ATLANTIS PRESS
Advances in Social Science, Education and Humanities Research, volume 253
3rd Asian Education Symposium (AES 2018)

Collaborative Inquiry Learning to Improve Students' Mathematical Reflective Thinking Ability

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Abstract—**Mathematical reflective thinking ability is one of the mathematical abilities needed for students because it provides an opportunity for students to think again and determine the best strategy in achieving goals. Therefore, efforts are needed to improve the ability of mathematical reflective thinking by creating of learning environment that focuses on students as the center of activities in the learning process. One of the lessons that can be applied is use inquiry learning in collaborative groups. This study aims to determine the improvement of students' mathematical reflective thinking skills through collaborative inquiry learning. The subject of the research was the eighth-grade students of SMP in Pringsewu, Lampung, Indonesia. The results show that students' mathematical reflective thinking skills using collaborative inquiry learning are higher than students' mathematical reflective thinking skills that do not use collaborative inquiry learning. This study recommends teachers to use collaborative inquiry learning to improve students' mathematical reflective thinking skills.**

Keywords—inquiry; collaborative; reflective thinking

I. Introduction

The learning of mathematics requires students to reflect consciously on their own mental structures and procedures [1]. Reflective thinking ability is a person's abilities to review, monitor and display the solution process in solving a problem [2]. Reflective thinking provides opportunities for students to solve problems with logical reasons, maintain their opinions, analyze and think back when responding or choose solutions that are useful in solving problems [3]. This shows that reflective thinking is a meaningful thinking process, based on reason and purpose [4].

The important role of reflective thinking is as a means to encourage thinkers during problem solving situations because it provides an opportunity to step back and think about the best strategies for achieving goals [5]. The most important factor that separates reflective thinking from all types of thinking is that reflective thinking emerges as a solution to interpret, delay, translate, get and understand the issues of thinking in prediction and decision making for the future [6].

The theory of reflective thinking ability began with John Dewey's thinking that reflective thinking is an active thinking, continuous and careful thought in a belief or presumptive form of knowledge with clear reasons that support and to lead to further conclusions [7]. Therefore, reflective thinking is a directed process and appropriate activities when individuals can realize, analyze, evaluate, and motivate their own learning processes [6]. In this case, reflective thinking asks the students to think about their thinking processes, such as asking what is known and what is needed, because this is very important to bridge the gap in the learning situation.

Reflection helps students to develop higher-order thinking skills through their encouragement to: a) link new knowledge to their previous understanding; b) thinking with abstract and concrete terminology; c) implementing specific strategies for new tasks; and d) understanding their own thinking processes and learning strategies [8].

Based on the description of reflective thinking that has been described, it can be concluded that the ability of mathematical reflective thinking is the ability to examine the thinking process, which is in the form of thinking activities that can make students try to connect the knowledge gained to solve new problems related to their old knowledge to get a conclusion. Indicators of the ability to think reflectively are: *reacting* (reacting with personal attention to events /situations/problems), *comparing* (comparing reactions with other experiences, such as referring to a general principle, a theory), and *contemplating* (prioritizing the development of self-understanding the depth of the problem, such as prioritizing learning issues, training methods, the next goal, attitude, ethics, focusing in the process of describing, informing, contrasting, and reconstructing situations) [8].

Some explanations have shown the importance of students' mathematical reflective thinking skills. The efforts that can be done to improve the ability of mathematical reflective thinking are to create a learning environment that focuses the students as the center of activities in the learning process. The constructivism learning theory emphasizes students as active learners so that in its application constructivism theory is often referred to the student-centered teaching strategy *(student-centered instruction).*

One of the learning models based on constructivism learning theory and emphasizing process skills and guiding students to high-level thinking is the inquiry model. Inquiry learning is learning that requires investigation activities as part of scientific work involving process skills and based on scientific attitudes [9]. Students learn to investigate problems and find information through the five senses, the activities that involve the surrounding environment, or the abstraction process.

In Vygotsky's theory of constructivism, knowledge is built through social interaction. It can be established in collaborative groups that allow students to help each other in constructing their knowledge. How constructivism are influences learning in groups? In the study group, students can express how he sees the problem and what will be done to the problem [10].

In solving problems, students with diverse backgrounds and abilities can collaborate to achieve common goals. It is possible that students have different views or perceptions about a problem. The views that look different need to be equalized or reconciled through social interaction media, namely discussion activities in collaborative groups, so that each individual takes a new position towards mutual understanding [11].

Through inquiry learning in collaborative groups, students can help each other in reviewing their thinking processes so that they will provide opportunities for each individual to communicate ideas logically and choose the right solution. This is related to reflective thinking that provides opportunities for students to solve problems with logical reasons, maintain their opinions, analyze and think again when responding or choosing solutions that are useful in solving problems [3]. Therefore, the collaborative inquiry learning model is intended to improve the ability of mathematical reflective thinking and *self-efficacy* of students.

II. Methodology

Subjects for the study were taken from students of class VIII 4 (32 students) and class VIII 5 (33 students) from one of junior high school in Pringsewu Lampung Indonesia using *purposive random sampling* technique. In this category, students are selected on the assumption that they are mature enough to form independent opinions about mathematics in relation to their mathematical reflective thinking abilities. The instruments in this study are tests of mathematical reflective thinking skills.

The validity used for the mathematical reflective thinking test instrument is based on content validity and empirical validity. The content validity of this test of mathematical reflective thinking ability can be known by comparing the content contained in the test of mathematical reflective thinking ability with predetermined learning indicators. The instruments that have been made are validated by experts in the field of mathematics. After the instrument is declared valid, the instrument is tested to determine the validity, level of difficulty, power difference, and reliability empirically.

The technique used to test empirical validity is done using the product moment correlation formula [12]:

$$r_{XY} = \frac{N\sum XY - (\sum X)(\sum Y)}{\sqrt{\{((N\sum X^2) - (\sum X)^2)\,((N\sum Y^2) - (\sum Y)^2)\}}}$$

Information:

r_{XY} = Correlation coefficient between variables X and variable Y

N = Number of respondents

$\sum X$ = Number of student scores in each item

$\sum Y$ = Total number of student scores

$\sum XY$ = Total results of multiplying student scores on each item in total student score.

Interpretation of correlation prices is done by comparing using r_{XY} table prices which are 0.3494. This means that if $r_{XY} \geq 0.3494$ then the item number is said to be valid.

The technique used to calculate the difficulty level of a question item is using Sudijono's formula and interpretation [13]:

$$TK = \frac{J_T}{I_T}$$

Information:

TK = Index of the level of difficulty of an item

J_T = Number of scores obtained by students on the items obtained

I_T = Total maximum score obtained by students in an item

TABLE I. Interpretation of the Difficulty Level of the Question Item

Difficulty Level Index	Interpretation
0,00 TK ≤ 0,15	Very Difficult
0,16 TK ≤ 0,30	Difficult
0,31 TK ≤ 0,70	Moderate
0,71 TK ≤ 0,85	Easy
0,86 TK ≤ 1,00	Very Easy

The technique of calculating differentiation is determined by the formula and interpretation below [14]:

$$DP = \frac{\bar{X}_A - \bar{X}_B}{SMI}$$

Information:

DP = the distinguishing index of a particular item

X_A = the average score of the answers of the upper group students

X_B =the average score of the answers of the lower group students

SMI =Ideal Maximum Score, which is a score student will get if they answer the item correctly (perfect).

TABLE II. INTERPRETATION OF DISTINGUISHING VALUE

DP	Interpretation
0,71 - 1,00	Very Good
0,41 – 0,70	Good
0,21 – 0,40	Enough
0,01 – 0,20	Bad
DP ≤ 0,00	Very Bad

Table 3 presents the results of validity, level of difficulty, and the value of distinguishing values of mathematical reflective thinking test instruments.

TABLE III. VALIDITY, LEVEL OF DIFFICULTY, VALUE OF DISTINGUISHING INSTRUMENT THE TEST OF MATHEMATICAL REFLECTIVE THINKING ABILITY

Num.	Validity		TK		DP	
	r_{xy}	*Int*	*Indeks*	*Int*	*Value*	*Int*
1	0.83	Valid	0.46	Moderate	0.318	Enough
2	0.75	Valid	0.66	Moderate	0.311	Enough
3	0.77	Valid	0.32	Moderate	0.402	Enough
4	0.79	Valid	0.46	Moderate	0.727	Very Good

Techniques for finding reliability coefficients (r_{11}) description type questions using Alpha formula [15]:

$$r_{11} = \left(\frac{n}{n-1}\right)\left(1 - \frac{\sum \sigma_i^{2}}{\sigma_t^2}\right)$$

Information:

r_{11} = Instrument reliability coefficient (test)

n = Number of items

$\sum \sigma_i^{2}$ = Amount of variance score for each question

σ_t^{2} = Total score variance

A test is said to be good if it has a reliability value of ≥ 0.70 [13]. The coefficient of *Cronbach's alpha* instrument is 0.76. This shows that the instrument tested has good reliability so that this test instrument can be used to measure students' mathematical reflective thinking skills.

III. RESEARCH RESULTS AND DISCUSSION

TABLE IV. INITIAL SCORE DATA OF STUDENTS' MATHEMATICAL REFLECTIVE THINKING ABILITY

Research of Group	Average	Standard Deviation	The Lowest Value	The Highest Score
Experiment	52,929	11,335	31,250	70,833
Control	59,244	15,598	33,333	83,333

Ideal maximum value = 100

Table 4 Shows that the average ability of students' mathematical reflective thinking skills in an experimental class is lower than the average ability of control class mathematical reflective thinking. Furthermore, to test whether there are differences in the ability of mathematical reflective thinking of the two sample classes that also apply to the population, data analysis is carried out.

TABLE V. T TEST RESULTS IN INITIAL SCORES MATHEMATICAL REFLECTIVE THINKING SKILLS

Research of Group	Average	t	*Df*	*Sig. (2-tailed)*
Experiment	52,929	-1,960	63	0,054
Control	59,244			

Ideal maximum value = 100

Based on table 5, it can be seen that the probability value *(Sig.)* is more than 0.05. This means that the null hypothesis is accepted. Therefore, it can be concluded that there is no significant difference between the initial ability of students' mathematical reflective thinking that uses collaborative inquiry learning and students who do not use collaborative inquiry learning.

TABLE VI. FINAL SCORE DATA ON STUDENTS' MATHEMATICAL REFLECTIVE THINKING ABILITY

Research of Group	Average	Standard Deviation	The Lowest Value	The Highest Score
Experiment	88,020	6,545	70,833	95,833
Control	81,439	11,181	54,133	100

Table 6 Shows that the average of the final ability mathematical reflective thinking of students in the experimental class is higher than the average ability of the final mathematical reflective thinking of the control class students. Furthermore, to test whether there is a difference in the final ability of the mathematical reflective thinking of the two sample classes which also applies to the population, data analysis is carried out.

TABLE VII. FINAL SCORE T TEST RESULTS MATHEMATICAL REFLECTIVE THINKING ABILITY

Research of Group	Average	Z	*Df*	*Sig. (2-tailed)*
Experiment	38,92	-2,498	32	0,012
Control	27,26			

Based on table 7, it can be seen that the probability value (Sig.) is less than 0.05. This means that the null hypothesis is rejected, so it can be concluded that there is a significant difference between the final ability of students' mathematical reflective thinking using collaborative inquiry learning and students who do not use collaborative inquiry learning.

TABLE VIII. DATA ON THE GAIN INDEX OF STUDENTS' MATHEMATICAL REFLECTIVE THINKING ABILITY

Class	Average	Xmin	Xmax	$\bar{x}$	Rerata N-gain
Experiment	*Pretest*	31,250	70,833	52,930	0.745
	Posttest	56,250	95,833	88,020	
Control	*Pretest*	33,33	83,33	59,244	0,544
	Posttest	54,167	97,917	81,439	
Ideal Maximum Valuel = 100					

Table 8 shows that the average gain index of students' mathematical reflective thinking ability using collaborative inquiry learning (experimental class) is higher than the average gain index of students' mathematical reflective thinking ability who do not use collaborative inquiry learning (control class). This means that the improvement of students' mathematical reflective thinking skills using collaborative inquiry learning is included in the improvement with high criteria, while the improvement of students' mathematical reflective thinking skills that do not use collaborative inquiry learning are included in the improvement with moderate criteria.

The factor that became an observation in this study was collaborative inquiry learning and its influence on the ability of mathematical reflective thinking so that this collaborative inquiry learning model was effective to improve the ability of mathematical reflective thinking.

The first factor is the formulation of inquiry learning strategies in collaborative groups with heterogeneous members, so, students with different backgrounds and abilities can learn from each other and work together to achieve common goals. It was done by combining students who have high, medium, and low math skills, passive students with easy communication, and low self-optimistic students. Collaborative learning is for each group member can learn from one another [16]. For that reason, group formation is carried out by considering that each member can discuss to achieve their goals and build an effective working relationship.

When students construct their understanding through the process of inquiry, students can help each other in looking over their thinking processes so they will provide opportunities for each individual to communicate ideas logically and choose the right solution. This is related to reflective thinking that provides opportunities for students to solve problems with logical reasons, maintain their opinions, analyze and think again when responding or choose solutions that are useful in solving problems [3].

The second factor is the inquiry process which begins with a problem which certainly gives different views or perceptions to each individual in solving the problem. According to Vygotsky, social interaction has an important (fundamental) role in the development of understanding (knowledge) [17]. Thus, there is a connection between students' knowledge and their social interaction. Learning is successful or effectual if it involves the whole sense of students [11]. Active communication and collaboration process between students and teachers are essential to produce quality learning.

The third factor, at the stage of formulating the problem, students in the control class have to understand the problems contained in the textbook. Whereas students who use collaborative inquiry in learning process have to convey the problems and start carrying out of inquiry activities through collaboration in their respective groups and each group member has a role to interpret the problem. The problems given were to relating the environment and they have the basic concept of previous material, so it will spur on students to their personal attention of the problems. React with personal attention toward of the events, situations or mathematical problems was focused on the nature of the situation (reacting) it is one phase of the ability to think mathematically reflective [8].

The fourth factor, at the stage of formulating a hypothesis is a temporary answer to the problem that it is actuality studied. Students in the control class formulated hypotheses together with the help of the teacher through several questions. In classes with collaborative inquiry learning, the teacher asks each group members to give ideas to each other in their group. This allows each student to have different perceptions of formulating temporary answers to the problems that it is studied so that they will compare the reactions with other experiences. Analyzing and clarifying individual experiences, as well as meanings and information to evaluate anything that it is believed with comparing reactions with other experiences, such as referring to a general principle or theory is reflectively to think for evaluation (comparing) [8].

The fifth factor is at the stage of collecting data. At this moment, students in the control class were collecting data by finding the information needed to test the hypothesis. Whereas students who use collaborative inquiry learning are given by the opportunity to find information from various sources that are needed. Next, the teacher asks students to write down all of the ideas to solve the problem so each group member compares the information that they get with their friends. In this case, students will assess their abilities based on their own experience and the experiences of their group friends. Comparing with other experiences, such as general principles or theory (comparing) is one phase of the ability of mathematical reflective thinking.

The sixth factor is in the stage of testing the hypothesis. Students in the control class test hypotheses based on information obtained based on data collection. While the collaborative inquiry learning class, the teacher gives direction to students to determine the answers that are considered acceptable toward the data otherwise information obtained based on data collection. The teacher asks students to discussed ideas by each student in solving the problems that it was presented with the group and writing down the problem solving that it was agreed by the group members based on data collection. At this stage, students will focus on a personal level in processes such as describing, informing, considering, and reconstructing a situation or problem (contemplating). According to Surbeck, Han, and Moyer, this is a phase of reflective thinking for a critical inquiry [8].

The seventh factor is at the stage in making conclusions. Students in the control class describe the problems based on the results of testing hypotheses. Whereas students with collaborative inquiry learning conclude that the results of the discussion based on the inquiry activities, it have been carried out with their group friends. In this case, the teacher guides students in the process of describing, informing, contrasting, and reconstructing situations based on testing hypotheses. At this stage, students will prioritize the building of a deep self-understanding of the problem, such as prioritizing learning issues, training methods, goal, attitudes, and ethics, it focused in the process of describing, informing, contrasting, and reconstructing situations (contemplating). This is an indicator of the ability of mathematical reflective thinking, so it can stimulate students' mathematical reflective thinking skills.

The eighth factor is in the evaluation stage. The evaluation phase consists of individual assessment and individual performance evaluation in collaborative groups. Individual assessment is carried out by the teacher that it was giving several questions to students, it is done individually related to the concept of material that was found during group discussion. In solving problems, of course, students will react to problems (reacting), to compare the reactions with their experiences in inquiry activities that have been carried out with their group friends (comparing), and it is focused on describing these problems (contemplating). This is a phase and indicator of mathematical reflective thinking skills.

IV. Conclusion

The factors that influence the effectiveness of collaborative inquiry models to improve students' mathematical reflective thinking skills include the formation of collaborative groups with heterogeneous members, the mathematical problems are given by related to the surrounding environment and constructivist, each stage of collaborative inquiry learning always have facilitates to indicate of thinking ability mathematical reflective.

These result of the factor of the students' mathematical reflective thinking abilities using collaborative inquiry learning models more effective than everything who do not use collaborative inquiry learning. It is recommended for all teachers to have to use collaborative inquiry learning as one of the mathematics learning that it was expected to improve students' mathematical reflective thinking skills.

Acknowledgment

I would like to say thanks to the dean of the Magister Mathematics Education and the director of postgraduate Lampung University that facilitates this research.

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