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Deterministic Quantum-Controlled Teleportation of Arbitrary Multi-Qubit States Via Three-Qubit Entangled States

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Abstract. In this paper, we propose an efficient proposal of quantum-controlled teleportation for transmitting arbitrary multi-qubit states via maximally three-qubit entangled states, in which the sender transmits arbitrary multi-qubit states to the remote receiver under control of the supervisor. This proposal is performed with 100% successful probability by using only Bell-state mutually orthogonal measurement basis and single-qubit measurement basis without the introduction of auxiliary particle. The concrete implementation process is given in detail and the total classical information cost required in the process of this proposal is also calculated.

Keywords: Index Terms—Quantum controlled teleportation, Multi-qubit states, Successful probability, Three-qubit entangled states.

1. Introduction

Quantum teleportation [1]– [4], originally proposed by Ben-nett et al. in 1993 [1], is one of the branches of quantum communication, which refers to a communication protocol teleporting the unknown quantum state from a sender to a separate receiver via Einstein-Podolsky-Rosen (EPR) pair with the help of classical information. The process requires two channels, namely, quantum channel and classical channel, which are used to transmit quantum information and classical information respectively. The quantum channel is composed of multi-qubit entangled states, reconstructing the information at the receiving end, and there is no transmission of information during the whole process.

Quantum controlled teleportation was originally presented by Karlsson and Bourennane [5], which was similar to the ideas of quantum secret sharing (QSS) scheme proposed by Hillery et al. [6]. In [7]– [13], an unknown state could be teleported from the sender to the remote receiver via a three-particles entangled states of GHZ state under the control of the third party. Compared with the original quantum telepor-tation [1], its main difference is that the initial state cannot be teleported unless all third parties agree to cooperate. After that, a number of schemes for quantum-controlled teleportation have been presented. Dai et al. [14]– [16] proposed theoretical schemes for transmitting unknown two- and three-qubit en-tangled states from a sender to either one of two receivers. Gao et al. [17] further put forward a protocol of controlled teleportation and secure direct communication, which used GHZ state as quantum channel. Zha et al. [18] proposed bidirectional quantum-controlled teleportation scheme via the five-qubit cluster state. With the further improvement of the scheme, on the one hand, the scale of quantum state that is to be transmitted has increased from single-qubit state to arbitrary multi-qubit states, and on the other, quantum channel could be GHZ state, W state, or other multi-particles entangled states [19], [20].

In this paper, we propose an efficient proposal of quantum-controlled teleportation for transmitting arbitrary multi-qubit states via three-qubit maximally entangled states, in which the GHZ state is used as quantum channel. The concrete implementation steps of transmitting single-qubit state, two-qubit states, and multi-qubit states are elaborated in section 2, section 3, section 4, respectively. After that, we also calculate successful probability and total classical information cost required in the process of this proposal.

2. The Controlled Teleportation of a Single-Qubit State

Suppose that the sender Alice, the receiver Bob and the controller Charlie are very far apart from each other. For achieving teleportation of a single-qubit state, we use one GHZ state as quantum channel. The arbitrary single-qubit state that to be transported from Alice to Bob can be expressed as

$$|\psi\rangle_I = a_0|0\rangle_I + a_1|1\rangle_I \tag{1}$$

where I is used to represent the unknown particle, and the complex numbers ai (i = 0, 1) satisfy the normalization condition |a0|2 + |a1|2 = 1. Quantum channel is composed of the three-particle entangled states of GHZ state, which can be described as

$$|\psi\rangle_{ABC} = \frac{1}{\sqrt{2}}|000\rangle_{ABC} + \frac{1}{\sqrt{2}}|111\rangle_{ABC} \tag{2}$$

here the three-qubit entangled states are shared by Alice, Bob and Charlie, and subscript A, B and C are used to represent the particle they have. To teleport the initial state given by Eq. (1), Alice needs to perform the Bell-state measurements on particles I and A and inform Bob and Charlie of the results. The Bell-state measurements are as follows:

$$|\beta_{m_A}^{m_I}\rangle \quad = \quad \frac{1}{\sqrt{2}} \; [\; |0\; m_A\rangle + (-1)^{m_I}|1\; \overline{m_A}\rangle \;] \tag{3}$$

here $\{|\beta_{m_A}^{m_I}\rangle \mid m_I m_A = 00, 01, 10, 11\}$.. Only when Charlie knows the measurement results and agrees to cooperate, he will perform a measurement on the particle C based on vectors $\{|+\rangle, |-\rangle\}$.

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle) \tag{4}$$

The whole system including particles I, A, B and C can be expressed as

$$\begin{aligned}
&|\psi\rangle_{IABC} = |\psi\rangle_I \otimes |\psi\rangle_{ABC} \\
&= (a_0|0\rangle_I + a_1|1\rangle_I) \otimes \frac{1}{\sqrt{2}}\left(|000\rangle_{ABC} + |111\rangle_{ABC}\right) \\
&= \sum_{m_I m_A=00}^{m_I m_A=11} |\beta_{m_A}^{m_I}\rangle_{IA} \otimes |+\rangle_C \otimes \frac{1}{2\sqrt{2}} \{\langle m_A|0\rangle\, [a_0|0\rangle_B \\
&\quad +(-1)^{m_I} a_1|1\rangle_B] + \langle m_A|1\rangle\, [(-1)^{m_I} a_1|0\rangle_B + a_0|1\rangle_B]\} \\
&+ \sum_{m_I m_A=00}^{m_I m_A=11} |\beta_{m_A}^{m_I}\rangle_{IA} \otimes |-\rangle_C \otimes \frac{1}{2\sqrt{2}} \{\langle m_A|0\rangle\, [a_0|0\rangle_B \\
&\quad -(-1)^{m_I} a_1|1\rangle_B] + \langle m_A|1\rangle\, [(-1)^{m_I} a_1|0\rangle_B - a_0|1\rangle_B]\} \\
&= \sum_{m_I m_A=00}^{m_I m_A=11} \sum_{m_C=|\pm\rangle} |\beta_{m_A}^{m_I}\rangle_{IA} \otimes |m_C\rangle_C \otimes \frac{1}{2\sqrt{2}} \left(\sigma_Z^{\langle m_C|-\rangle}\right)_B \\
&\quad \{\langle m_A|0\rangle\, [a_0|0\rangle_B + (-1)^{m_I} a_1|1\rangle_B] \\
&\qquad\qquad +\langle m_A|1\rangle\, [(-1)^{m_I} a_1|0\rangle_B + a_0|1\rangle_B]\} \\
&= \sum_{m_I m_A=00}^{m_I m_A=11} \sum_{m_C=|\pm\rangle} |\beta_{m_A}^{m_I}\rangle_{IA} \otimes |m_C\rangle_C \otimes \\
&\qquad \frac{1}{2\sqrt{2}} \left(\sigma_Z^{m_I} \sigma_X^{m_A} \sigma_Z^{\langle m_C|-\rangle}\right)_B (a_0|0\rangle_B + a_1|1\rangle_B)
\end{aligned} \tag{5}$$

(

Here, m_I, m_A and mC are used to represent the measurement results of the particles I, A and C, respectively. Meanwhile, σX and σZ are known as the Pauli matrices. Furthermore, when Bob receives the results mImAmC transmitted from Alice and Charlie to Bob via classical channel, he can perform the corresponding unitary transform $\left(\sigma_Z^{m_I}\sigma_X^{m_A}\sigma_Z^{\langle m_C|-\rangle}\right)$ to reconstruct the initial state of the particle I. There may be eight measurements and the probability of getting every result $m_I m_A m_C$ all equal to $P_i = \left(\frac{1}{2\sqrt{2}}\right)^2$ Thus, the successful probability of our scheme can be calculated as

$$P_{total} = \sum_{m_I m_A=00}^{m_I m_A=11} \sum_{m_C=|\pm\rangle} \left(\frac{1}{2\sqrt{2}}\right)^2 = 100\% \tag{6}$$

Meanwhile, the total classical information cost S required in the process of this proposal is

$$S = -\sum_{i=1}^{8} P_i \log P_i = 3 \tag{7}$$

From the above discussions, our scheme can realize the controlled teleportation of a single-qubit state by only us-ing the usual Bell-state measurements, single-qubit projective measurement and single-qubit local unitary operations with 100% successful probability.

3. The Controlled Teleportation of Arbitrary Two-Qubit States

For achieving the teleportation of arbitrary two-qubit states, we use two GHZ state as quantum channels. The arbitrary two-qubit states that to be teleported from Alice to Bob can be expressed as

$$|\psi\rangle_{I_1 I_2} = (a_0|00\rangle + a_1|01\rangle + a_2|10\rangle + a_3|11\rangle)_{I_1 I_2} \tag{8}$$

Pwhere ai (i = 0; 1; 2; 3) are the complex numbers, and $\sum_0^3 |a_i|^2 = 1$ Similar to the above, the quantum channels are as follow:

$$\begin{aligned} |\psi\rangle_{A_1 B_1 C_1} &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_1 B_1 C_1} \\ |\psi\rangle_{A_2 B_2 C_2} &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_2 B_2 C_2} \end{aligned} \tag{9}$$

Here, the sender Alice, the receiver Bob and the controller Charlie have particles A_k, B_k and C_k, (k = 1, 2), respectively. The detailed realization process of transmitting arbitrary two-qubit states is described below.

Step 1: To teleport the initial state given by Eq. (8), Alice needs to perform the Bell-state measurements $\{|\beta_{m_{A_k}}^{m_{I_k}}\rangle \mid m_{I_k} m_{A_k} = 00, 01, 10, 11\}$ on particles Ik and Ak, and inform Bob and Charlie of the results.

$$\begin{aligned} &|\beta_{m_{A_k}}^{m_{I_k}}\rangle = \frac{1}{\sqrt{2}}\ [\ |0\ m_{A_k}\rangle + (-1)^{m_{I_k}}|1\ \overline{m_{A_k}}\rangle\ ] \\ &(k = 1, 2) \end{aligned} \tag{10}$$

Step 2: When Charlie gets the measurement results and agrees to communicate, he will measure particles Ck based on vectors $\{|\pm\rangle||\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)\}$ in sequence. The state of the whole system can be showed as

$$
\begin{aligned}
&|\psi\rangle_{I_1I_2A_1A_2B_1B_2C_1C_2} = |\psi\rangle_{I_1I_2} \otimes |\psi\rangle_{A_1B_1C_1} \otimes |\psi\rangle_{A_2B_2C_2} \\
&= \sum_{m_{I_1},m_{A_1},m_{I_2},m_{A_2}=0}^{1} \sum_{m_{C_1},m_{C_2}=|\pm\rangle} \Big\{ |\beta_{m_{A_1}}^{m_{I_1}}\rangle_{I_1A_1} \\
&\quad \otimes |\beta_{m_{A_2}}^{m_{I_2}}\rangle_{I_2A_2} \otimes |m_{C_1}\rangle_{C_1} \otimes |m_{C_2}\rangle_{C_2} \otimes \\
&\quad \left(\frac{1}{2\sqrt{2}}\right)^2 \cdot \left(\sigma_Z^{m_{I_1}} \sigma_X^{m_{A_1}} \sigma_Z^{\langle m_{C_1}|-\rangle}\right)_{B_1} \\
&\quad \otimes \left(\sigma_Z^{m_{I_2}} \sigma_X^{m_{A_2}} \sigma_Z^{\langle m_{C_2}|-\rangle}\right)_{B_2} \\
&\quad \cdot (a_0|00\rangle + a_1|01\rangle + a_2|10\rangle + a_3|11\rangle)_{B_1B_2} \Big\}
\end{aligned}
\tag{11}
$$

here mP are used to represent the measurement results of the particles P, and $P \in \{I_k, A_k, C_k \mid k = 1, 2\}$.

Step 3: There may be sixty-four me ure nts, and the probability of each of them is $P_i = \left(\frac{1}{2\sqrt{2}}\right)^4$ According to the measurement results of the particles Ik, Ak and Ck, _Bob needs to perform the corresponding unitary transform $\left(\sigma_Z^{m_{I_k}} \sigma_X^{m_{A_k}} \sigma_Z^{\langle m_{C_k}|-\rangle}\right)$ on particles Bk (k = 1; 2) to reconstruct the initial state in Eq. (8). Furthermore, the total successful probability of our scheme can be expressed as

$$
P_{total} = \sum_{m_{I_1},m_{A_1},m_{I_2},m_{A_2}=0}^{1} \sum_{m_{C_1},m_{C_2}=|\pm\rangle} \left(\frac{1}{2\sqrt{2}}\right)^4 = 100\% \tag{12}
$$

the total classical information cost S of teleporting arbitrary two-qubit states is

$$
S = -\sum_{i=1}^{64} P_i \log|P_i = 6 \tag{13}
$$

Thus, our proposal can realize the controlled teleportation of arbitrary two-qubit states with the successful probability of 100%.

4. The Controlled Teleportation of Arbitrary Multi-Qubit States

The scheme for transmitting single-qubit state and two-qubit states from Alice to Bob has been validly achieved in Section 2 and 3, respectively. Without loss of generality, we talk about the process of teleporting arbitrary n-qubit states, which can be given by

$$
\begin{aligned}
&|\psi\rangle_{I_1I_2\cdots I_n} = \sum_{x=0}^{2^n-1} a_x |d_n \ldots d_2 d_1\rangle_{I_1I_2\cdots I_n} \\
&d_i \in \{0,1\}; x = \sum_{i=1}^{n} d_i \cdot 2^{i-1}.
\end{aligned}
\tag{14}
$$

Here the parameters a_x $(x = 0, 1, \cdots, 2^n - 1)$ are the complex numbers, and satisfy the condition $\sum_{x=0}^{2^n-1} |a_x|^2 = 1$. In order to realize controlled teleportation of n-qubit states, we need to use n GHZ state and distribute it to Alice, Bob and Charlie, respectively. The quantum channels are expressed as follows:

$$|\psi\rangle_{A_kB_kC_k} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{A_kB_kC_k} \quad (k = 1, 2 \cdots n) \tag{15}$$

the particles A_k, B_k and Ck belong to the sender Alice, the receiver Bob and the controller Charlie, separately. Similar with the above, Alice needs to perform the measurements $\{ |\beta_{m_{A_k}}^{m_{I_k}}\rangle \mid m_{I_k}m_{A_k} = 00, 01, 10, 11 \}$ on particle pairs of Ik and Ak, and inform Bob and Charlie of the measurement results.

$$|\beta_{m_{A_k}}^{m_{I_k}}\rangle = \frac{1}{\sqrt{2}} [|0\ m_{A_k}\rangle + (-1)^{m_{I_k}}|1\ \overline{m_{A_k}}\rangle] \quad (k = 1, 2 \ldots n) \tag{16}$$

Then Charlie can decide whether to proceed with the com-munication or not, and only when he agrees to communicate, can the teleportation be realized. Assuming Charlie agrees to communicate, he needs to perform single-qubit measurement on particles Ck based on vectors$\{\langle\mp|\}$ in sequence. The concrete process is implemented as follows:

$$\begin{aligned}
&|\psi\rangle_{I_1\cdots I_nA_1\cdots A_nB_1\cdots B_nC_1\cdots C_n} \\
&= |\psi\rangle_{I_1I_2\cdots I_n} \otimes |\psi\rangle_{A_1B_1C_1} \otimes |\psi\rangle_{A_2B_2C_2} \cdots \otimes |\psi\rangle_{A_nB_nC_n} \\
&= \sum_{m_{I_1},\cdots,m_{I_n},m_{A_1},\cdots,m_{A_n}=0}^{1} \sum_{m_{C_1},\cdots,m_{C_n}=|\pm\rangle} \left(\frac{1}{2\sqrt{2}}\right)^n \\
&\cdot|\beta_{m_{A_1}}^{m_{I_1}}\rangle_{I_1A_1}|\beta_{m_{A_2}}^{m_{I_2}}\rangle_{I_2A_2}\cdots|\beta_{m_{A_n}}^{m_{I_n}}\rangle_{I_nA_n} \otimes \\
&|m_{C_1}\rangle_{C_1}|m_{C_2}\rangle_{C_2}\cdots|m_{C_n}\rangle_{C_n} \cdot \left(\sigma_Z^{m_{I_1}}\sigma_X^{m_{A_1}}\sigma_Z^{\langle m_{C_1}|-\rangle}\right)_{B_1} \\
&\otimes \left(\sigma_Z^{m_{I_2}}\sigma_X^{m_{A_2}}\sigma_Z^{\langle m_{C_2}|-\rangle}\right)_{B_2} \cdots \otimes \left(\sigma_Z^{m_{I_n}}\sigma_X^{m_{A_n}}\sigma_Z^{\langle m_{C_n}|-\rangle}\right)_{B_n} \\
&\cdot \left(\sum_{x=0}^{2^n-1} a_x|d_n\ldots d_2d_1\rangle_{B_1B_2\cdots B_n}\right)
\end{aligned} \tag{17}$$

There could be 23n measurement results, and the probability of each result is

$$P_i = \frac{1}{2^{3n}}, (i = 1, 2, \cdots, 2^{3n}) \tag{18}$$

Furthermore, when Bob receives the measurement re-sults, for constructing the initial multi-qubit states, he needs to perform the corresponding unitary transform$\left(\sigma_Z^{m_{I_k}}\sigma_X^{m_{A_k}}\sigma_Z^{\langle m_{C_k}|-\rangle}\right)$ on particles B_k $(k = 1, 2, \cdots, n)$, respectively. The total successful probability of this proposal can be calculated as

$$P_{total} = \sum_{m_{I_1},\cdots,m_{I_n},m_{A_1},\cdots,m_{A_n}=0}^{1} \sum_{m_{C_1},\cdots,m_{C_n}=|\pm\rangle} P_i = 100\% \tag{19}$$

Meanwhile, the total classical information cost S required in the process of this proposal is

$$S = -\sum_{i=1}^{2^{3n}} P_i \log P_i = \log 2^{3n} = 3n \tag{20}$$

Therefore, our scheme can successfully implement con-trolled teleportation of arbitrary multi-qubit states, whose successful probability can reach to 100%.

5. Conclusion

In summary, we have improved the proposal for quantum-controlled teleportation of an arbitrary multi-qubit states by using GHZ state as quantum channel. We have shown the con-crete implementation process of transmitting arbitrary single-qubit state, two-qubit states and multi-qubit states, and we also calculate the total successful probability of this proposal, which all can reach to 100%. Compared with the previous schemes, our proposal has the following advantages. First of all, our scheme is a highly inductive summary of previous schemes, and the derivation process is more convenient for researchers to understand. Second, in our scheme, we only need to conduct Bell-state mutually orthogonal basis measure-ment and single-qubit measurement to realize the invisible transmission of an unknown multi-qubit states. Third, this proposal can achieve 100% successful probability without the introduction of auxiliary particles. Thus, because of the poten-tial application value of our scheme, it will greatly promote the development of quantum communication technology.

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