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Memristor-based H_∞ Synchronization Control for Fractional-order Neural Networks with Time-delays

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***Abstract*—The H_∞ synchronization control of fractional-order neural networks (FONNs) is investigated in this paper, where the FONNs are affected by time-delays. First, based on memristor, a novel adaptive control is proposed. And then, by taking the Mittag-Leffler stability theory and robust control, sufficient conditions are proposed to achieve H_∞ synchronization of FONNs. Finally, the correctness of the obtained theoretical results are verified by some numerical examples.**

***Keywords*—Memristor; H_∞ synchronization; FONNs, time-delays**

I. INTRODUCTION

The stability of systems has always been an important research topic. In reality, the stability of the systems may be affected by various of uncertainties such as time delays. Recently, robust H_∞ control methods are found have the good ability of compensating the influence of external disturbance, and some good results were reported, such as, the distributed robust H_∞ consensus control in directed networks of agents with time-delay was investigated in [1], the robust H_∞ synchronization control of complex networks with uncertainties was investigated in [2] and so on [3, 4].

Recently, notice that the memristor is similar to the neurons in the human brain, the human brain was emulated by constructing a memristor-based neural networks which use the memoristor. The concept of memristor was first proposed in [5], and then, based on memristor, various of synchronization problem of neural networks have been investigated, such as [6][7]. Furthermore, the fractional calculus is found to have many good properties, such as memory and heredity, its meaningful to make an investigation for the synchronization problem of FONNs. As far as we known, numerous of good results have been obtained, such as finite-time synchronization [8]. However, it is a pity to find that there are no research on H_∞ synchronization of FONNs with time-delays.

Therefore, we first make an investigation for the H_∞ synchronization control for FONNs with time delays based on memristor. This paper has the following contributions. First, based on meristor, a novel distributed adaptive control protocol is given. Then, sufficient conditions are proposed to achieve H_∞ synchronization control of FONNs affected by time-delays.

The rest of this paper has the following organizations. Section 2 presents several preliminaries concerning fractional-order calculus and system models. Section 3 gives the main results we obtained. Section 4 gives two numerical examples to demonstrate the correctness of the control law. Section 5 makes a conclusion.

II. PRELIMINARIES AND SYSTEM MODELS

In this section, several useful preliminaries and lemmas about fractional-order calculus, and the system models are given.

A. Fractional-order Preliminaries

Definition 1. [9] Define the Mittag-Leffler function with two-parameter as

$$E_{\mu,\gamma}(x)=\sum_{i=0}^{\infty}\frac{x^i}{\Gamma(\mu i+\gamma)}, \tag{1}$$

where $\mu>0,\gamma>0$ and $x\in C,\Gamma(z)=\int_0^{+\infty}\xi^{z-1}e^{-\xi}d\xi$. When $\gamma=1$,

$$E_{\mu,1}(x)=E_\mu(x)=\sum_{i=0}^{\infty}\frac{x^i}{\Gamma(1+\mu i)}. \tag{2}$$

Definition 2. [9] Define the Riemann-Liouvile fractional-order integral of order $\mu>0$ for $h(t)\in C^n([0,+\infty),R)$ as

$${}_0I_t^\mu h(t)=\frac{1}{\Gamma(\mu)}\int_0^t(t-\xi)^{\mu-1}h(\xi)d\xi. \tag{3}$$

Definition 3. [9] Define the Caputo's fractional-order derivative of order μ for $h(t)\in C^n([0,+\infty),R)$ as

$${}_0^CD_t^\alpha h(t)=\frac{1}{\Gamma(n-\mu)}\int_0^t(t-\xi)^{n-\mu-1}h^{(n)}(\xi)d\xi, \tag{4}$$

where $n=[\mu]+1$. Particularly, when $\mu\in(0,1)$,

$${}_0^CD_t^\alpha h(t)=\frac{1}{\Gamma(1-\alpha)}\int_0^t(t-\xi)^{-\mu}\dot{h}(\xi)d\xi. \tag{5}$$

Lemma 1. [10] Assume that $h(x)\in R^n$ continuous and derivable with respect to x. Then,

$$ {}_0D_x^{\mu}h^T(x)h(x)\le 2h^T(x){}_0D_x^{\mu}h(x), \tag{6}$$

where $\mu\in(0,1)$.

Lemma 2. [11] Suppose that $q=0$ is an equilibrium point for ${}_{t_0}D_t^{\mu}q(t)=h(q)$, $\Omega\subset R^n$ be a domain which contains the origin. Let $U(t):[0,\infty)\times\Omega\to R$ be a continuously differentiable function and satisfies the locally Lipschitz with respect to t such that

$$\upsilon_1\|q(t)\|^r\le U(t)\le\upsilon_2\|q(t)\|^{rb}, \tag{7}$$

$$ {}_0^CD_t^{\alpha}U(t)\le-\upsilon_3\|q(t)\|^{rb}, \tag{8}$$

where $t\ge 0, q\in R, \mu\in(0,1), \upsilon_1,\upsilon_2,\upsilon_3,r,b>0$. Then $q=0$ is Mittag-Leffler stable.

B. System Models

The FONNs with *n neurons and time-delays* is investigated in this paper. *The drive system is given as follows:*

$$\begin{aligned} {}_0^CD_t^{\mu}z_i(t)&=-k_iz_i(t)+\sum_{k=1}^{n}c_{ik}(z_k(t))h(z_k(t))\\ &+\sum_{k=1}^{n}d_{ik}(z_k(t))h(z_k(t-\sigma_k))+w_i(t), \end{aligned} \tag{9}$$

where $i,k=1,2,\ldots,n,k_i>0$ *is a positive constant* $z_i(t)\in R$ and $h_i(\cdot)$ respectively represent the drive state and nonlinear activation function *of the i th neuron,* $w_i(t)$ *represents the unknown external disturbances,* $\sigma_k>0$ *is the time delay,* define $c_{ik}(x_k(t))$ *and* $d_{ik}(x_k(t))$ *as* the *connection memristive weights,*with $c_{ik}(z_k(t))=\begin{cases}\hat{c}_{ik}, & |z_k(t)|>T_k,\\ \check{c}_{ik}, & |z_k(t)|\le T_k,\end{cases}$

$d_{ik}(z_k(t))=\begin{cases}\hat{d}_{ik}, & |z_k(t)|>T_k,\\ \check{d}_{ik}, & |z_k(t)|\le T_k,\end{cases}$ *and* $c_{ik}(\pm\sigma_k)=\hat{c}_{ik}$ *or* $\check{c}_{ik}$, $d_{ik}(\pm\sigma_k)=\hat{d}_{ik}$ *or* $\check{d}_{ik}$, *where switching jumps* $\sigma_k>0$, *and* $\hat{c}_{ik},\check{c}_{ik},\hat{d}_{ik},\check{d}_{ik}$ *are all constants. A*ssume that *the response system h*as the following dynamics*:*

$$\begin{aligned} {}_0^CD_t^{\mu}s_i(t)&=-k_is_i(t)+\sum_{k=1}^{n}\alpha_{ik}(s_k(t))h(s_k(t))\\ &+\sum_{k=1}^{n}\beta_{ik}(s_k(t))f(s_k(t-\sigma_k))+u_i(t), \end{aligned} \tag{10}$$

where $s_i(t)\in R$ and $u_i(t)$ is *the response state* and control input *of the i th neuron,* respectively. $\alpha_{ik}(s_k(t))$ *and* $\beta_{ik}(s_k(t))$ *are connection memristive weights, which are defined as*

$\alpha_{ik}(s_k(t))=\begin{cases}\hat{\alpha}_{ik}, & |s_k(t)|>T_k,\\ \check{\alpha}_{ik}, & |s_k(t)|\le T_k,\end{cases}$ $\beta_{ik}(s_k(t))=\begin{cases}\hat{\beta}_{ik}, & |s_k(t)|>T_k,\\ \check{\beta}_{ik}, & |s_k(t)|\le T_k,\end{cases}$ *and* $\alpha_{ik}(\pm\sigma_k)=\hat{\alpha}_{ik}$ *or* $\check{\alpha}_{ik}$, $\beta_{ik}(\pm\sigma_k)=\hat{\beta}_{ik}$ *or* $\check{\beta}_{ik}$. $\hat{\alpha}_{ik},\check{\alpha}_{ik},\hat{\beta}_{ik},\check{\beta}_{ik}$ *are all constants. The initial values of F*ONNs are $z_i(v)=\chi_i(v), s_i(v)=\delta_i(v)$ *respectively,* $v\in(-\infty,0], \chi_i(v)$, $\delta_i(v)\in C((-\infty,0],R)$. $i=1,2,\ldots,n$.

Assumption 1. There exists $L_i>0$ such that $|h_i(x)-h_i(y)|\le L_i|x-y|$.

Assumption 2. There exists a constant $\eta>0$ such that $\int_0^t\sum_{i=1}^{n}w_i^T(t)w_i(t)dt<\eta$.

Definition 4. The FONNs are said to achieve global asymptotically synchronization, if there control protocols $u_i(t)$ such that

$$\lim_{t\to\infty}|s_i(t)-z_i(t)|=0,$$

$$\forall i=1,2,\ldots,n.$$

Definition 5. [12] The error system is said to satisfy H_∞ perfo-rmance, if

$$\|T_{zw}(s)\|_\infty<\gamma=(\sum_{i=1}^{n}e_i(t)^2)/(\sum_{i=1}^{n}w_i(t)^2), \tag{11}$$

with zero initial condition, $T_{zw}(s)$ is the transfer function between the error system and external disturbances, and $\sum_{i=1}^{n}w_i(t)^2\ne 0$.

III. Main Results

Let $\varepsilon_i(t)=s_i(t)-z_i(t)$. In this section, based on the adaptive feedback control, the following novel control algorithms are proposed.

$$u_i(t)=-l_i(t)s_i(t)-\mathrm{sgn}(\varepsilon_i(t))m_i(t)|e_i(t-\sigma_i)|, \tag{12}$$

$$ {}_0^CD_t^{\mu}l_i(t)=c_i|\varepsilon_i(t)|-p_i(l_i(t)-l_i), \tag{13}$$

$$ {}_0^CD_t^{\mu}m_i(t)=n_i|\varepsilon_i(t-\sigma_i)|-q_i(m_i(t)-m_i), \tag{14}$$

where $l_i>0, c_i>0, n_i>0$ are all positive constants that will be determined in the following analysis. Then we could obtain that

$$\begin{aligned} {}_0^CD_t^{\mu}\varepsilon_i(t)&=u_i(t)-k_i\varepsilon_i(t)-w_i(t)\\ &+\sum_{k=1}^{n}[\alpha_{ik}(s_k(t))h_k(s_k(t))-c_{ik}(z_k(t))h_k(z_k(t))]\\ &+\sum_{k=1}^{n}[\beta_{ik}(s_k(t))h_k(s_k(t-\sigma_k))-d_{ik}(z_k(t))h_k(z_k(t-\sigma_k))]. \end{aligned} \tag{15}$$

Theorem 1. If the external disturbance does not exist, i.e., $w_i(t)=0$ for all neurons. The FONNs (9) and (10) can achieve globally asymptotically synchronization by taking the adaptive control algorithms (12) (13) (14), if the parameters satisfy:

$$k_i + l_i - \sum_{k=1}^{n} c_{ik}^{\rho} L_k > 0, \tag{16}$$

$$m_i - \sum_{k=1}^{n} d_{ik}^{\rho} L_k > 0. \tag{17}$$

Proof. Construct the following candidate Lyapunov function:

$$U(t) = q[\sum_{k=1}^{n} |\varepsilon_i(t)| + \sum_{k=1}^{n} \frac{1}{2c_i}(l_i(t)-l_i)^2 + \sum_{k=1}^{n} \frac{1}{2n_i}(m_i(t)-m_i)^2]. \tag{18}$$

Noted that $D^{\alpha}|\cdot| \le \mathrm{sgn}(\cdot)D^{\alpha}(\cdot)$ and according to Lemma 1, we have

$$\begin{aligned}
&{}_0^C D_t^{\mu} U(t) \\
&= q[\sum_{k=1}^{n} {}_0^C D_t^{\mu} |\mu_i(t)| + \sum_{k=1}^{n} \frac{1}{2c_i} {}_0^C D_t^{\mu} (l_i(t)-l_i)^2 + \sum_{k=1}^{n} \frac{1}{2n_i} {}_0^C D_t^{\mu} (m_i(t)-m_i)^2] \\
&\le q[\sum_{k=1}^{n} \mathrm{sgn}(\varepsilon_i(t))\{-l_i(t)\varepsilon_i(t) - \mathrm{sgn}(\varepsilon_i(t))m_i(t)|\varepsilon_i(t-\sigma_i)| \\
&- k_i\varepsilon_i(t) - w_i(t) + \sum_{k=1}^{n}[\alpha_{ik}(s_k(t))h_k(s_k(t)) - c_{ik}(s_k(t))h_k(z_k(t))] \\
&+ \sum_{k=1}^{n}[\beta_{ik}(s_k(t))h_k(s_k(t-\sigma_k)) - d_{ik}(z_k(t))h_k(z_k(t-\sigma_k))]\} \\
&+ q\sum_{k=1}^{n} \frac{1}{c_i}(l_i(t)-l_i)[c_i|\varepsilon_i(t)| - p_i(l_i(t)-l_i)] \\
&+ \sum_{k=1}^{n} \frac{1}{n_i}(m_i(t)-m_i)[n_i|\varepsilon_i(t-\sigma_i)| - q_i(m_i(t)-m_i)] \\
&\le q\sum_{k=1}^{n}\{-k_i|\varepsilon_i(t)| + \sum_{k=1}^{n} c_{ik}^{\rho} L_k|\varepsilon_k(t)| + \sum_{k=1}^{n} d_{ik}^{\rho} L_k|\varepsilon_k(t-\sigma_k)| - l_i(t)|\varepsilon_i(t)| \\
&- m_i(t)|\varepsilon_i(t-\sigma_i)| + w_i(t)\} \\
&+ q\sum_{k=1}^{n}\{l_i(t)|\varepsilon_i(t)| - l_i|\varepsilon_i(t)| - \frac{p_i}{c_i}(l_i(t)-l_i)^2\} \\
&+ q\sum_{k=1}^{n}\{m_i(t)|\varepsilon_i(t-\sigma_i)| - m_i|\varepsilon_i(t-\sigma_i)| - \frac{q_i}{n_i}(m_i(t)-m_i)^2\} \\
&= q\sum_{k=1}^{n}(-k_i - l_i + \sum_{j=1}^{n} c_{ik}^{\rho} L_k)|\varepsilon_i(t)| + q\sum_{k=1}^{n}(-m_i + \sum_{j=1}^{n} d_{ik}^{\rho} L_k)|\varepsilon_k(t-\sigma_k)| \\
&- q\frac{p_i}{c_i}(l_i(t)-l_i)^2 - q\frac{q_i}{n_i}(m_i(t)-m_i)^2 + q\sum_{k=1}^{n}|w_i(t)|.
\end{aligned} \tag{19}$$

From the conditions (16) and (17), and take $p_i = q_i = \frac{1}{2}(k_i + l_i - \sum_{k=1}^{n} c_{ik}^{\rho} L_k)$, then we have

$$\begin{aligned}
{}_0^C D_t^{\mu} U(t) \le{}& q\sum_{k=1}^{n}(-k_i - l_i + \sum_{k=1}^{n} c_{ik}^{\rho} L_k)|\varepsilon_i(t)| + \frac{1}{2c_i}(l_i(t)-l_i)^2 \\
&+ \frac{1}{2n_i}(m_i(t)-m_i)^2 + q\sum_{k=1}^{n}|w_i(t)|
\end{aligned}. \tag{20}$$

Take $\lambda_i = k_i + l_i - \sum_{k=1}^{n} c_{ik}^{\rho} L_k > 0, \underline{\lambda} = \min\{\lambda_1, \lambda_2, ..., \lambda_n\}$. Then

$$\begin{aligned}
{}_0^C D_t^{\mu} U(t) \le{}& -\underline{\lambda} q\sum_{k=1}^{n}|\varepsilon_i(t)| + \frac{1}{2c_i}(l_i(t)-l_i)^2 + \frac{1}{2n_i}(m_i(t)-m_i)^2 \\
&+ q\sum_{k=1}^{n}|w_i(t)|.
\end{aligned} \tag{21}$$

When $w_i(t)=0$, we have ${}_0^C D_t^{\mu} U(t) \le -\underline{\lambda} U(t)$.

Therefore, according to Lemma 2, the synchronization of the FONNs is achieved.

Theorem 2. Consider the FONNs (9) and (10). Given $\gamma > 0$, then the H_∞ synchronization of the FONNs can be achieved by taking the adaptive control algorithms (12) (13) (14).

Proof. When $w_i(t) \ne 0$, from Theorem 1, we obtain ${}_0^C D_t^{\mu} U(t) \le -\underline{\lambda} U(t) + q\sum_{k=1}^{n}|w_i(t)|$. Under the initial values $e_i(t_0)=0, l_i(t_0)=l_{i0}, m_i(t_0)=m_{i0}$, where $l_{i0}>0, m_{i0}>0$ are all positive constants, and define $z_i(t)=\varepsilon_i(t)$, then we have

$$\begin{aligned}
&{}_0 I_t^{\mu}[{}_0^C D_t^{\mu} U(t) + \sum_{k=1}^{n}|z_i(t)| - \gamma\sum_{k=1}^{n}|w_i(t)|] \\
&\le {}_0 I_t^{\mu}[-\underline{\lambda} q\sum_{k=1}^{n}|\varepsilon_i(t)| - \underline{\lambda} q\sum_{k=1}^{n}\frac{1}{2c_i}(l_i(t)-l_i)^2 - \underline{\lambda} q\sum_{k=1}^{n}\frac{1}{2n_i}(m_i(t)-m_i)^2 \\
&+ q\sum_{k=1}^{n}|w_i(t)| + \sum_{k=1}^{n}|z_i(t)| - \gamma\sum_{k=1}^{n}|w_i(t)|] \\
&= {}_0 I_t^{\alpha}[-(\underline{\lambda} q - 1)\sum_{k=1}^{n}|\varepsilon_i(t)| - \underline{\lambda} q\sum_{k=1}^{n}\frac{1}{2c_i}(l_i(t)-l_i)^2 \\
&- \underline{\lambda} q\sum_{k=1}^{n}\frac{1}{2n_i}(m_i(t)-m_i)^2 - (\gamma - q)\sum_{k=1}^{n}|w_i(t)|].
\end{aligned} \tag{22}$$

If $\underline{\lambda} q - 1 > 0, \gamma - q > 0$, then we have

$${}_0 I_t^{\mu}[{}_0^C D_t^{\mu} U(t) + \sum_{k=1}^{n}|z_i(t)| - \gamma\sum_{k=1}^{n}|w_i(t)|] \le 0. \tag{23}$$

Noted that $U(0)=0$, we have

$$U(t) - U(0) + {}_0 I_t^{\mu}(\sum_{k=1}^{n}|z_i(t)| - \gamma\sum_{k=1}^{n}|w_i(t)|) \le 0. \tag{24}$$

Then we have

$$\sum_{k=1}^{n} {}_0I_t^{\mu} \,|\, z_i(t) \,| \le \gamma \sum_{k=1}^{n} {}_0I_t^{\mu} \,|\, w_i(t) \,|. \tag{25}$$

Therefore, we could arrive

$$\alpha\tau \sum_{k=1}^{n} \int_0^t |\, z_i(\xi) \,|\, d\xi \le \gamma \sum_{k=1}^{n} \int_0^t |\, w_i(\xi) \,|\, d\xi. \tag{26}$$

That is, the H_∞ performance can be satisfied.

IV. SIMULATIONS

Numerical examples are provided to demonstrate that the control protocol we designed is effective in achieving globally asymptotically synchronization when $w_i = 0$ and H_∞ synchronization when $w_i(t) \ne 0$.

A. Example1

For drive system (9) and response system (10), taking the adaptive control algorithms (12)(13)(14). Take $\mu = 0.95$, $i, k = 1,2,\ldots,n.\sigma_i = 1$, $k_i = 1, l_1 = 53, l_2 = 20, l_3 = 26, c_1 = 1.4, c_2 = 2.1, c_3 = 1.3$, $p_1 = 2$, $p_2 = 3$, $p_3 = 1.5, n_1 = 0.7, n_2 = 0.8, n_3 = 0.88, q_i = 1$, $m_1(0) = 0, m_2(0) = 3, m_3(0) = 1.9, w_i(t) = 0, h_k(z_k(t)) = \cos(z_k(t))$, $h_k(z_k(t-\sigma_k)) = \tanh(z_k(t-\sigma_k)), h_k(s_k(t)) = \cos(s_k(t))$, $h_k(s_k(t-\sigma_k)) = \tanh(s_k(t-\sigma_k))$. The states trajectories of drive system and response system without controller are shown in Fig.1, which implies that the two systems have not achieved synchronization. Fig. 2 shows the states trajectories by using our designed controller, therefore, the synchronization of the FONNs is realized.

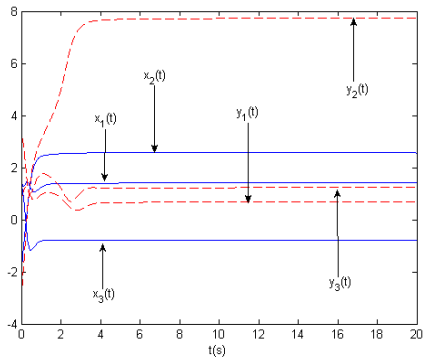

FIGURE I. STATES OF (9) AND (10) WITHOUT CONTROLLER.

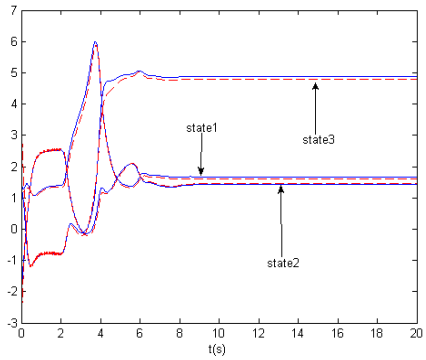

FIGURE II. STATES OF DRIVE SYSTEM AND RESPONSE SYSTEM WITH CONTROLLER.

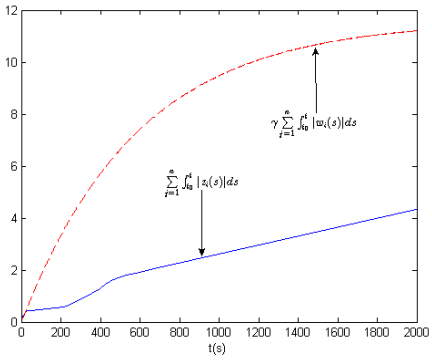

FIGURE III. ENERGY TRAJECTORIES OF $\sum_{j=1}^{n} \int_{t_0}^{t} |\, z_i(s) \,|\, ds$ AND $\gamma \sum_{j=1}^{n} \int_{t_0}^{t} |\, w_i(s) \,|\, ds.$

B. Example 2

On the basis of the Example 1, $w(t) = (0.6\sin(t), 0.5\cos(t), 2\sin(t))^T$ are considered. We get Fig. 3, which show that $\sum_{j=1}^{n} \int_{t_0}^{t} |\, z_i(s) \,|\, ds \le \gamma \sum_{j=1}^{n} \int_{t_0}^{t} |\, w_i(s) \,|\, ds$ as time tends to infinity, that is, the adaptive H_∞ synchronization is satisfied.

V. CONCLUSIONS

In this paper, based on memristor, the H_∞ synchronization problem of FONNs with time-delays was studied. A novel adaptive controller and some sufficient conditions were given to make the closed-loop system achieve H_∞ synchronization. Furthermore, the correctness of the obtained theoretical results were verified by numerical examples.

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