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The Influence of Quantum Field Fluctuations on Chaotic Dynamics of Yang-Mills System II. The Role of the Centrifugal Term

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Abstract

We have considered $SU(2)\bigotimes U(1)$ gauge field theory describing electroweak interactions. We have demonstrated that centrifugal term in model Hamiltonian increases the region of regular dynamics of Yang-Mills and Higgs fields system at low densities of energy. Also we have found analytically the approximate relation for critical density of energy of the order to chaos transition on centrifugal constant M. It is necessary to note that mentioned increase of the region of regular dynamics has linear dependance on the value M.

A steady interest to chaos in gauge field theories [3] is connected with the fact that all four fundamental particle interactions have chaotic solutions [7]. There are a lot of footprints of chaos in HEP [10, 13], nuclear physics (energy spacing distributions) [4, 5], quantum mechanics [8].

Much attention has been paid in the last decade to chaos in quantum field theory. Non-abelian Yang-Mills gauge fields were investigated without spontaneous symmetry breaking. It was analytically and numerically shown that classical Yang-Mills theories are inherently chaotic ones[12, 15]. The further research has shown for spatially homogeneous field configurations [1] that spontaneous symmetry breakdown leads to appearance of order-chaos transition with rise of density of energy of classical gauge fields [2, 16], whereas dynamics of gauge fields in the absence of spontaneous symmetry breakdown is chaotic at any density of energy [15].

In the work [11] it was shown that the "switching on" of quantum fluctuations of vector gauge fields leads to ordering at low densities of energy, order-to-chaos transition with the rise of density of energy of gauge fields. Also it was noted that if the ratio of the coupling constants of Yang-Mills and Higgs fields is larger than some critical value then quantum corrections do not affect the chaotic dynamics of gauge and Higgs fields.

In this paper we investigate the influence of the centrifugal term in model Hamiltonian on chaotic dynamics of Yang-Mills and Higgs gauge fields. We demonstrate numerically and analytically that centrifugal term increases the energy region of regular dynamics of this fields.

Consider $SU(2)\bigotimes U(1)$ gauge field theory which is describing electroweak interactions with real massless scalar field ρ with the Lagrangian

$$L = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{1}{8}g^2\rho^2\left(W_1^2 + W_2^2 + \frac{W_3^2}{\cos^2\theta_w}\right) + \frac{1}{2}\partial_\mu\rho\partial^\mu\rho - \frac{1}{4!}\lambda\rho^4, \quad a = 1,2,3, \quad \mu = 1,2,3,4. \tag{1}$$

We used the following denotations

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\varepsilon^{abc}W^b_\mu W^c_\nu; \tag{2}$$

$$H_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{3}$$

Here W^1_μ, W^2_μ describe $W^{\pm}$-bosons and W^3_μ - neutral Z-boson, A_μ corresponds electromagnetic field, g denotes a self-coupling constant of non-abelian gauge fields, λ - self-coupling constant of scalar field, θ_w is Weinberg angle.

To study the dynamics of classical gauge fields from the viewpoint of chaos we simplify the problem using spatially homogeneous solutions [1]. So we will investigate fields of the following form

$$W^a_i = e^a_i q_a(\tau), \quad a = 1,2; \quad i = 1,2,3; \quad \overrightarrow{e^a} = (e^a_1, e^a_2, e^a_3), \quad \left(\overrightarrow{e^a}\right)^2 = 1. \tag{4}$$

Here $\overrightarrow{e^a}$ are constant unit vectors which is describing the linear polarization of the gauge fields $\overrightarrow{W^a}$. Also we use simplification:

$$\overrightarrow{e^1}\overrightarrow{e^2} = 1, \tag{5}$$

which means that the fields of W^+ and W^- gauge bosons have the same linear polarization. Other classical gauge fields for simplicity are put to be equal to zero

It was shown that the dynamics of classical gauge fields in the classical vacuum of scalar field $\rho = 0$ is chaotic at any densities of energy [15]. Situation qualitatively changes if we take into account quantum fluctuations of vector fields. It is caused by the known fact that the state $<\rho>= 0$ is not a vacuum state in this case. Here $<\rho>$ denotes vacuum quantum expectation value of scalar field related with its classical vacuum value ρ as follows $<\rho>=\rho + quantum \quad corrections$. To find a true vacuum of scalar field in this case we use the method of the effective potential [6].

One loop effective potential generated by the Lagrangian (1) has the form (see also [11], [9])

$$U(<\rho>) = \frac{1}{4!}\lambda <\rho>^4 + \frac{3g^4<\rho>^4}{128\pi^2}\left(-\frac{1}{2} + \ln\frac{g^2<\rho>^2}{2\mu^2}\right) + \frac{3g^4<\rho>^4}{256\pi^2\cos^4\theta_w}\left(-\frac{1}{2} + \ln\frac{g^2<\rho>^2}{2\mu^2\cos^2\theta_w}\right). \tag{6}$$

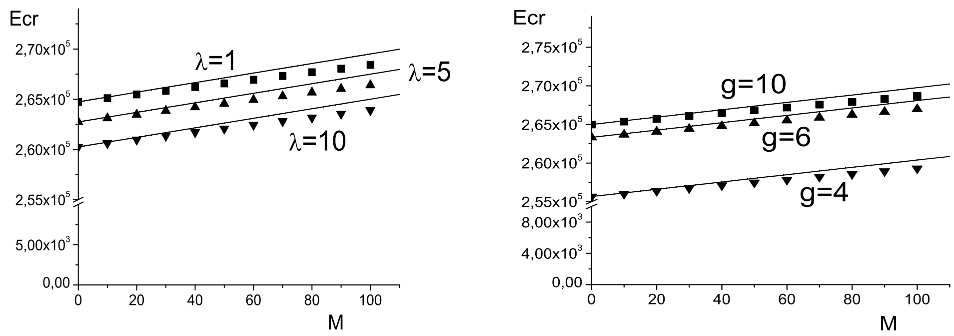

Figure 1. Relation between critical density of energy and centrifugal constant M at various values of self-coupling constants of scalar field(λ) and non-abelian gauge fields(g).

Where μ^2 is a renormalization constant. Here we took into account contributions of all Feynman diagrams with one loop of any (W_1, W_2, W_3 or A) gauge field and external lines of Higgs field. This potential leads to spontaneous symmetry breaking and non-zero vacuum expectation value of scalar $\rho-$field appears. Classical vacuum of scalar field is $\rho = 0$, but it is not so in quantum case, because of Coleman-Weinberg effect [6]. In simplifications (4) we put classical gauge fields $\vec{W}^3$ and $\vec{A}$ to be equal to zero, but their quantum fluctuations are included and give contribution to the potential (6).

Hamiltonian describing dynamics of the model field system has the following form

$$H = \frac{1}{2}(p_r^2 + p^2 + \frac{M^2}{r^2}) + \frac{1}{8}g^2 < \rho >^2 r^2 + U(< \rho >), \tag{7}$$

where we make the substitution $q_1 = r\cos\varphi$ and $q_2 = r\sin\varphi$. It is clarify that φ is a cyclical variable and therefore its conjugate momentum $p_\varphi = r^2\dot{\varphi} = M$ is a constant of motion. Also we used denotations $p_r^2 = \dot{q}_1^2 + \dot{q}_2^2, \quad r^2 = q_1^2 + q_2^2$. Here p_r is a momentum of gauge fields and p is a momentum of Higgs field.

In contrast to article [11] we didn't neglected here by the term M^2/r^2 and investigated the influence of the centrifugal term M^2/r^2 on the dynamics of the model field system.

Using well known technique based on Toda criterion of local instability [14] one can obtain dependence of critical density of energy of order to chaos transition (minus vacuum density of energy which is non-zero) on centrifugal constant M and coupling constants of Yang-Mills and Higgs fields. Numerical calculations allow us to build two required plots. First plot (figure 1) demonstrates dependence of critical density of energy on value M at various values of a self-coupling constant of scalar field λ(other constants have following values $\mu = 100, g = 10$). Second plot (figure 1) demonstrates dependence on value of another self-coupling constant $g(\mu = 100, \lambda = 0.495)$. Points on this plots are in accordance to obtaining numerical results.

From obtained plots we can make next conclusions. First, it is clarified that taking into account centrifugal term M^2/r^2 increases the region of regular dynamics of Yang-Mills and Higgs fields system at low densities of energy. And second, dependence of value of critical density of energy on value M is approximate linear at any prescribed values of self-coupling constants of Yang-Mills and Higgs fields. And third, it is seen that self-coupling constant g more intensively then λ influences on dynamics of considered system. It is adjust with results obtaining in work[11].

Further, basing on Toda criterion of local instability [14] and prerequisite $U\left(<\rho>\right)'' = 0$ [11] we have found analytically the approximate relation for critical density of energy in the case $M \neq 0$

$$E_{cr} = \frac{1}{\sqrt{2}}\mu \exp\left(\frac{1}{2}\alpha_w - \frac{1}{2}\frac{\lambda}{g^4}\beta_w - \frac{1}{3}\right)M + E_c. \tag{8}$$

Were we used the following denotations [11]

$$\alpha_w = \frac{2\ln\cos\theta_w}{1+2\cos^4\theta_w}, \qquad \beta_w = \frac{32\pi^2\cos^4\theta_w}{9(1+2\cos^4\theta_w)} \tag{9}$$

and value E_c describes critical density of energy in case of $M = 0$[11]

$$E_c = \frac{3\mu^4}{64\pi^2}\exp\left(2\alpha_w - \frac{2\lambda}{g^4}\beta_w\right)\left(1+\frac{1}{2\cos^4\theta_w}\right)\left(1-\frac{7}{3}e^{-\frac{4}{3}}\right). \tag{10}$$

To check-up relation (8) we have made a comparison between results obtaining with mentioned relation and acquired numerically(figure 1). Points on this plots accordance to numerical results and lines to analytical ones. From mentioned plots it is seen that errors are proportional to the value M and obtained relation (8) describes closely numerical results in a region of small value M.

In conclusion, we have considered $SU(2)\bigotimes U(1)$ gauge field theory describing electroweak interactions. We have demonstrated that centrifugal term increases the region of regular dynamics of Yang-Mills and Higgs fields system at low densities of energy. It is necessary to note that mentioned increase of the region of regular dynamics has linear dependance on the value M. The approximate expression for critical density of energy(8) was also analytically found. Obtained results show us Higgs field influences on the stability of Yang-Mills fields dynamics.

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