ATLANTIS PRESS
Advances in Intelligent Systems Research, volume 165
International Conference on Modeling, Simulation, Optimization and Numerical Techniques (SMONT 2019)

Analytical Expressions of the Magnetic Field Generated by Horizontal Time-harmonic Electric Dipole in Sea-air Model

Chunhua Zhang[1,*], Bing Li[1], Ning Gao[1], Wenbin Zhao[1] and Song Li[2]
[1]P. O. Box 5111, Beijing, 100094, China
[2]Unit 91977, Beijing, 100161, China
[*]Corresponding author

***Abstract*—The time-harmonic electric dipole itself can directly simulate some field sources. And for some complex field sources, they can also be described by superposition of electric dipoles. In the deep sea, the influence of the seabed is often ignored. When solving the magnetic field generated by an electric dipole, the sea-air model is adopted. Firstly, the magnetic vector potential is solved from Maxwell's equation with the boundary conditions. Secondly, the expression of the electric field is derived by means of image method. Finally, the analytical solution is obtained by the numerical method. The simulation results show that magnetic fields generated by the time-harmonic electric dipole have strong regional character and decrease quickly with the increase of the distance.**

Keywords—magnetic field; horizontal time-harmonic electric dipole; Sea-air medium; image method

I. INTRODUCTION

The electromagnetic field generated by time-harmonic dipole in stratified media and its propagation law have always been concerned. In the detection of metal ore bodies under the seabed, the time-harmonic dipole can be used to represent the detection source. The extremely low frequency electromagnetic field formed before and after the earthquake can also be simulated by electric dipole.

References[1] gives discussion of the problem of how much the propagation of radio waves near the ground will be affected by the ground, and gives an example of solving electromagnetic field excited by electric dipole in layered media. Using the Hertz potential plane wave expansion method, reference [2] discusses the electromagnetic field generated by the electric dipole in two-layer medium systematically and comprehensively. Some in-depth studies have been also carried out in references [3,4,5,6]. In reference [7], dyadic green's function method was used to obtain the expression of Z component of electric field in three-layer media. The analytical expressions of the magnetic field generated by horizontal time-harmonic electric dipole in sea-water medium are obtained based on the image method.

II. THE EXPRESSIONS OF MAGNETIC FIELD CAUSED BY THE HORIZONTAL ELECTRIC DIPOLE IN THE SEA WATER[8]

According to Maxwell's equations, vector magnetic potential equation and magnetic induction intensity are as follows in a homogeneous, linear, isotropic medium.

$$\begin{gathered}\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}_S \\ \boldsymbol{B} = \nabla \times \boldsymbol{A} \\ k^2 = -j\omega\mu\sigma + \omega^2 \mu\varepsilon\end{gathered} \tag{1}$$

In the macroscopic electromagnetic field, vector A does not have direct physical significance, it is just an auxiliary quantity introduced for the convenience of analysis.

A. The Boundary Conditions of the Magnetic Vector Potential

According to the uniqueness theorem of electromagnetic field, the boundary conditions in the inner boundary surface can be shown as follows:

$$\begin{gathered}\frac{\nabla \cdot A_i}{k_0^2} - \frac{\nabla \cdot A_j}{k_1^2} = 0 \\ \frac{A_{in}}{\mu_i} - \frac{A_{jn}}{\mu_j} = 0 \\ A_{it} - A_{jt} = 0 \\ \frac{\partial}{\partial n}[h_t(\frac{A_{it}}{\mu_i} - \frac{A_{jt}}{\mu_j})] = h_t h_n K_{ijt}\end{gathered}$$

Where, i and j represent two kinds of medium, n and t represent the normal and tangent vectors. C_1 and C_2 are constants.

In two-layer model, according to the boundary conditions, A_{ix}, A_{jx}, A_{iz} and A_{jz} satisfy the following conditions:

$$\lim_{z\to+0} A_{ix} = \lim_{z\to-0} A_{jx} \tag{2}$$

$$\lim_{z\to+0} \frac{1}{\mu_i}\frac{\partial A_{ix}}{\partial z} = \lim_{z\to-0} \frac{1}{\mu_j}\frac{\partial A_{jx}}{\partial z} \tag{3}$$

$$\lim_{z\to+0} \frac{A_{iz}}{\mu_i} = \lim_{z\to-0} \frac{A_{jz}}{\mu_j} \tag{4}$$

$$\lim_{z\to+0} \frac{1}{k_i^2}\left(\frac{\partial A_{ix}}{\partial x} + \frac{\partial A_{iz}}{\partial z}\right) = \lim_{z\to-0} \frac{1}{k_j^2}\left(\frac{\partial A_{jx}}{\partial x} + \frac{\partial A_{jz}}{\partial z}\right) \tag{5}$$

In order to satisfy the boundary conditions, the horizontal and vertical magnetic vector potential must be garneted by the horizontal electric dipole, which is different from the vertical electric dipole.

B. The Magnetic Vector Potential of Horizontal Electric Dipole

Assuming that the dipole in the sea water is located at the coordinates $(0,0,-z_0)$ .Then the image of the dipole is located at $(0,0,z_0)$. The locations of the electric dipole and its image are shown in figure 1. In the first layer, the dielectric constant is μ_0 , the magnetic permeability is ε_0 and the conductivity is σ_0 . In the second layer, the dielectric constant is μ_1 , the magnetic permeability is ε_1 and the conductivity is σ_1 . r is the distance from the projection on the xoy plane to central point. θ is the angle between the projection line and x-axis.

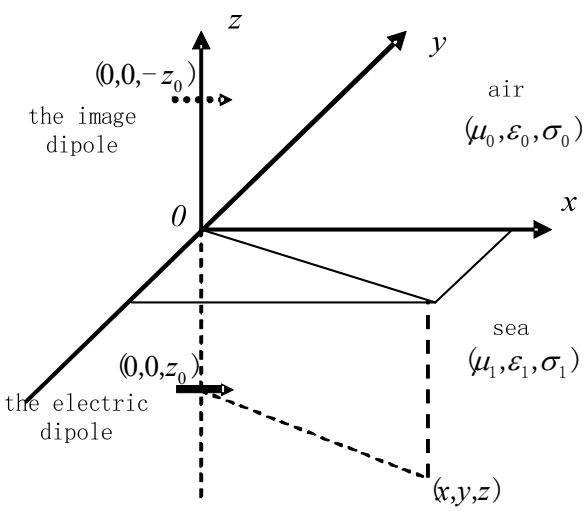

FIGURE I. THE LOCATIONS OF THE ELECTRIC DIPOLE AND ITS IMAGE

If the whole space is overflowed by the sea water, the horizontal magnetic vector potential generated by the real source and image in the sea water can be expressed as follows:

$$A_{1x} = \frac{\mu_1 Il}{4\pi}\int \frac{\xi}{\upsilon_1} J_0(\rho\xi)\left(e^{-\upsilon_1|z-z_0|} + C(\xi)e^{-\upsilon_1|z+z_0|}\right)d\xi$$

Where, $\rho = \sqrt{x^2+y^2}$ $\upsilon_1 = \sqrt{\xi^2 - k_1^2}$. $J_0(\rho\xi)$ is the first kind Bessel function and $C(\xi)$ is the undefined coefficient.

If the whole space is overflowed by the air, the horizontal magnetic vector potential generated by the real source and image in the air can be expressed as follows:

$$A_{0x} = \frac{\mu_0 Il}{4\pi}\int \frac{\xi}{\upsilon_0} D(\xi) J_0(\rho\xi) e^{-\upsilon_0|z-z_0|}d\xi$$

Where, $D(\xi)$ is the undefined coefficient.

According to the boundary conditions (2)~(5), $C(\xi)$ and $D(\xi)$ can be expressed as follows:

$$C(\xi) = \frac{\mu_0\upsilon_1 - \mu_1\upsilon_0}{\mu_0\upsilon_1 + \mu_1\upsilon_0},\ D(\xi) = 2\frac{\mu_1\upsilon_0}{\mu_0\upsilon_1 + \mu_1\upsilon_0}e^{(\upsilon_1-\upsilon_0)z_0}$$

Where, $\upsilon_0 = \sqrt{\xi^2 - k_0^2}$, $\upsilon_1 = \sqrt{\xi^2 - k_1^2}$.

The vertical magnetic vector potential can be expressed as follows with the same method:

$$A_{1z} = \frac{\mu_1 Il}{2\pi}\cos\theta\int_0^\infty MJ_1(\xi\rho)e^{\upsilon_1(z+z_0)}d\xi$$

Where, $\theta = \arctan\frac{y}{x}$,

$$M = \frac{\mu_0\mu_1\left(k_0^2 - k_1^2\right)\xi^2}{\left(\mu_0\upsilon_1 + \mu_1\upsilon_0\right)\left(\mu_0 k_1^2\upsilon_0 + \mu_1 k_0^2\upsilon_1\right)}.$$

According to the equation $A = A_{1x}i + A_{1z}k$, the relation of the magnetic vector potential A and the magnetic field B can be expressed as follows:

Using the following the partial differential operators, the analytical solutions of the magnetic field can be obtained with equation (1).

$$A=\frac{\mu_1 Il}{4\pi}\int\frac{\xi}{\upsilon_1}J_0\left(\rho\xi\right)\left(e^{-\upsilon_1|z-z_0|}+C\left(\xi\right)e^{-\upsilon_1|z+z_0|}\right)d\xi+\frac{\mu_1 Il}{2\pi}\cos\theta\int_0^\infty MJ_1\left(\xi\rho\right)e^{\upsilon_1(z+z_0)}d\xi$$

$$B_{1x}=\frac{\mu_0 Il}{4\pi}\sin\theta\cos\theta\left(\frac{2}{k_1^2}\frac{\partial^3[\frac{\exp\left(-\mathrm{j}k_1\bar{R}\right)}{\bar{R}}]}{\partial z\partial r^2}-\frac{k_0^2+k_1^2}{k_1^4}\frac{\partial^3 V}{\partial z\partial r^2}\right)$$

$$B_{1y}=\frac{\mu_0 Il}{4\pi}\left[\frac{\partial}{\partial z}\left(\frac{\exp\left(-\mathrm{j}k_1R\right)}{R}-\frac{\exp\left(-\mathrm{j}k_1\bar{R}\right)}{\bar{R}}\right)-\frac{1}{k_1^2-k_0^2}\left(\frac{\partial^3 V}{\partial z\partial r^2}+\frac{1}{r}\frac{\partial^2 V}{\partial z\partial r}+2\frac{\partial^3[\frac{\exp\left(-\mathrm{j}k_1\bar{R}\right)}{\bar{R}}]}{\partial z^3}\right)\right]$$

$$-\frac{\mu_0 Il}{4\pi}\left(\frac{2}{k_1^2}\frac{\partial^3[\frac{\exp\left(-\mathrm{j}k_1\bar{R}\right)}{\bar{R}}]}{\partial z\partial r^2}-\frac{k_0^2+k_1^2}{k_1^4}\frac{\partial^3 V}{\partial z\partial r^2}\right)\cos^2\theta$$

$$B_{1z}=-\frac{\mu_0 Il}{4\pi}\sin\theta\left[\frac{\partial}{\partial r}\left(\frac{\exp\left(-\mathrm{j}k_1R\right)}{R}-\frac{\exp\left(-\mathrm{j}k_1\bar{R}\right)}{\bar{R}}\right)-\frac{1}{k_1^2-k_0^2}\left(\frac{\partial^3 V}{\partial r^3}+\frac{1}{r}\frac{\partial^2 V}{\partial r^2}+2\frac{\partial^3[\frac{\exp\left(-\mathrm{j}k_1\bar{R}\right)}{\bar{R}}]}{\partial z^2\partial r}\right)\right]$$

where,

$R=\sqrt{x^2+y^2+(z-h^2)}$, $\bar{R}=\sqrt{x^2+y^2+(z+h)^2}$,

$$U=\int_0^\infty\frac{2\xi}{\upsilon_1+\upsilon_0}\exp\left[-\upsilon_1(z-h)\right]\mathrm{J}_0(r\xi)\mathrm{d}\xi,$$

$$W=\int_0^\infty\frac{2\xi^2\left(k_1^2-k_0^2\right)\mathrm{e}^{-\upsilon_1(z-h)}}{(\upsilon_0+\upsilon_1)(\upsilon_0k_1^2+\upsilon_1k_0^2)}\mathrm{J}_1(r\xi)\cos\theta\mathrm{d}\xi,$$

$$V=2\int_0^\infty\xi\frac{k_1^2}{\upsilon_0k_1^2+\upsilon_1k_0^2}\exp\left[-\upsilon_1\left(z+h\right)\right]\mathrm{J}_0(r\xi)\,\mathrm{d}\xi$$

The expressions of the magnetic fields include the Generalized Sommerfeld Integrals which can be calculated by the numerical calculation method in the reference[9].

III. Computational Example

In order to study the magnetic field generated by extremely low frequency time-harmonic horizontal electric dipole located in the seawater, the distribution of magnetic fields is calculated by the image method. The electromagnetic parameters of air and seawater are assumed respectively: $\mu_0=4\pi\times10^{-7}H/m$, $\varepsilon_0=(1/36\pi)\times10^{-9}F/m$, $\sigma_0=0$; $\mu_0=\mu_1$, $\varepsilon_1=80\varepsilon_0$, $\sigma_1=4\Omega/m$. The frequency of the dipole which is located at (0, 0, 3m) is 2Hz and the magnitude is 5Am. The distributions of magnetic fields are shown in Figure 2 to Figure 7.

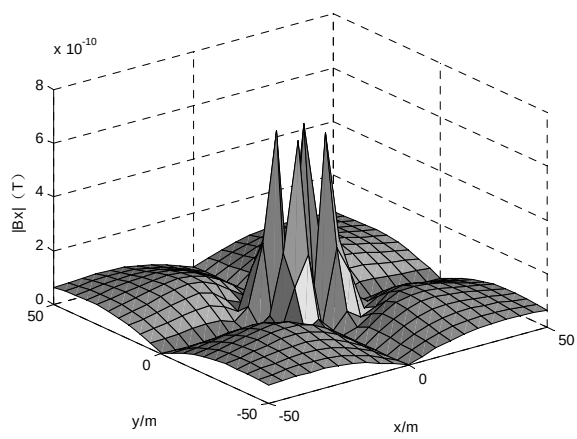

FIGURE II. THE THREE-DIMENSIONAL SPATIAL DISTRIBUTIONS OF THE MAGNETIC IN THE X DIRECTION CAUSED BY THE HORIZONTAL DIPOLE.

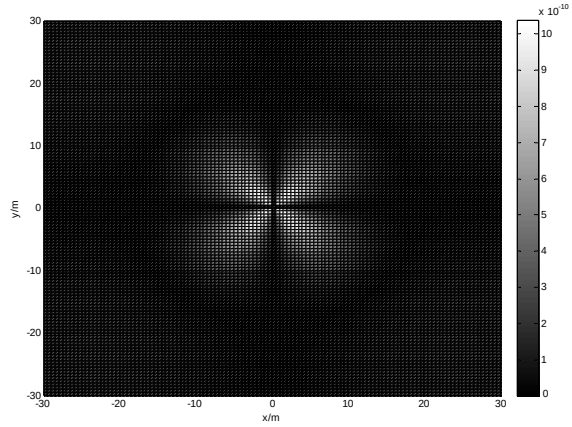

FIGURE III. THE TWO-DIMENSIONAL SPATIAL DISTRIBUTIONS OF THE MAGNETIC IN THE X DIRECTION CAUSED BY THE HORIZONTAL DIPOLE.

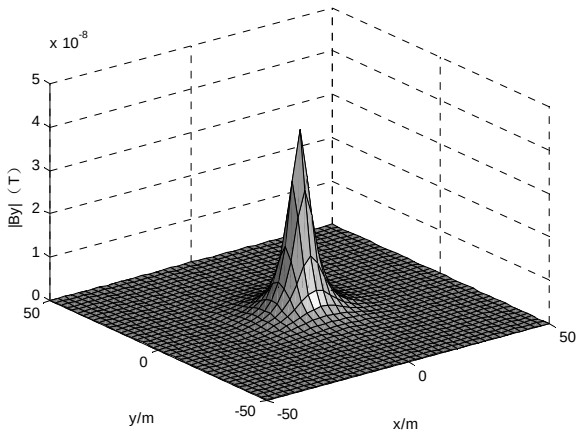

FIGURE IV. THE THREE-DIMENSIONAL SPATIAL DISTRIBUTIONS OF THE MAGNETIC IN THE Y DIRECTION CAUSED BY THE HORIZONTAL DIPOLE.

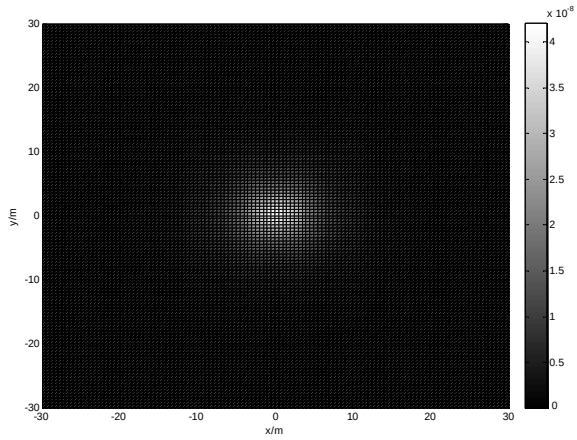

FIGURE V. THE TWO-DIMENSIONAL SPATIAL DISTRIBUTIONS OF THE MAGNETIC IN THE Y DIRECTION CAUSED BY THE HORIZONTAL DIPOLE.

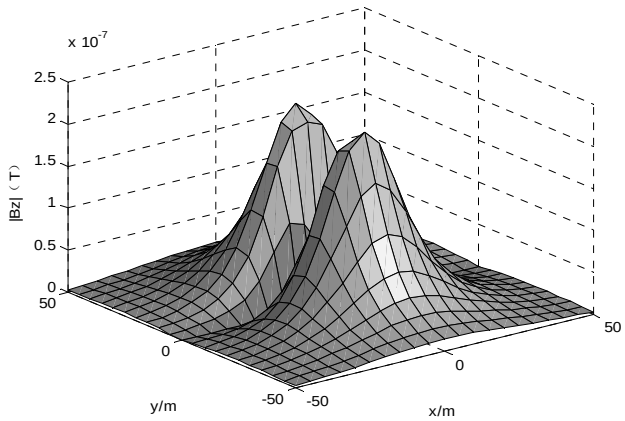

FIGURE VI. THE THREE-DIMENSIONAL SPATIAL DISTRIBUTIONS OF THE MAGNETIC IN THE Z DIRECTION CAUSED BY THE HORIZONTAL DIPOLE.

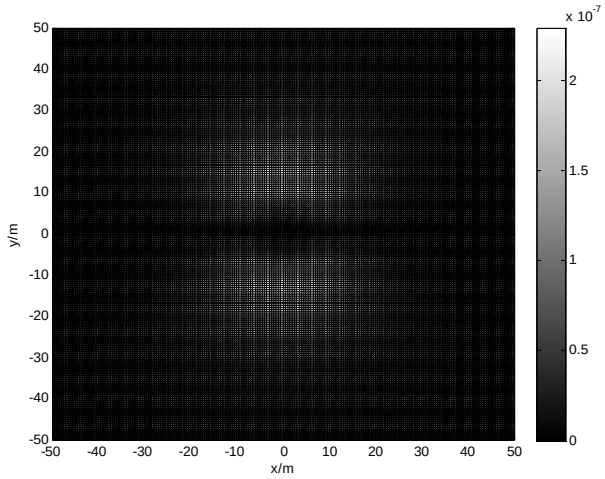

FIGURE VII. THE TWO-DIMENSIONAL SPATIAL DISTRIBUTIONS OF THE MAGNETIC IN THE Z DIRECTION CAUSED BY THE HORIZONTAL DIPOLE.

IV. CONCLUSION

The image method is a classical method to solve some electromagnetic boundary value problems with conductor boundary according to the uniqueness principle. The expressions of the magnetic field generated by time-harmonic electric dipole embedded in sea-water model are deduced. It can be seen from the simulation results that the amplitude of magnetic field generated by electric dipole is regional obviously, and the magnetic field value decays seriously with the increase of distance.

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