 5. add manganese chip subsystem of multi-state BN model

In the statistical analysis of the fault occurred in the system running, we draw a priori probability of each event occurrence as in Table 1:

TABLE I. NODE PRIOR PROBABILITY TABLE

code	probability of state 0	probability of state 1	probability of state 2
X1	99.71%	0.06%	0.23%
X2	99.52%	0.00%	0.48%
X3	99.69%	0.00%	0.31%
X4	99.43%	0.00%	0.57%
X5	99.88%	0.12%	0.00%
X6	99.81%	0.19%	0.00%
X7	99.97%	0.01%	0.02%

In add manganese chip subsystem BN model, according to the priori probability of *X1* to *X7* and node *G1*, *G2* and CPT of the *TE* by applying accurate reasoning Bucket elimination algorithm to calculate the probability, we figure out the probability of the node *TE* is *P* (*TE* = FAULT1) = 0.38%, *P*(*TE* = FAULT2) = 1.59%, and the probability of the node *G1* and *G2*, as shown in Figure 6. Thus, in the case of no evidence, we can predict the probability of node *TE*, *G1* and *G2*.

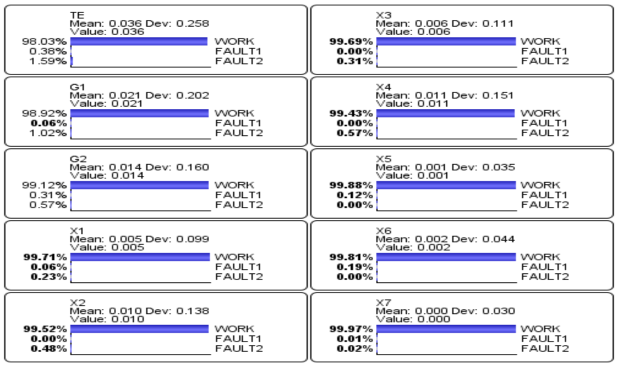


Figure 6. Probability of node TE, G1 and G2 without evidence

Similarly, we can deduce the posterior probability of each node, when the *TE* = FALUT1 and *TE* = FALUT2, as is shown in Figure 7, Figure 8.

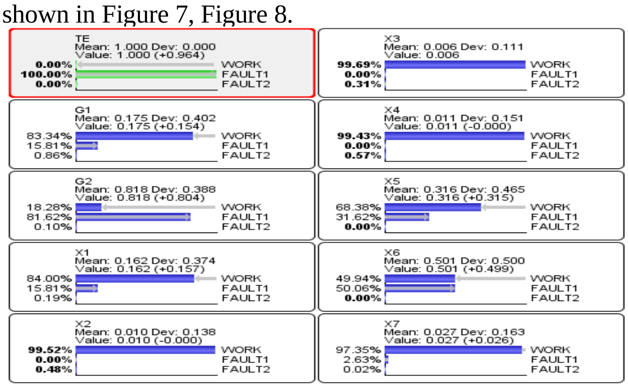


Figure 7. Posterior probability of each node when TE = FALUT1

Thus, in the case of given evidence (*TE* = FALUT1 or *TE* = FALUT2), we can speculate the probability of the other nodes. *X1* to *X7* conditional probability in turn can be calculated, according to the probability, the weak link of the system can be judged out, and so we can identify the main reason that affects the reliability of the system.

Through analysis when system is in FAULT1 state, the most influential factor is the *X6*. When the system is in FAULT2 state, the greatest impact factor is *X4*. As FAULT1 state is re-fault state, the impact on the system is bigger, so the *X6* are particularly being concerned. In the design and maintenance of the system it is necessary to take more effective measures to improve the reliability of the system.

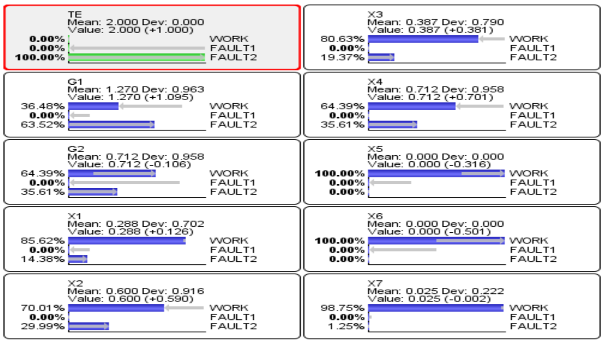


Figure 8. Posterior probability of each node when TE = FALUT2

V. CONCLUSIONS

The advantages of the Bayesian network in a multi-state system reliability analysis are obvious: a flexible modeling framework, strict mathematical formula derivation, the precise reasoning, strong uncertainty and multi-state skills, detailed and comprehensive analysis and so on. Bayesian network (BN) is ideal for multi-state system reliability analysis. It can intuitively and clearly express relationship between the multi-state systems and components, identify the weak links of the system, finally realize the analysis of the system reliability. It provides a very promising path to a complex multi-state system reliability analysis and modeling.

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REFERENCES

[1] Huang X. "Fault tree analysis method of a system having components of multiple failure modes," Microelectronics and Reliability, vol.23, no. 2, pp. 325-328, 1983.

[2] Huang X. "The generic method of the multistate fault tree analysis," Microelectronics Reliability, vol.24,no.4, pp. 617-622,1984.

[3] Bossche A. "The top-event failure frequency for non-coherent multistate fault trees," Microelectronics and Reliability, vol.24,no.4, pp. 707-715,1984.

[4] Yi-Kuei Lin, John Yuan. "A new algorithm to generate d-minimal paths in a multistate flow network with non integer arc capacities," International Journal of Reliability, Quality and Safety Engineering, vol. 05,no. 03, pp. 269-285, September 1998.

[5] Ramirez-Marquez J E, Coit D W, Tortorella M. "Multi-state two-terminal reliability: a generalized cut-set approach," Rutgers University IE Working Paper, 03-135,2003.

[6] Zaitseva E, Levashenko V, Matiasko K, et al. "Dynamic reliability indices for k-out-of-n multi-state system," The 35th International Symposium on Multiple-Valued Logic, pp.264-269, 2005.

[7] Zaitseva E, Levashenko V. "Dynamic reliability indices for parallel, series and k-out-of-n multi-state systems," The Annual Reliability and Maintainability Symposium (RAMS 2006), California, USA, pp.253-259, 2006.

[8] Levitin G. "Reliability of multi-state systems with two failure-modes," IEEE Transactions on Reliability,vol.52, no.3, pp.340-348, September 2003.

[9] Helge Langseth, Luigi Portinale. "Bayesian networks in reliability," Reliability Engineering System Safety,vol.92, no.1, pp.92-108, January 2007.

[10] X. W. Yin, W. X. Qian, and L. Y. Xie. "Multi-state system reliability modeling and assessment based on bayesian networks," Chinese Journal of Mechanical Engineering, vol.45, no.2, pp.206-212,2009.

[11] Alyson G. Wilson,Aparna V. Huzurbazar. "Bayesian networks for multilevel system reliability," Reliability Engineering and System Safety, vol. 92, no. 10, pp.1413–1420 ,October 2007.

[12] Nima Khakzada, Faisal Khana, Paul Amyotteb."Safety analysis in process facilities: Comparison of fault tree and Bayesian network approaches," vol. 96, no. 8, pp. 925–932, August 2011.

[13] Baoping Cai,Yonghong Liu. Zengkai Liu, Xiaojie Tian, Xin Dong, Shilin Yu. "Using Bayesian networks in reliability evaluation for subsea blowout preventer control system," Reliability Engineering and System Safety, vol.108, pp. 32-41,2012.

[14] Marcelo Ramos Martins, Marcos Coelho Maturana. "Application of Bayesian Belief networks to the human reliability analysis of an oil tanker operation focusing on collision accidents," Reliability Engineering and System Safety, vol.110, pp. 89-109,2013.

[15] R. Barlow , A. S. Wu, "Coherent systems with multi-state elements," Journal Mathematics of Operations Research, vol. 3,pp. 275-281, 1978.

[16] A. Lisnianski, G. Levitin, "Multi-state System Reliability, Assessment, Optimization and Application," Singapore: World Scientific Publishing, 2003.

[17] Jensen FV, Nielsen TD. "Bayesian networks and decision graphs," 2nd ed, New York, Springer, 2007.

[18] Darwiche A. "Modeling and reasoning with Bayesian networks," New York, Cambridge University Press, 2009.