An Improved Channel Estimation Method in OFDM System

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***Abstract*—In this paper, we will focus on channel estimation (CE) in orthogonal frequency-division multiplexing (OFDM) systems. The time-varying (TV) channels are modeled by a basis expansion model (BEM). Due to the time-variation, the channel matrix in the frequency domain is no longer diagonal, but approximately banded. We use a pilot-aided algorithm for estimation of rapidly varying wireless channels in OFDM systems. The performs is good when the channels vary on the scale of a single OFDM symbol duration, which occurs in mobile communication scenarios such as WiMAX, WAVE, and DVB-T. We recover Fourier coefficients of the channel taps by the pilot information. We then estimate the BEM coefficients of the channel taps from their respective Fourier coefficients using a recently developed inverse reconstruction method. We compare some BEM models in inverse methods to find out the best ones in certain conditions.**

Keywords-OFDM, channel estimation, Basis Expansion Model (BEM), doubly selective, inverse reconstruction method

I. INTRODUCTION

A. Motivation

OFDM is a multicarrier modulation technique with several advantages, e.g., high spectral efficiency and robustness against multipath propagation. OFDM is increasingly used in high-mobility wireless communication systems, such as the mobile WiMAX, WAVE, and DVB-T. OFDM over doubly selective channels are affected by intercarrier interference (ICI), which is caused by Doppler effects and carrier frequency offset. In the case of scalable OFDM, the required bandwidth grows with the number of subcarriers. Increasing the bandwidth means higher sampling frequency, which in turn proportionally increases the number of resolvable discrete multipath. For example, Mobile WiMAX with K subcarriers typically exhibits a discrete path delay of $K/8$ [X1].Moreover, OFDM is a likely candidate for future aeronautical communication systems, see [X2]. In such applications, substantial relative Doppler shifts are possible. A large number of channel taps in the above cases makes accurate CE much hard.

B .Previous Work

In the case of doubly-selective channels, the channel taps change with time. BEM is commonly used to model doubly selective channels, see [X3], [X4], [X5], [X6].The BEM approximates the channel taps by linear combinations of prescribed basis functions. In this approach, channel estimation reduces to estimation of the basis coefficients of the channel taps. Some bases have been used for modeling doubly selective channels. The BEM with complex exponential (CE-BEM) [X7], [X8], which uses a truncated Fourier series, can recover a banded frequency-domain channel matrix. Unfortunately, the CE-BEM resulting the Gibbs phenomenon and is not sufficiently accurate for doubly selective channels, see [X4]. The polynomial BEM (P-BEM) is presented in [X10]. Definitive references on pilot-aided transmission in doubly-selective channels are [X11], [X3].the BEM with discrete prolate spheroidal sequences (DPSSs) is presented in [X4].The papers [X13] and [X14] focus on channel estimation in extreme regimes, where the channel taps noticeably fluctuate within a single OFDM symbol duration. For a channel with L taps, the method presented in [X13] requires $O(L^2)$ flops and memory to estimate the BEM coefficients. The method of [X14] requires only $O(L\log L)$ operations and $O(L)$ memory for the same task.

C. Contributions

In [X14], the authors present a pilot-aided method to compute the Fourier coefficients of the channel taps, which requires $O(L)$ operations and $O(L\log L)$ memory. In [X15], the author uses the inverse method to estimate the PSWFs-BEM coefficients from the estimated Fourier coefficients and reconstruct the channel taps. In this paper we use the inverse methods[X15],[X16] to estimate the DCT-BEM, P-BEM, DPSS-BEM coefficients from the estimated Fourier coefficients and reconstruct the channel taps. We compute the MSE in computer simulation to find out the best one in particularly conditions.

The paper is organized as follows. In Section II, we describe our OFDM transmission model, introduce the method of [X14] for estimation of the Fourier coefficients of the channel taps and the basis expansion model of the wireless channel. In Section III, we introduce the inverse methods[X15][X16] and DCT-BEM, P-BEM, DPSS-BEM. Section IV estimate the DCT-BEM, P-BEM, DPSS-BEM coefficients from the estimated Fourier coefficients and reconstruct the channel taps. We present our simulation results in Section V and our conclusion in Section VI.

III. SYSTEM MODEL

A. Transmitter-Receiver Model

We consider a baseband single-antenna CP-OFDM

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system with K subcarriers. The cyclic prefix is used in every OFDM symbol. The length of cyclic prefix is l_{cp}. We choose $l_{cp}T_s$ exceeds the channel's maximum delay to avoid inter symbol interference (ISI). We will deal with one OFDM symbol at a time. Each subcarrier is used to transmit one symbol $X[k]$ $(k = 0, \ldots, K-1)$ from a finite symbol constellation e.g. 4QAM. The OFDM modulator uses the inverse discrete Fourier transform (IDFT) of size K to map the frequency-domain symbols $X[k]$ to the time-domain signal $x[n]$:

$$x[n] = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X[k] e^{j2\pi \frac{nk}{K}}, \quad n = -L_{cp}, \ldots, K-1, \tag{1}$$

The received signal is

$$y[n] = \sum_{l=0}^{L-1} h_l[n] x[n-l] + \omega[n], \quad n = 0, \ldots, K-1, \tag{2}$$

CP has been removed in $y[n]$, $\omega[n]$ denotes complex additive noise of variance N_0, $h_l[n]$ is the complex channel tap with the delay l, and L is the maximum discrete-time channel length. Thus, the channel's maximum delay is $(L-1)T_s$. We make the worst case assumption where $L = L_{cp}$. The OFDM demodulator performs a discrete Fourier transform (DFT) of size K to gain the frequency-domain receive signal $Y[k]$,

$$Y[k] = \frac{1}{\sqrt{K}} \sum_{n=0}^{K-1} y[n] e^{-j2\pi \frac{nk}{K}}. \tag{3}$$

Combining (2) with (3), we get

$$Y[k] = \sum_{l=0}^{L-1} (H_l * X_l)[k] + W[k], \tag{4}$$

where $*$ denotes the cyclic convolution of length K, and $k = 0, \ldots, K-1$. In this expression, the $Y[k]$, $H_l[k]$, $X_l[k]$, and $W[k]$ respectively denote the DFTs of $y[n]$, $h_l[n]$, $x[n-l]$, and $w[n]$.

we can get:

$$X_l[k] = e^{-j2\pi \frac{nk}{K}} X[k] \tag{5}$$

for $k = 0, \ldots, K-1$ and $l = 0, \ldots, L-1$

And we can also get:

$$H_l[k] = \frac{1}{\sqrt{K}} \sum_{n=0}^{K-1} h_l[n] e^{-j2\pi \frac{nk}{K}} \tag{6}$$

are the Fourier coefficients of the channel tap with delay l.

We can obtain the channel taps with their D term Fourier series approximately, with a fixed positive integer D as follow:

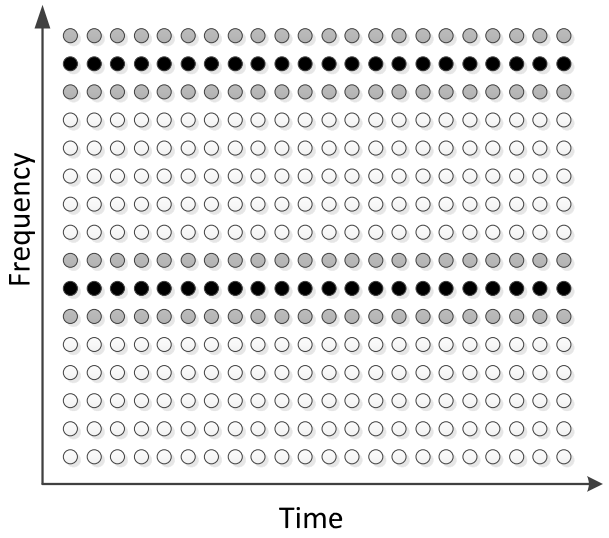
$$h_l[n] \approx \sum_{d=D^-}^{D^+} H_l[d] e^{j2\pi \frac{dn}{K}} \tag{7}$$

where $D^- = -\lfloor (D-1)/2 \rfloor$, $D^+ = \lfloor D/2 \rfloor$ ($\lfloor \bullet \rfloor$ denotes the floor operation).We can know, $D^- \le D^+$, and $D^+ - D^- = D - 1$. We set $H_l[-d] = H_l[K-d]$, For a negative index $-d$. $H_l[K-d]$ is defined in (5).

Combining (2), (4), (6) and (7), we obtain

$$Y[k] = \sum_{d=D^-}^{D^+} X[k-d] \sum_{l=0}^{L-1} H_l[d] e^{-j2\pi \frac{l(k-d)}{K}} + \tilde{W}[k] \tag{8}$$

B. Pilot Set



an example of FDKD pilot arrangement with K=16,L=2,D=2;'○' represents data symbols,'◍' represents zero pilot symbols,'●' represents non-zero pilot symbols

We set that $I = K/L$ is an integer, which can easily achieved by the choice of L. Typically, K and L are integer powers of 2. We use an FDKD pilot arrangement scheme. In each OFDM symbol, we distribute L pilot groups in one OFDM symbol and the groups are uniformly spaced with size $2D-1$. It is only possible if $2D-1 \le I$. The location of the first pilot subcarrier is $k_0, 0 \le k_0 \le I - (2D-1)$, and the pilot location

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is formed as follow:

$$k_0 + q + iI \tag{9}$$

where $q = 0, \ldots, 2D-2, i = 0, \ldots, L-1$.

Fig. 2 is an example of FDKD arrangement. In each group, all the pilot values are zero except for the central pilot, which is a_0. Under such an arrangement, we can find

$$X[k_0 + D - 1 + iI] = a_0, i = 0, \ldots, L-1$$

which means only L symbols carry non-zero pilots.

C. Estimation of Fourier Coefficients

An accurate, pilot-aided, FFT-based estimation method for the Fourier coefficients of the channel taps is presented in [X14].

At the receiver end, we can estimate the Fourier coefficients from the received signal and the known pilots

$$\hat{H}_I[d] = \frac{1}{a_0\sqrt{L}} e^{j2\pi \frac{I(k_0+D-1)}{K} \tilde{y}(d-D^-)[I]}, \tag{10}$$

where

$$\tilde{y}_d[I] = \frac{1}{\sqrt{L}} \sum_{i=0}^{L-1} \tilde{Y}d[i] e^{-j2\pi \frac{iI}{L}}$$

$$= a_0\sqrt{L} H_I[d + D^-] e^{-j2\pi \frac{I(k_0+D-1)}{K}} + \tilde{\omega}_d[I] \tag{11}$$

$$\tilde{Y}_d[i] = Y[k_0 + D^+ + d + iI] \tag{12}$$

Reconstruction of the channel taps from the estimated Fourier coefficients corresponds to the CE-BEM [X7]. However, because of the Gibbs phenomenon, the CE-BEM is not accurate for estimation of doubly selective channels. In the next section, we describe the inverse method for the computation of the BEM coefficients from the estimated Fourier coefficients of the channel taps. Particularly, we apply different types of BEM to this method.

III. INVERSE RECONSTRUCTION METHOD WITH DIFFERENT BEMS

The inverse reconstruction method [X16] is to reconstruct a function as a linear combination of given basis functions from a finite number of Fourier coefficients of this function.

If we have a function $f(x)$ defined on $[-1,1]$, we consider the first D Fourier coefficients of $f(x)$ are $\hat{f}(d)$:

$$\hat{f}(d) = \int_{-1}^{1} e^{-jd\pi x} f(x) dx. \quad D^- \le d \le D^+ \tag{13}$$

We approximately reconstruct the function $f(x)$ as a linear combination of fixed $M \le D$ basis functions $\Phi_0, \ldots, \Phi_{M-1}$

$$f \approx f_M(t) = \sum_{m=0}^{M-1} a_m \Phi_m(t) \tag{14}$$

We need to find a_m which minimizes the norm of the difference between the D lowest Fourier coefficients of f and f_M

$$\left(\sum_{D^- \le d \le D^+} \left| \hat{f}(d) - \hat{f}_M(d) \right|^2 \right)^{\frac{1}{2}} \tag{15}$$

where $\hat{f}_M(d)$ is the Kth Fourier coefficient of f_M

The minimization problem in the expression (14) is converted to the following over-determined least squares problem

$$\min_{a \in C^M} \left\| \mathbf{P}\mathbf{a} - \hat{\mathbf{f}} \right\| \tag{16}$$

where

$\mathbf{a} = [a_0, \ldots, a_{M-1}]^T$, $\hat{\mathbf{f}} = [\hat{f}(D^-), \ldots, \hat{f}(D^+)]$, and $\mathbf{P}$ is the $D \times M$ matrix whose entries are the respective Fourier coefficients of the basis functions

$$P_{dm} = \hat{\Phi}_m(d), \tag{17}$$

$m = 0, \ldots, M-1, d = D^-, \ldots, D^+$. If $\mathbf{P}$ has full rank, then (15) has a unique solution given by

$$\mathbf{a} = \mathbf{P}^+ \hat{\mathbf{f}} \tag{18}$$

where $\mathbf{P}^+$ is the Moore–Penrose pseudoinverse of the matrix $\mathbf{P}$.

We note that the inverse reconstruction method can be used to compute approximate expansion coefficients of a function with respect to an arbitrary basis from the Fourier coefficients of the function if only we know the matrix whose entries are the respective arbitrary basis functions. Next we will introduce some basis functions.

A. BEM With the Legendre Polynomials

Legendre Polynomials basis functions is

$$p_m[n] = p_m(nT_s)$$

$$p_m(t) = P_m\left(\frac{2t}{KT_s} - 1\right), \quad 0 \le t \le KT_s$$

According to [a27, Sec. 22.11.5], we can obtain

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]$$

B. BEM with the Discrete Cosine Transform (DCT)

Matrix whose entries are the respective *DCT-BEM* is:

$$u_m(n) = \begin{cases} \sqrt{1/N} & m = 0 \\ \sqrt{2/N} \cos[\pi m/N)(n+1/2)] & 0 < m < M \end{cases}$$

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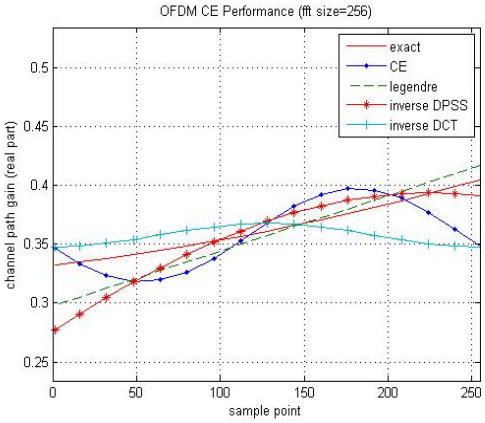
C. BEM With the discrete prolate spheroidal sequences

The discrete prolate spheroidal sequences are optimum waveforms in many communication and signal processing applications because they comprise the most spectral efficient set of orthogonal sequences possible. Generation of the sequences has proven to be difficult in the past due to the absence of a closed form solution. Method of easily generating any single discrete prolate spheroidal sequence is presented in [X17].



IV. COMPUTER SIMULATIONS

·A SIMULATIONPARAMETERS

(a)

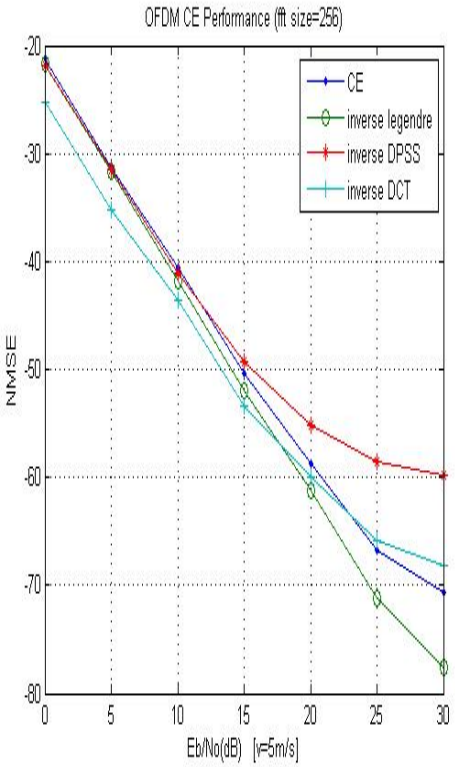
number of subcarriers (K)	256
intercarrier spacing (f_s)	10.9 kHz
bandwidth ($B = Kf_s$)	2.8 MHz
sampling time ($T_s = 1/B$)	0.357μs
cyclic prefix ($L_{cp} = K/8$)	32
symbol duration $(K + L_{cp})T_s$	102.9μs
carrier frequency (f_c)	5.8GHz
error correcting code	1/2-conv.
constellation	4QAM

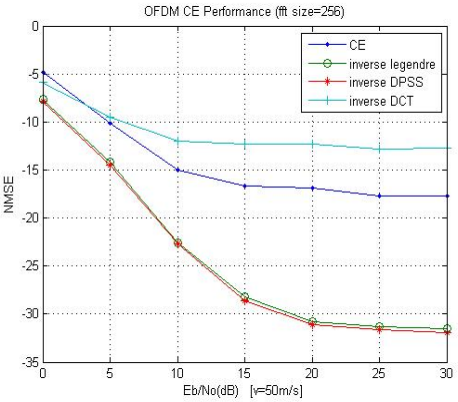
·(a) TRANSMISSION SIMULATION PARAMETERS

(b)

max. path delay	11.4μs
max. Doppler shift	16%f_s, 62%f_s
average path gain	-2 dB
fading	Rayleigh
Doppler spectrum	Jakes
E_b/N_0	$5 - 30$ dB

·(b) CHANNEL SIMULATION PARAMETERS





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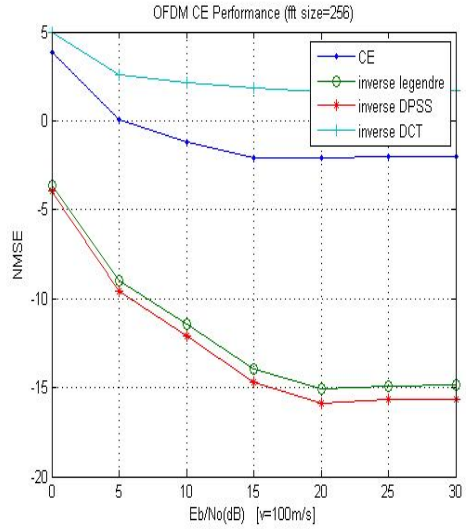

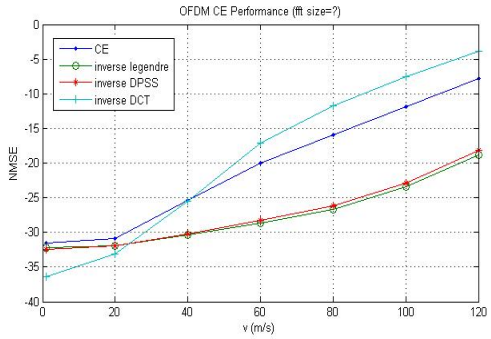

V.CONCLUSION

An accurate, pilot-aided, FFT-based estimation method for the Fourier coefficients of the channel taps [X14] is applied in this paper. We recover Fourier coefficients of the channel taps by the pilot information. We then estimate the BEM coefficients of the channel taps from their respective Fourier coefficients using a recently developed inverse reconstruction method. We apply different types of BEM to this method. We observe in our numerical simulations that the DCT-BEM works better in the low speed conditions (v<15m/s, fd<2.66%fs),the P-BEM works better in the intermediate speed conditions (15m/s <v<65m/s, 2.66%fs< fd<11.53%fs), the DPSS-BEM works better in the high speed conditions (65m/s <v, 11.53%fs< fd). The method of [X14] requires only $O(L \log L)$ operations and $O(L)$ memory to estimate the CE-BEM coefficients, which can be used to reconstruct the channel taps directly. In addition, we can use the inverse method and different types of BEM in certain conditions to reconstruct the channel taps accurately.

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