Numerical Simulation of Laminar Free Convection and Heat Transfer of Al₂O₃-Water Nanofluid with Temperature-Dependent Physical Properties

Liangcai Zhong¹,³,* and Deyi Shang²
¹School of Metallurgy, Northeastern University, Shenyang, China
²136 Ingersoll Cres, Ottawa ON, Canada K2T 0C8
³Key Laboratory of Eco-utilization of polymetallic ore of Ministry of Education, Northeastern University, Shenyang, China
*Corresponding author

Abstract—Newly developed similarity variables were first put forward in a research on free laminar convection heat transfer of nanofluid with nanofluid's temperature-dependent physical properties. The research results indicated that these similarity variables can be used to transform the governing partial differential equations of nanofluid with temperature-dependent physical properties into the governing ordinary differential equations. It was known from the numerical calculation that the velocity and temperature distributions are influenced with nanofluid's temperature-related physical properties and nano-particle's volume fraction in nanofluid. The influence of wall temperature on velocity and temperature boundary layers of nanofluid's laminar natural convection is not greater than that of bulk temperature.

Keywords—nanofluids; natural convection; heat transfer; variable thermophysical properties; numerical simulation

I. INTRODUCTION

Since nanomaterial was put forward, researchers started to mix different nano-particles and fluids to form different nanofluid in order to improve fluids’ convection heat transfer and develop a new and efficient heating or cooling technology. Choi is the first scientist to propose the concept of nanofluids [1]. He presented that thermal conductivity of nanofluids could be improved by such nanofluids. Then, many researchers [2-4] have conducted research in such field to improve the convection heat transfer of nanofluids. It was found in most past literatures that the convection heat transfer cold be improved with the nanofluids, while the other researches, for example references [5-6] obtained inverse results, indicating that the nanofluids’ convection heat transfer became decreased with increase in nano-particle content in nanofluids.

The present study was to theoretically investigate the influence of nano-particle content in nanofluid with temperature-dependent physical properties on nanofluid’s convection heat transfer. In order to understand the influence of nano-particle addition on the convection heat transfer of nanofluids, laminar natural convection heat transfer of Al₂O₃-water nanofluid was taken as the research objective in this work.

II. THEORETICAL MODEL

A. Assumptions and Governing Equations

The schematic of the physical phenomenon of the nanofluid convection heat transfer and the co-ordinate system are presented in Fig. I. An inclined flat plate with constant wall surface temperature \( T_w \) is located in a still nanofluid with bulk temperature \( T_\infty \). If the plate surface temperature, \( T_w \) is different from the nanofluid’s bulk temperature, \( T_\infty \), a natural convection heat transfer boundary layer will formed on the plate surface. The assumptions in this study were as follows: (i) Assume that the nanofluid can be treated as single phase. (ii) Assume that the nano-particles are fine particles with their sphericity equal to one. (iii) Assume that the nano-particle’s thermophysical properties are reasonably considered to be constant. (iv) Assume that the nanofluid’s boundary layer flow is steady, natural and laminar flow.

![FIGURE I. PHYSICAL MODEL AND COORDINATE SYSTEM](image-url)
physical properties into account include the mass, momentum and energy conservation equations and are as the follows:

\[
\frac{\partial}{\partial x} (\rho_w w_x) + \frac{\partial}{\partial y} (\rho_w w_y) = 0
\]  

(1)

\[
\rho_{pf} \left( \frac{\partial w_i}{\partial x} + \frac{\partial w_j}{\partial y} \right) = \frac{\partial}{\partial y} (\mu_{pf} \frac{\partial w_j}{\partial y}) + g \cdot \cos \alpha (\rho_{w, pf} - \rho_{pf})
\]

(2)

\[
\rho_{pf} \left[ w_x \frac{\partial (c_{pf} T)}{\partial x} + w_y \frac{\partial (c_{pf} T)}{\partial y} \right] = \frac{\partial}{\partial y} (\lambda_{pf} \frac{\partial T}{\partial y})
\]

(3)

The boundary conditions can been written as:

\[
y=0: w_x=0, w_y=0, T=T_w.
\]

(4)

\[
y \to \infty: w_x \to 0, T=T_x.
\]

(5)

When angle \( \alpha \) in Eq. (2) is equal to zero, the above governing equations can be used to describe the laminar natural boundary layer flow and heat transfer on a flat plate.

**B. Similarity Variables**

In order to transform the above governing partial differential equations into ordinary differential equations, the following similarity variables [7] were brought up for natural laminar boundary layer convection of nanofluid on an inclined flat plate:

\[
w_x = 2\sqrt{gx} \cos \alpha \left( \frac{\rho_{w, pf}}{\rho_{w, pf}} \right)^{1/2} W_x
\]

(6)

\[
w_y = 2\sqrt{gx} \cos \alpha \left( \frac{\rho_{w, pf}}{\rho_{w, pf}} \right)^{1/2} \left( \frac{1}{4} \frac{Gr_{x, \infty, pf}}{W_x} \right)^{1/4} W_y
\]

(7)

\[
\eta_{pf} = \frac{y}{x} \left( \frac{1}{4} \frac{Gr_{x, \infty, pf}}{W_x} \right)^{1/4}
\]

(8)

\[
Gr_{x, \infty, pf} = \frac{g \cos \alpha \left( \frac{\rho_{w, pf}}{\rho_{w, pf}} \right)^{1/4}}{v_{w, pf}^2}
\]

(9)

\[
\theta = \frac{T - T_\infty}{T_w - T_\infty}
\]

(10)

where, \( \rho_{w, pf} \) presents nanofluid density at plate surface and \( \eta \) indicate at plate surface and in infinity, respectively, and \( Gr_{x, \infty, pf} \) is defined as local Grashof number in natural convection of nanofluid.

**C. Similarity Transformation of the Governing Equations**

With the above similarity variables, the equations (1) to (3) for the boundary layer convection and heat transfer were transformed to the following dimensionless ordinary differential equations, respectively:

\[
2W_x \eta_{pf} \frac{dW_x}{d\eta_{pf}} + 4 \frac{dW_y}{d\eta_{pf}} + \frac{1}{\eta_{pf}} \frac{d\rho_{pf}}{d\eta_{pf}} (\eta_{pf} W_x - 4W_y) = 0
\]

(11)

\[
2W_x \eta_{pf} \frac{dW_x}{d\eta_{pf}} + 4 \frac{dW_y}{d\eta_{pf}} + \frac{1}{\eta_{pf}} \frac{d\rho_{pf}}{d\eta_{pf}} (\eta_{pf} W_x - 4W_y) = 0
\]

(12)

\[
2W_x \eta_{pf} \frac{dW_x}{d\eta_{pf}} + 4 \frac{dW_y}{d\eta_{pf}} + \frac{1}{\eta_{pf}} \frac{d\rho_{pf}}{d\eta_{pf}} (\eta_{pf} W_x - 4W_y) = 0
\]

(13)

Eqs. (4) and (5) were also transformed to the following dimensionless ones:

\[
\eta_{pf}=0: W_x=0, W_y=0, \theta=1.
\]

(14)

\[
\eta_{pf} \to \infty: W_x \to 0, \theta \to 0.
\]

(15)

**D. Factors of Physical Properties of Nanofluid**

Equations (11) to (13) demonstrate that all temperature-dependent physical properties of nanofluid were transformed into the related temperature-related physical property factors which exist in the dimensionless similarity ordinary differential equations. These temperature-dependent factors of nanofluid have to be further treated mathematically in order to simultaneously solve the dimensionless similarity equations (11) to (13). According to the physical properties of nanofluid and the temperature-dependent thermodynamic properties of the base fluid water [8], these factors are expressed as:

\[
\frac{1}{\eta_{pf}} \frac{d\rho_{pf}}{d\eta_{pf}} \frac{(1-f_p)}{\rho_{pf}} \left[ -2 \times 4.48 \times 10^{-3} (T - 273)(T_w - T_\infty) \right] \frac{d\theta}{d\eta_{pf}}
\]

(16)
\[ \frac{1}{\mu_{pf}} \frac{d\mu_{pf}}{d\eta_{pf}} = \frac{1}{\mu_f} \frac{d\mu_f}{d\eta_{pf}} = \frac{1150}{T^2} - 2 \times \frac{689.58^2}{T^3} (T_w - T_\infty) \frac{d\theta}{d\eta_{pf}} \]  
\tag{17}

\[ \frac{1}{\lambda_{pf}} \frac{d\lambda_{pf}}{d\eta_{pf}} = \left[ \frac{1}{\lambda_f} + \frac{2}{\lambda_p + 2\lambda_f + 2(\lambda_p - \lambda_f)f_p} \right] \left[(\lambda_p + 2\lambda_f - (\lambda_p - \lambda_f)f_p)x(T - 273) + 1.94 \times 10^{-3}(T_w - T_\infty)\right] \frac{d\theta}{d\eta_{pf}} \]  
\tag{18}

\[ \frac{1}{c_{pf}} \frac{dc_{pf}}{d\eta_{pf}} = 0 \]  
\tag{19}

Equation (19) means that the specific heat of nano-particle and water is constant in this study.

### III. Numerical Simulation

The dimensionless equations (11) to (13) must be numerically solved with the correlations of the temperature-related factors Eqs. (16), (17), (18) and (19) for the numerical simulation of laminar natural convection with taking temperature-dependent physical properties of Al\(_2\)O\(_3\)-water nanofluid into account. The numerical shooting approach with combination of fifth-order Runge-Kutta integration was used in this study. In the solution of the very strongly nonlinear boundary value problem, a changeable mesh method was utilized in the numerical integration. Some of the simulation results are shown in Figs. II to IX.
From Figs. II to IX some phenomena can be found for the laminar natural boundary layer convection and heat transfer of Al₂O₃-water nanofluid. When the Al₂O₃ nano-particle volume fraction increases, the convection velocity and temperature of the Al₂O₃-water nanofluid increase, because of the variation of nanofluid density. As the bulk temperature of the Al₂O₃-water nanofluid, \( T_\infty \), increase, the flow velocity and temperature of the Al₂O₃-water nanofluid and boundary layer width increase obviously. The influence of wall temperature of the plate, \( T_w \), on the velocity profiles and boundary layer thickness is slight in comparison of the influence of bulk temperature of the Al₂O₃-water nanofluid, \( T_\infty \). This phenomenon shows that the influence of \( T_\infty \) on the boundary layer convection and heat transfer of Al₂O₃-water nanofluid, caused from the temperature field, is greater than that of the plate surface temperature, \( T_w \).

### IV. Conclusions

From this work, the following conclusions are obtained from the in-depth investigation of laminar natural boundary layer convection and heat transfer of nanofluids with temperature-dependent physical properties.

1. Nanofluid’s temperature-dependent physical properties should be considered in theoretical investigation of heat transfer of nanofluid’s convection and heat transfer.

2. The similarity transformation used in this research for obtaining dimensionless governing ordinary differential equations is rigorous theoretically. Its application makes it possible to transmute the nanofluid’s temperature-dependent physical properties into the temperature-dependent physical property factors. It is advantageous to simplify the handling of temperature-dependent physical properties, numerical calculation, and heat transfer analysis.

3. With increase in nano-particle volume fraction, the velocity and temperature distributions of nanofluid change, due to the variation of nanofluid density. With increase of bulk temperature, \( T_\infty \), the flow velocity and temperature of nanofluid’s boundary layer and boundary layer width are enlarged obviously. The influence of wall temperature, \( T_w \), on the velocity level and boundary layer width is slight. This
phenomenon shows that the influence of $T_w$ on the boundary layer convection and heat transfer of $\text{Al}_2\text{O}_3$-water nanofluid, caused from the temperature field, is greater than that of the plate surface temperature, $T_w$.

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