

Optimal LQ45 Stock Allocation and Normal Contribution in a Defined Benefit Pension Plan

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Abstract—This study aims to determine the model for optimal stocks allocation and normal contribution that can minimize the funding variation based on stocks returns and dynamic mortality rates in a Defined Benefit Pension Plan. In this study, assets are allocated to the stock market, as investments in the stock market can increase funding variation that lead to high risk of decreasing funds as well as lack of funds in paying Pension Benefits to participants. The optimization model used in this study is a model which the objective function is a quadratic function. The stocks used in this study were SMRA, PWON, GGRM, INTP, UNTR, UNVR, BBTN, PTBA, SCMA and ANTM, in addition to mortality rates using probability death data of female civil servants with age ranges from age 52 to 60 in 2008 to 2015. By using the optimization model, the proportion of SMRA is 6.59%, PWON is 19.42%, GGRM is 3.54%, INTP is 7.32%, UNTR is 8.03%, UNVR is 18.87%, BBTN is 16.71%, PTBA is 6.51%, SCMA is 9.07%, and ANTM is 3.94% and also normal contribution is Rp 20,976,310.

Index Terms—defined benefit pension plan, stock allocation, normal contribution, optimization

I. INTRODUCTION

A Defined Benefit Pension Plan is said to be in a state of funds, if the amount of its net assets is sufficient to meet its liabilities. If underfunding, then the sponsor is obliged to make a payment of some additional funds in order to achieve the condition of the funds are met. Therefore, investment and financial risks in the Pension Fund are the responsibility of the sponsor of the Pension Fund. The Defined Benefit Pension Plan is definitely vulnerable to significant financial downside risks triggered by market volatility and/or a significant increase in mortality.

According to Cox, Lin, Tian, and Yu, the Defined Benefit Pension Plan can reduce these risks through a stable contribution strategy [1]. Stability in contributions means there are no significant fluctuations in the payment of fees per period. The contribution paid per period by the Pension Fund is a normal contribution. Normal contribution fluctuations are affected by differences in additional contributions that must be paid by the Pension Fund each period. In addition, risks can also be overcome by optimizing the allocation of assets by investing assets in certain types of investments.

In practice, not all Pension Fund assets are fully invested in money markets or capital markets or other instruments. The realization of optimal asset allocation in the stock market

according to Cox, Lin, Tian and Yu [1]: (i) If the proportion of investment in the stock market is lower than others to reduce the risk of underfunding, this means that the Pension Fund must set higher requirements to Total Pension Costs. Low rates of return along with significant changes in mortality require a large additional contribution to meet the liabilities that cause the Total Pension Costs to increase; (ii) If the proportion of investment in the stock market is greater than others to reduce the risk of underfunding, this increases the funding variation and causes a higher risk of decreasing funds. Investing in stocks results in higher returns in the long-term but is riskier. Therefore, to get an optimal asset allocation strategy, the Pension Fund must consider the risk of decreasing funds and Total Pension Costs.

Based on BPS publications on Indonesia's population projections for 2010-2035, there is an increase of life expectancy in population for ages 65 and above [2]. This means that the increase in life expectancy of the population can causes the risk of longevity. This high risk of longevity causes higher Pension Costs since the payment of Benefits longer than it planned. Therefore, the Pension Fund must consider the risk of longevity, so that the obligation to pay Benefits in the future does not exceed the present value of the funds.

Financial risk in the Defined Benefit Pension Plan, whether funding downside risk and higher Pension Costs caused by investments in the stock market and also the lack of funds to meet the payment of Benefits over longevity risks can be overcome by the Pension Fund. According to Cox, Lin, Tian and Yu, Pension Fund can reduce these risks by minimizing the variation in the amount of funding in the Defined Benefit Pension Plan throughout the life of the retired participant group [1]. In this study, to minimize total funding variation, Pension Fund can use a method that pursue a strategy with stable normal contribution in each period and the proportion of allocation into stocks is optimal. Thus, the Pension Fund can anticipate the risk of funding variation in order to meet the Pension Obligation for active participants when they retire and participants who have retired.

II. LITERATURE REVIEW

The Defined Benefit Pension Plan is fully responsible for the fulfillment of the Pension Benefit Obligation regardless

of the financial market risk and how long the participants remain alive after retirement. There are two types of funding in Pension Fund which come from Normal Contribution and Investment. These two types of funding are generally Pension Fund concerned since it can cause high risk in financial health. The volatility of capital markets and longevity risk forces the Pension Plan to manage pension risk. According to Lee, the risk of the Defined Benefit Pension Plan may be reduced by applying a stable contribution strategy [3]. Associated with Lee, several studies have been conducted on the management of the pension risk [3].

Haberman, Butt and Megaloudi, investigate an optimization problem with an objective to minimize the variance of periodic contributions [4]. In their study, the risk of pension is divided into two, namely the contribution fee and solvency risk associated with the stability of contribution and also funding. To minimize these two risks, Haberman, Butt and Megaloudi uses the unfunded liability or spread method as a control variable [4]. In addition, it also applies asset allocation adjustments to funding the pension schemes to avoid overfunding and for investment returns using stochastic processes. The method in this research is using spread period as control variable to minimize these two risks.

In the study of Colombo and Haberman, the membership of the Pension Fund of the Defined Benefit program is dynamic, in which the number of participants and the age of the participants is random [5]. This can thus be shown the balance between the cash flows arising from the changing condition of the participants. A mismatch between an asset and a liability may increase the total pension cost due to the additional contribution required to retire obligations. In this study, Colombo and Haberman focused on the dynamic membership of the Defined Benefit Pension Fund by determining the optimal dues that can minimize the variance of underfunding and overfunding.

Furthermore, Maurer, Mitchell and Rogalla identified strategies of contributions and investments that minimized the risk of pension contributions and also applied CVaR restrictions to the total pension costs in accordance with public pension schemes [6]. By focusing on the CVaR of the present stochastic value of the pension cost, it can be compared various contributions and investment strategies that are interlinked. In addition, this approach makes it possible to explicitly incorporate budget risk budgets, by determining the maximum acceptable CVaR of retirement costs.

In this study, the focus attention is the optimization of stock allocation and normal contribution in Defined Benefit Pension Plan. It is assumed that assets are invested only in stock market because its return is higher in the long term and the allocation into several stock can increase the funding variation that can leads to the risk that Pension Fund assets fall for below its liabilities. As in Chang, Tzeng and Miao, the contribution risk is measured by the square of deviation between the employer's contribution and target contribution rate [7]. Contribution made by Pension Fund called normal contribution and the payment is periodic. According to Cox, Lin, Tian and Yu, the

unstable normal contribution may raise total pension cost that leads to funding downside risk. To derive optimal proportion of stocks and stable normal contribution, focus of this study to minimize the variance of plan contributions and also impose a constraint to specify expected total pension cost and to control funding downside risk.

III. RESEARCH METHODOLOGY

A. Pension Fund Optimization Model

The optimization model is intended to minimize funding variation that occur due to investments in the stock market. Referring to Cox, Lin, Tian and Yu [1], this optimization model is to overcome the problem of stock weights $w = w_1, w_2, \dots, w_n$ and normal contribution C , so as to minimize the funding variation J throughout the age of the retirement group:

$$\begin{aligned}
 \text{Minimize}_{w,C} \quad & E \left[\sum_{t=1}^{\text{infy}} \left(\frac{UL_t}{(1+\rho)^t} \right)^2 \right] \\
 \text{subject to} \quad & E(TUL) = 0 \\
 & E(TPC) = \zeta \\
 & CVaR_{\alpha}(TUL) \leq \tau \\
 & 0 \leq w_i \leq 1, i = 1, 2, \dots, n \\
 & \sum_{i=1}^n w_i = 1 \\
 & C \geq 0
 \end{aligned} \tag{1}$$

where, UL_t is Unfunded Liability occurred in period t , TUL is Total Unfunded Liability, TPC is Total Pension Costs, ρ is valuation rate that reflects the expectation of future investment return and ζ and τ is constant.

This optimization model can control funding variation, Total Pension Costs and also the risk of underfunding. The risk of funding variation in this optimization model is modeled by the quadratic objective function J which is stated as follows:

$$J = E \left[\sum_{t=1}^{\infty} \left(\frac{UL_t}{(1+\rho)^t} \right)^2 \right]. \tag{2}$$

The Unfunded Liability in the objective function J describes the circumstances in which the available funds are not sufficient to fulfill Actuarial Obligations. When Unfunded Liability is very large, this can result underfunding, whereas the Unfunded Liability is negative, or it can be said that there is no Unfunded Liability, this can results overfunding. Based on Cox, Lin, Tian and Yu, the objective function can minimize variance from two funding status, underfunding and overfunding [1].

Contributions paid regularly are the Total Pension Costs in the Defined Benefit Pension Plan. The Total Pension Cost is defined as the present value of all normal contribution C , supplementary contribution SC_t and withdrawal W_t . To control the Total Pension Cost risk, the Total Pension Costs are modeled with TPC stated as follows:

$$TPC = \sum_{t=1}^T \frac{C}{(1+\rho)^t} + \sum_{t=1}^{\infty} \frac{SC_t(1+\psi_1) - W_t(1-\psi_2)}{(1+\rho)^t} \quad (3)$$

where $SC_t = \max(k \cdot UL_t, 0)$ and $W_t = \max(-k \cdot UL_t, 0)$. Constants ψ_1 and ψ_2 are penalty factors on additional contributions SC_t and withdrawal W_t . The constant k is the amortization factor. The penalty factor ψ_1 takes into account the opportunity cost that arises due to an additional mandatory additional contribution. While the penalty factor ψ_2 takes into account the tax Benefits that must be incurred. The Total Unfunded Liability until the final age of the Pension group, which is notated by TUL is stated as follows:

$$TUL = \sum_{t=1}^{infy} \frac{UL_t}{(1+\rho)^t} \quad (4)$$

Unfunded Liability occur when actuarial liabilities exceed the available funds. Notated as UL_t , can be stated as follows:

$$UL_t = \frac{1}{(1-k)} (PBO_t - \sum_{i=1}^n S_{i,t}), t = 1, 2, \dots, T, \quad (5)$$

where, PBO_t is Pension Benefit Obligation at period t , PA_t is accumulated funds at period t , $S_{i,t}$ is amount invested in stock i at period t , k is amortization factor equals where $k = 1/\ddot{a}_{\overline{m}|r}$. Assumed that the regulation allows for amortization of the actuarial liability insufficiency for $m > 1$ period at the periodic discount rate r . PBO_t which is defined as the discounted value of actuarial liabilities in the future, as

$$PBO_t = \begin{cases} \frac{B(a(x(T)))}{(1+\rho)^{T-t}} & t = 1, 2, \dots, T \\ B(a(y(t))) & y = x + 1, x + 2, \dots \\ & t = T + 1, T + 2, \dots \end{cases} \quad (6)$$

where $(a(x(T)))$ is the discounted conditional expected value of payments of 1 per year as long as the retiree survives, as

$$(a(x(T))) = \sum_{s=1}^{\infty} v^s s\hat{p}_{x,T} \quad (7)$$

Notation $v = \frac{1}{1+r}$ is a discount factor at the discount rate r . Notation $s\hat{p}_{x,T}$ is conditional expected s -year survival rate for age x at retirement T as

$$s\hat{p}_{x,T} = E \left[{}_s\tilde{p}_{x,T} \left| \tilde{p}_{x,T}, \tilde{p}_{x+1,T+1}, \dots, \tilde{P}_{x+s-1,T+s-1} \right. \right] \quad (8)$$

where $s\tilde{p}_{x,T}$ is probability that the participant of the Defined Benefit Pension Plan age x at T can survive to age of $x + s$ at the beginning of the year $T + s$ with $s = 1, 2, \dots$ and gets a Benefit payment.

The initial accumulation of funds at $t = 0$ is $PA_0 = M$ invested in different stocks. M is the initial fund in the Defined Benefit Pension Plan in the form of a constant. Then the accumulated value of funds PA_t at $t, t = 0, 1, \dots$, depends

on the amount invested in stock i at $t-1$, $S_{i,t-1}$, and its return in period t , $r_{i,t}$ is

$$PA_t = \sum_{i=1}^n S_{i,t-1} (1 + r_{i,t}), t = 1, 2, \dots \quad (9)$$

Assume that the Pension Fund invest in stocks i , with a proportion of w_i . Then the value of the stock i at t is

$$S_{i,t} = \begin{cases} (1-k)S_{i,t-1}(1+r_{i,t}) + (1-k)C \cdot w_i + \\ k \cdot PBO_t \cdot w_i; t = 1, 2, \dots, T \\ (1-k)S_{i,t-1}(1+r_{i,t}) - (1-k)B \cdot {}_{t-\tau}\hat{p}_{x,t} \cdot w_i + \\ k \cdot PBO_t \cdot w_i; t = T + 1, \dots \end{cases} \quad (10)$$

When $t = 0$, $S_{i,0} = w_{i.M}$ for $i = 1, 2, \dots, n$.

B. Stock Price Dynamics Model

In this study, assets are allocated in the form of investment into several stocks. It is assumed that these stocks follow the Geometric Brownian Motion (GBM) because it can describe the random behavior of the stock price level of $S(t)$ over time. According to Reddy and Clinton [8], GBM assumes that stock prices have a lognormal distribution with certain mean and standard deviation, as

$$S_{i,t+\Delta t} = S_{i,t} \exp \left[\left(\alpha_i - \frac{1}{2} \sigma_i^2 \right) \Delta t + \sigma_i \Delta W_{it} \right], \quad (11)$$

where $S_{i,t}$ is the stock price at time t , Δt is the time interval for the stock price prediction, α_i is the expectation of return, σ_i is the expectation of volatility. Equation (11) shows that the stock price that follows GBG has a lognormal distribution, while the stock return is normally distributed.

According to Brigo, Dallesandro, Neugebauer and Triki, two important characteristics in the GBM case is the log return must be normally distributed and independent [9]. Independent assumptions in this case mean that there is no autocorrelation in the log return. After testing for normality and independence, then the parameters for α and σ in equation (11) can be calibrated. To find α and σ that correspond to historical data, the maximum likelihood estimation method is used. By knowing the parameters α and σ , the stock price can be estimated for the next period.

According to Bodie, Kane, Marcus and Jain, the Capital Asset Pricing Model (CAPM) provides an accurate prediction of the relationship between asset risk and expected return [10]. CAPM is used to predict the balance of the expected return and risk of an asset in equilibrium conditions. In equilibrium the level of required return by investors for a stock is affected by the risk of the stock. In this case the risk calculated is only systematic risk or market risk as measured by beta coefficient (β). An approach to making a single operational factor model is to assert that the rate of return on a broad security index is a valid proxy for general macroeconomic factors. This approach leads to equations similar to single factor models, called single-index models because it uses market indexes to proxy common factors.

In the single-index model, the market index is denoted by M where the excess return, $R_M = r_M - r_f$, and the standard deviation σ_M . Because the single-index model is linear, it can be estimated that the sensitivity (beta) of stock in the market index uses a linear regression, by regressing the excess return from stock i , $R_i = r_i - r_f$ on the excess return from the market index R_M . For regression estimation, historical sample data from $R_i(t)$ and $R_M(t)$ are used, where t is a daily observation. That way, the regression equation is

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t), \quad (12)$$

with intercept α_i is the expected excess return from stock when the excess return from the market is zero. The slope coefficient β_i , is a stock beta. Beta coefficient is the sensitivity of stock with a market index, which is the number of stock returns tend to increase or decrease by the value of the beta coefficient for each increase or decrease of 1% in market index returns.

C. Research Design

- Assets is allocated in stock market. Stocks selection based on the concept of a single-index model. It is assumed that the individual stock is chosen by its high and low risk. To measure the risk, beta coefficient is used. To make a diversified stock portfolio, the number of individual stock and its beta coefficient criterion are determined in advance. After the stocks are chosen, assume that stock prices follow GBM by testing normality and independent. After the GBM assumption is fulfilled, using historical data, the parameters can be estimated using the maximum likelihood estimation method.
- Mortality rates using mortality tables based on Hangarawati's research [11].
- Set case study illustration for optimization problems. This provides some information to make basic optimization model.
- Giving initial condition, assume that the proportion of each stock invested equal and normal contribution is assumed to have been determined in advance to make the initial model condition. To get the optimal solution, a simulation is conducted to generate stock returns and Pension Benefit Obligation in the future. The stock returns and Pension Benefit Obligation are used in the case study illustration to determine the basic framework of the optimization model in the initial conditions
- Using the basic framework model in solving optimization problems with certain constraints to determine the optimal proportion of stock allocation and normal contribution.

IV. RESULTS

A. Stock Price Dynamics Data

In this study, LQ45 are used in determining the individual stock used in the optimization model. From LQ45, stock that has high and low risk are chosen. In determining these risks,

the beta coefficient used as a risk determinant of individual stock on LQ45. The period in determining the beta coefficient is from January 1, 2017 to January 1, 2018. From 45 stock, ten stocks are selected which is five stocks with large beta coefficients and five stocks with small beta coefficients. With this selection, it can form a diversified stock portfolio. Thus, the allocation of stocks in this study is divided into ten individual stock which is SMRA; PWON; GGRM; INTP; UNTR; UNVR; BBTN; PTBA SCMA; and ANTM.

In describing the random behavior of the price movements of each of these stocks, it is assumed that each stock follows GBM. In this study, the historical data of each stock used is the daily closing price data from January 1, 2012 to January 1, 2018. In order for the stock movement to be assumed to follow GBG, the return of each stock must be normally distributed and independent. By using the concept of the central limit theorem, it can be assumed that the daily return of each stock is normally distributed. For independence, the autocorrelation function is used and the results for each stock is there is no significant lag. That way it can be concluded that the daily return of each of these stocks is independent.

The random behavior of stock price movements that follow GBM is illustrated by equation (11), where there are parameters α and σ . The maximum likelihood estimation method is used for finding α and σ .

After obtaining the estimated parameters α and σ , the stock price of each stock can be obtained for the future. In this study, one period is considered one year. Therefore, the daily return of stocks obtained from the sample path simulation will be converted to return in annual terms. For 1000 simulations, a 2016-day stock return was obtained. It is assumed that for one year there are 252 days, so that eight periods are obtained in a year.

B. Case Study Illustration for Optimization

In order to see the difference between optimal stock allocation and normal contributions and those that are not, the initial condition is given. Assume the base year of this study is 2007, when $t = 0$. The retirement group that join the Defined Benefit Pension Plan, all join at the age $x_0 = 52$ years and enter retirement when $t = T = 4$ that is at the age of 56. Assume that x_T will survive for $s = 4$ years at the age of 60. Assume the initial accumulation funds in the Pension Plan is Rp. 50 million at $t = 0$. The valuation rate $\rho = 0.08$ and the discount rate $r = 0.05$. This Pension Plan amortizes Unfunded Liability for $m = 3$ years. It is assumed that the penalty factor for supplementary contribution and withdrawals is $\psi_1 = \psi_2 = 0.2$. Assume the proportion of stock allocation is equal $w_1 = w_2 = \dots = w_{10} = \frac{1}{10}$. Then assume the normal contribution is $C = Rp.20$ million. In this study, to solve the optimization problem, the 1000 simulations are performed to generate annual returns and generate Pension Benefit Obligation for the next period.

1) *Initial Condition:* By using the available information, it can be determined that the Unfunded Liability for eight years. After knowing the Unfunded Liability for each year,

TABLE I
NORMAL CONTRIBUTION AND PROPORTION OF STOCK ALLOCATION

	Optimization
C	Rp. 20,976,310
SMRA	6.59%
PWON	19.42%
GGRM	3.54%
INTP	7.32%
UNTR	8.03%
UNVR	18.87%
BBTN	16.71%
PTBA	6.51%
SCMA	9.07%
ANTM	3.94%

a Total Unfunded Liability can be obtained. Thus, expectation of Total Unfunded Liability until the final age of the Pension group is $E(TUL) = Rp.598,942,394$. To manage the downside risk of Total Unfunded Liability, the 95th percentile is given, so $CVaR_{95\%}(TUL) = Rp.812,497,206$. For Total Pension Costs is, $E(TPC) = Rp.227,291,173$. By using information that has been obtained previously, it can be determined the quadratic objective function J , which is $J = Rp.102,518,229,882,095,000$.

2) *Optimization*: The proportion of stock allocation and normal contribution in initial condition has been determined in advance. In this subsection, the quadratic objective function that can minimize funding variation in the Defined Benefit Pension Plan is obtained through the optimal proportion of stock allocation and normal contribution.

To get the optimal solution, the constraints are given. For $E(TPC) = \zeta$, it is a constant that is assumed to be the same value as initial condition, $E(TPC) = \zeta = Rp.227,291,173$. Then, given the constraint $CVaR_{95\%}(TUL) = \tau = Rp.812,497,206$. In addition, it is assumed that the expectations of Total Unfunded Liability $E(TUL) = 0$. That way, the optimization problem can be solved by toolbox optimization in matlab where the optimization type is quadratic programming because the objective function of the optimization model is a quadratic function. In order to obtain a minimum funding variation, the proportion of stock allocation and normal contributions is obtained as follows:

Using information from Table I, it can be determined the quadratic objective function J , which is $J = Rp.89,376,223,584,270,700$. The proportion of stock allocations and normal contributions is the result of using all ten stock data, female civil servant mortality data aged 52 to 60 years from 2008 to 2015 and case study illustrations. By using the optimization model framework obtained from the initial conditions, and given certain constraints, the optimization results are obtained from the proportion of stock allocation and normal contributions as in Table I. In order for the normal contribution paid by the Pension Fund remain stable during this period, the normal contribution paid by the Pension Fund annually is Rp. 20,976,310. Thus, from the stock portfolio and also the mortality data used, the Pension Fund can estimate how much funds are needed to fulfill its actuarial obligations,

so that there will be no actuarial liabilities.

By knowing the optimal proportion of share allocation and normal contribution, the Defined Benefit Pension Plan can manage funds in accordance with the available funds, so there is no significant risk. A stable normal contribution for each year is expected to not require a large additional contribution in fulfilling Pension liabilities and is also expected to reduce the Total Pension Costs considering all costs and penalties related to normal contributions. Allocation in the stock market can increase the variation in funding which results in a high risk of a decrease in funds, because investing in stocks market results in higher investment returns for a long period of time but is more risky. By controlling the proportion of asset allocation in the stock market and also the payment of normal contributions every year, the Pension Fund is expected to minimize the risk of significant losses. Because the main funding source of the Pension Fund lies in contributions and also investment returns. Therefore, this optimization model is an efficient way for the Defined Benefit Pension Plan to face risks such as a decrease in funding and high Pension costs caused by investments in the stock market and also the lack of funds to meet the payment of Benefits over the risk of longevity. That way, the Defined Benefit Pension Plan can carry out its obligations to provide Benefits to its participants.

V. DISCUSSION

In its realization, publication of mortality data in Indonesia is only available for static models. Whereas for this study it is assumed that mortality rates is dynamic models, where the model considers the function of time other than age function. Since this study used the mortality rate of the research of Hanggarawati [11], the authors suggest for further study in order to consider the mortality rate of workers in all industrial sectors in Indonesia, not limited to civil servants as well as men and women.

In accordance with the regulations of the OJK in 2015, there are 17 types of investments that are allowed by the regulator [12]. The author only considers the type of investment in stocks because the purpose of this research is to minimize variations in the amount of funding. As is well known, investing in stocks results in higher investment returns for a long period of time but is more risky, so that it can increase the

variation in funding that can cause a high risk of decreasing funds. Thus, the authors suggest that further research consider the allocation of assets other than stock.

VI. CONCLUSION

To get the optimal proportion of stock allocation and normal contribution in Defined Benefit Pension Plan, an optimization model is determined with quadratic objective functions and certain constraints. The quadratic objective function can minimize the variance from two funding status (underfunding and overfunding). In this study, asset is only allocated in stock market, reasoning that stock is expected to earn higher return in the long term. Given case study illustration, a basic framework of optimization is obtained. The basic framework of this optimization is carried out in order to show the effect of using the optimal proportion of stock allocation and normal contribution strategies. That way, we get the proportion of optimal stock in SMRA, PWON, GGRM, INTP, UNTR, UNVR, BBTN, PTBA, SCMA and ANTM, accompanied by mortality rates for female civil servants aged 52 to 60 years from 2008 to 2015, the proportion of allocation for SMRA's is 6.59%, for PWON is 19.42%, for GGRM is 3.54%, for INTP is 7.32%, for UNTR is 8.03%, for UNVR is 18.87%, for BBTN is 16.71%, for PTBA is 6.51%, for SCMA is 9.07%, and for ANTM is 3.94% and obtained normal contribution paid annually is Rp. 20,976,310.

Given that the main funding sources for the Defined Benefit Pension Plan come from contributions and also investment returns, it is important for the Pension Fund to have a stable contribution strategy for each year. This stable contribution can limit the Total Pension Costs that must be incurred by the Pension Fund each year so that it can provide an upper limit to the Pension Fund in managing its resources. In addition, with the minimal amount of funding variation in the Defined Benefit Pension Plan, it is expected to help Pension Fund in facing market volatility that affects the risk of decreasing funds in the company. So this optimization model becomes an efficient way to deal with these risks.

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