

The Use of Copulas in Estimating the Value at Risk (VaR) of the IDX Development Board and Main Board Indices with Monte Carlo Simulation

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Abstract—This article discusses the use of copulas to estimate Value at Risk (VaR) with Monte Carlo simulation in the Indonesian stock market. As an illustration, we construct a portfolio which consists of the IDX Main Board and Development Board Indices in equal proportion (50% each). Based on Kupiec's Proportion of Failure Test (POF), the estimated 99% VaR with the Monte Carlo-Copula method is not rejected. Therefore, it can be concluded that the Monte Carlo-Copula method can be used to estimate VaR in the Indonesian stock market.

Index Terms—Value at Risk, Copula, Monte Carlo Simulation, Indonesian Stock Market

I. INTRODUCTION

According to Crouchy, Galai and Mark [1], one of the main focuses in risk management is risk measurement. This serves as a basis for company stakeholders to make decisions on risk mitigation. Without an accurate risk measurement, stakeholders will be unable to perform effective risk management and could potentially expose the company to higher risk than it can manage.

Crouchy et al. [1] state that VaR is the standard risk measure in the financial industry. Linsmeier and Pearson [2] define it as a single statistical measure of the maximum potential loss of a portfolio in normal market movements. In practice, VaR aggregates all portfolio risk into a single number, which is useful from a managerial perspective.

According to Jorion [3], there are two approaches in VaR estimation: the parametric and non-parametric. The parametric approach essentially uses probability distribution in estimating VaR. On the other hand, the non-parametric approach only uses historical data (without probability distribution modeling) in its estimation. In this article, we will focus on the parametric approach to estimating VaR.

Jorion [3] states that two methods are generally used in estimating VaR with the parametric approach. The first is the delta-normal, or variance-covariance. The delta-normal method assumes that asset returns are normally distributed, hence VaR estimation is relatively easy to compute. The second method is Monte Carlo simulation. This estimates VaR by generating random numbers from the known probability distribution of assets returns. Hence, it requires that the probability distribution of asset returns are modeled beforehand.

The main advantage of the Monte Carlo simulation method is its flexibility in choosing the appropriate distribution for the asset returns. While the delta-normal method must assume that asset returns are normally distributed, the Monte Carlo method does not require such an assumption.

It is relatively straightforward to estimate the VaR of a portfolio consisting of a single asset with the Monte Carlo simulation method; a probability distribution model for the asset returns needs to be found and random numbers generated from it to estimate VaR. A problem arises when the portfolio consists of several assets, because we need to find multivariate (joint) probability distributions in order to model all the asset returns in it.

One way to form a multivariate probability distribution is through the use of the copula function. A copula is basically a function which combines several probability distributions into joint distributions. Hence, the problem of modeling the joint distribution for the portfolio return is reduced to modeling the univariate distribution for each asset return in the portfolio and combining them using a copula to form a multivariate distribution for the portfolio return.

In this study, the Monte Carlo-Copula method is employed to estimate the VaR of a portfolio. The portfolio used consists of two components: the IDX Development Board Index and the IDX Main Board Index. The choice of these indices was made to illustrate a portfolio which combines stocks from companies with both high and low capitalization.

II. LITERATURE REVIEW

Value at Risk (VaR) is the risk measure that serves as the standard in the financial industry. Its popularity arises from the fact that it quantifies the risk of a portfolio into one single number, which is the maximum loss with a certain confidence [4].

In estimating VaR with the parametric approach, the main problem is to determine the joint probability distribution of all asset returns in the portfolio. Some studies assume that financial asset returns are normally distributed. Hendricks [5] uses the variance-covariance method to estimate the VaR of portfolios which consist of several currencies. However, Ang

and Chen [6] found that correlation among stocks in the US stock market is not symmetrical, while Bastianin [7] also found that there are non-normalities in the dependence structure among oil and gas company stocks. Therefore, the normality assumption for financial asset returns is not very suitable.

One way to construct joint probability distribution without assuming normality is through the use of copulas. According to Nelsen [8], a two-dimensional copula is defined as:

$$C : [0, 1]^2 \longrightarrow [0, 1] \quad (1)$$

From which it follows that:

$$C(0, y) = C(x, 0) = 0 \quad (2)$$

$$C(1, y) = y, C(x, 1) = x \quad (3)$$

$$C(x_1, y_1) + C(x_2, y_2) - C(x_1, y_2) - C(x_2, y_1) \geq 0 \quad (4)$$

for all $x, x_1, x_2, y, y_1, y_2 \in [0, 1]$ with $x_1 \leq x_2, y_1 \geq y_2$

The use of copulas to form the joint probability distribution of a portfolio's return can resolve the problems of non-normality in asset returns and in the dependence structure among asset returns in portfolios. Therefore, the VaR of a portfolio can be estimated by generating random numbers from copulas with the Monte Carlo simulation method.

Huang, Lee and Lin [9] used the copula-GARCH method to estimate the VaR of portfolios consisting of NASDAQ and TAIEX. In their research, they used various copula functions to model the multivariate distribution of the portfolios and found that the student t-copula gave the best VaR estimation. Moreover, they concluded that the VaR estimation results from the copula-GARCH method were more accurate than those from other classical VaR estimation methods (such as delta-normal and historical simulation).

Lu et al. [4] conducted research on estimating VaR with the Monte Carlo simulation method of portfolios consisting of oil and gas futures contracts. For the joint distribution of the portfolio return, they used the copula function. It was found that the student t-copula was the best method to model the dependence structure among oil and gas futures contracts.

In this paper, we conduct VaR estimation with a similar method to that of Lu et al. [4], with the Indonesian stock market as the object. As an illustration, we form a portfolio consisting of the IDX Development Board and Main Board Indices in the same proportion (50% each). To validate the VaR estimation, we use Kupiec's Proportion of Failure (POF) test

III. RESEARCH METHODOLOGY

There are four steps in estimating VaR with the Monte Carlo-Copula method. The first step is to model the marginal (univariate) distribution of each asset return, while the second step is to model the joint (multivariate) distribution of the

portfolio return using copulas to combine the marginal distribution from the first step. The third step is to estimate VaR by generating random numbers from the copula obtained in the second step using Monte Carlo simulation. The final step is to validate the VaR estimation result using a backtesting. In summary, the input of the Monte Carlo-Copula method is each asset return in the portfolio and the output is the VaR estimation of the portfolio.

A. Marginal Model

For a time series data of asset returns until time T such as:

$$r_t = r_1, r_2, \dots, r_T \quad (5)$$

according to Engle [10], it should be noted that return time series data have dependent characteristics among observations and volatility clustering. Volatility clustering is the phenomenon in which the conditional variance of the time series data is dependent on time, and is also known as conditional heteroskedasticity.

In order to obtain the probability distribution from the return time series data, it is first necessary to extract the random component from these data. In this study, the ARMA-GARCH model is used to extract this random component, a method also used by Lu et al. [4].

The ARMA-GARCH model is basically a combination of two models: ARMA (Autoregressive-moving-average) and GARCH (Generalized autoregressive conditional heteroskedasticity). The ARMA model is used to extract the random component in the form of an error term from the time series data. In cases when the conditional variance of the error term is dependent on time, it is necessary to further model the conditional variance of the error term by GARCH in order to obtain the random component in the form of standardized residuals. The ARMA-GARCH method used in this paper is as follows:

$$r_t = \mu + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \dots + \phi_3 r_{t-3} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_q \varepsilon_{t-q} \quad (6)$$

$$h_t = \omega_0 + \sum_{i=1}^s \mu_i \varepsilon_{t-1}^2 + \sum_{i=1}^r \beta_i h_{t-1} \quad (7)$$

$$\varepsilon_t = \sqrt{h_t} e_t \quad (8)$$

where $\mu, \phi_1, \phi_2, \dots, \phi_p, \theta_1, \theta_2, \dots, \theta_q$ are parameters of the ARMA model; $\omega_1, \omega_2, \dots, \omega_s, \beta_1, \beta_2, \dots, \beta_r$ are parameters of the GARCH model; ε_t are the residuals; h_t are the variance of ε_t ; and e_t are the standardized residuals with t distribution and v degrees of freedom. The parameters of the ARMA-GARCH model are estimated using the Maximum Likelihood Estimator (MLE). After all the ARMA-GARCH parameters have been estimated, it is possible to obtain for time $T + 1$ the forecast return \hat{r}_{T+1} and forecast variance \hat{h}_{T+1} using the Minimum Mean Square Error method (MMSE).

B. Bivariate Model

For two sets of return time series data, $r_{1,t}$ and $r_{2,t}$ where $t = 1, 2, \dots, T$ is time, we can apply the steps in the marginal model to obtain the standardized residuals $e_{1,t}$ and $e_{2,t}$, the Cumulative Distribution Function (CDF) for each standardized residual F_{e_1} and F_{e_2} and the Probability Density Function (PDF) f_{e_1} and f_{e_2} for each standardized residual. An attempt will now be made to find the best copula model as joint distribution for $e_{1,t}$ and $e_{2,t}$. Five copula functions are considered as joint distribution: Gumbel, Clayton, Frank, Gaussian and Student t. Based on Chaerubini, Luciano and Vecchiato [11], copula parameters can be estimated using the MLE method, with the log likelihood function, as in the example below:

$$L(\theta) = \sum_{t=1}^T \ln c(F_{e_1}(e_{1,t}), F_{e_2}(e_{2,t})) + \sum_{t=1}^T \ln f_{e_1}(e_{1,t}) + f_{e_2}(e_{2,t}) \tag{9}$$

where c is the copula Probability Density Function (PDF), f_{e_1} and f_{e_2} are the PDF of each standardized residual. The copula function that will be used as joint distribution is the one that is minimized below the AIC value:

$$AIC = -2(\log L) + 2(p) \tag{10}$$

where $\log L$ is the log likelihood function of each copula and p are the number of parameters in each copula function.

C. VaR estimation using Monte Carlo Simulation

After obtaining both the marginal and bivariate models, it is now possible to estimate the VaR of portfolio returns consisting of r_1 and r_2 for time $T + 1$ with Monte Carlo simulation. First, N couples of random values are generated from the copula function: $(\mu_1^1, \mu_2^1), (\mu_1^2, \mu_2^2), \dots, (\mu_1^N, \mu_2^N)$ Next, the inverse function $F^{-1}_{e_1}$ for u_1^j and inverse function $F^{-1}_{e_2}$ for u_2^j where $j = 1, 2, N$ are used to transform N couples of random values into N couples of standardized residuals:

$$(r_{1,T+1}^j, r_{2,T+1}^j) = (r_1, \hat{T} + 1 + e_1^j) \cdot \sqrt{\hat{h}_{1,T+1}^2}, \tag{11}$$

$$\hat{r}_{2,T+1} + e_2^j \cdot \sqrt{\hat{h}_{2,T+1}^2}$$

Subsequently, the N couples of standardized residuals (e_1^j, e_2^j) are transformed into N couples of return using the forecast mean and variance obtained in the marginal model:

$$(r_{1,T+1}^j, r_{2,T+1}^j) = (\hat{r}_{1,T+1} + e_1^j \cdot \sqrt{\hat{h}_{1,T+1}^2}, \tag{12}$$

$$\hat{r}_{2,T+1} + e_2^j \cdot \sqrt{\hat{h}_{2,T+1}^2})$$

The N couples of return can now be transformed into N portfolio returns:

$$r_{T+1}^j = p \cdot r_{1,T+1}^j + (1 - p) \cdot r_{2,T+1}^j \tag{13}$$

with p as the proportion of each asset in the portfolio. The estimated VaR with confidence level q for time $T+1$ is the $(1 - q) \cdot N$ ordered data from r_{T+1}^j

D. VaR backtesting

The estimated VaR is validated by running a backtest (comparing actual return with estimated VaR return). The test used in this study is Kupiec's Proportion of Failure test (POF). This test considers the accuracy of the estimated VaR as the parameter of rejection. The accuracy itself is defined by comparing the number of exceptions (the condition when actual loss is more severe than the estimated VaR) with the theoretical number of exceptions based on the confidence level used. The test statistics of the POF test are given by:

$$POF = -2 \log \left(\frac{(1 - p)^{N-x} p^x}{(1 - \frac{x}{N})^{N-x} (\frac{x}{N})^x} \right) \tag{14}$$

where x is the number of exceptions, N is the number of observations, and q is the confidence level of VaR. The POF test statistics are asymptotically distributed as a chi-square with 1 degree of freedom. The VaR model is rejected when POF exceeds the critical value, where the critical value depends on the VaR confidence level.

IV. RESULT

The data used in this paper are daily return data for the IDX Development Board Index and IDX Main Board Index for the period 2010-2016. They are split into two: an estimation sample (2010-2015) and a forecasting sample (2015-2016). The estimation sample is the period used to construct the marginal and bivariate models, while the forecasting sample is the period used for validating the estimated VaR by running a backtest. Using daily return data from both indices, a portfolio is then formed with equal proportions for each index.

As for the marginal model, ARMA(3,3)-GARCH (1,1) is used for the IDX Development Board Index return and ARMA(4,7)-GARCH(1,1) for the IDX Main Board Index return. Table I shows the parameter estimations for each marginal model.

After establishing the marginal model for the IDX Development Board and IDX Main Board indices, the bivariate model can then be constructed by using copulas. Based on the AIC calculation results shown in table II, the student t copula is chosen as the bivariate model.

Finally, the VaR of the portfolio can now be estimated using Monte Carlo simulation. The first estimated VaR for the portfolio is for 4 January 2016, based on the estimation sample from 4 January 2010-30 December 2015. The second estimated VaR for the portfolio is for 5 January 2016, based on the estimation sample from 5 January 2010 - 4 January 2016. The process is repeated until the estimated VaR is obtained for 30 December 2016. Figure 1 shows the estimated 99% VaR

TABLE I: ARMA-GARCH Parameter Estimation

Parameter	Value (IDX Main Board Index)	Value (IDX Development Board Index)
Constant ARMA	0.000336	0.000136
AR1	-0.00497	0.361711
AR2	0.777391	-0.37502
AR3	0.299825	0.787491
AR4	-0.43243	
MA1	0.000429	-0.37037
MA2	-0.85263	0.381802
MA3	-0.41198	-0.83401
MA4	0.4466	
MA5	0.116922	
MA6	-0.0137	
MA7	-0.00086	
Constant GARCH	4.83E-06	2.67E-06
GARCH1	0.856616	0.839991
ARCH1	0.113025	0.136237
Degree of Freedom	5.42847	5.33781

TABLE II: AIC Calculation Results

Copula	AIC	Parameter 1	Parameter 2
Gaussian	-895.4038891	0.6738	
T	-914.6241762	0.674	8.0374
Clayton	-845.5642499	1.2733	
Frank	-811.07115	5.2078	
Gumbel	-776.6301263	1.7988	

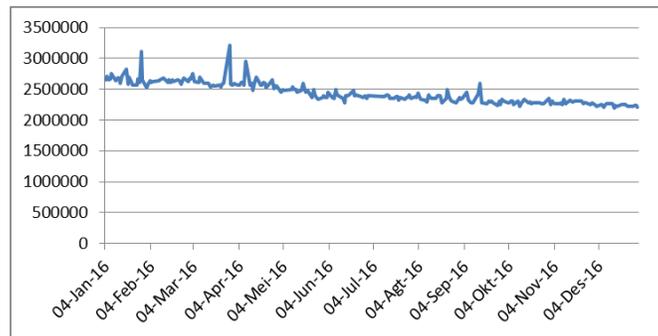


FIGURE 1: Estimated VaR (x108) for period 4 January 2016 - 30 December 2016

TABLE III: POF Test Results

Number of Excep-tions	Test Statistics	Critical Value	Result
1	1.14	6.635	VaR model is not rejected

of the portfolio for the period 4 January 2016 to 30 December 2016.

Based on the POF test results shown in table III, it can be concluded that the VaR estimation is not rejected. Hence the Monte Carlo-Copula method can be used to estimate portfolio VaR consisting of the IDX Development Board and Main Board Indices.

V. DISCUSSION

From a managerial perspective, VaR is used as the benchmark for maximum loss in a specific period. JP Morgan, for example, uses VaR to give an illustration of maximum potential loss in the next end of business day. Therefore, any method used to estimate VaR must result in an acceptable estimate of maximum portfolio losses. Based on table 3,

there is only one time that the actual loss is more severe than the VaR estimation. This means that the Monte Carlo-Copula method is able to give such a benchmark for maximum potential portfolio loss.

In order to improve the research conducted in this study, two future studies on the Monte Carlo-Copula method in estimating the VaR of the Indonesian stock market could be undertaken. The first is to change the VaR confidence level. The choice of confidence level in VaR estimation depends on the subject and condition. For example, in the case of high risk investment, a risk manager may want to take a conservative approach and estimate maximum risk by using 99% VaR. On the other hand, the same risk manager may want to take a more aggressive approach to low risk investment by using 90% VaR.

In this study, 99% VaR was used to estimate the maximum risk of the sample portfolio, which resulted in no rejection based on the POF test. However, it would be interesting to establish whether a similar result would be obtained by using another confidence level.

The second future study that could be conducted is by changing the period of study. It would be interesting to observe whether the Monte Carlo-Copula method can cope with more extreme market conditions, such as the 2007-2008 US financial crisis or the 1997-1998 Asian financial crisis.

VI. CONCLUSION

In this article, the use of copulas to estimate VaR with Monte Carlo Simulation in the Indonesian stock market has been presented. The use of copulas can be a solution to problems that arise from modeling the multivariate distribution of financial asset returns. By using copulas to form multivariate distribution, normality assumptions that are not suitable for financial asset returns can be avoided.

Based on the estimated VaR portfolio results consisting of the IDX Development Board and Main Board Indices, it can be concluded that the Monte Carlo-Copula method can be used to estimate VaR within the Indonesian stock market. Therefore, this method can be an alternative to other VaR estimation methods for used with the Indonesian stock market.

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