Estimation of the Contribution of Intensive Factors of Economic Growth of the USA

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Abstract— A difficult problem of estimation of the effectiveness of alternative options exists when choosing the direction of development of the national economy. To solve this problem, the author worked out a new tool in the form of a modified production function (PF) with variable parameters, based on the hypothesis of the variability of the rate of economic growth due to technical progress. Using modified PF lets reduce to a common base year the source data and estimate the PF for non-homogeneity periods which have both the periods of growth and decline. In the analysis, interest is not represented by the average annual contribution of production factors to product growth, but by the direct contribution of variable factors to product growth. The same result can be achieved both due to the extensive growth of production factors, and due to their qualitative growth. The developed by author tool lets evaluate intensive production factors contribution to output increase and determine the efficiency of development of the national economy. Verification of developed approach is shown by the example of the economic development of the USA. The development of the economy of the U.S. was predominantly intensive. Modified PF is universal, because under certain conditions, it is transformed into traditional static Cobb-Douglas’ PF and Tinbergen’s dynamic function.

Keywords— economic growth, the USA, production functions, technical progress, the factors contribution, the output increase

I. INTRODUCTION

The choice of effective development of the national economy is not possible without solving the problem of evaluating the effectiveness of alternative options. In traditional production functions, the parameters are assumed to be constant for the entire analyzed period. Because of the constancy of the parameters of the PF, it is possible to investigate not the variable but only the averaged contribution of factors to the rates of economic growth.

The transition to modified PF with variable parameters makes it possible to estimate PF not with constant, but with variable rates of economic growth.

Along with the estimation of the variable parameters of the modified PF, the problem of estimating of the variable contribution of factors to the increase of production is also topical.

II. PRODUCTION FUNCTIONS IN MODELING THE VARIABLE CONTRIBUTION OF FACTORS TO THE RATES OF ECONOMIC GROWTH

To study the problem of production efficiency, the author proposes to consider the PF with variable parameters:

\[ Y_t = A_t \cdot L_t^{\alpha_t} \cdot L_t^{\beta_t}. \]  

To estimate the variable parameters of the PF (1), it is necessary to go to a modified PF with constant parameters \( A_0 \), \( \alpha_0 \), \( \beta_0 \) and a variable rate of economic growth \( e^{\theta_t} \) due to technical progress and other unaccounted factors [13-19]:

\[ Y_t = A_0 \cdot K_t^{\alpha_0} \cdot L_t^{\beta_0} \cdot e^{\theta_t}, \]

\[ e^{\theta_t} = e^{\theta_0} \cdot e^{\lambda t}, \]

\[ \Theta_t = \ln \left( \frac{A_0 \cdot K_t^{\alpha_0} \cdot L_t^{\beta_0}}{A_0 \cdot K_0^{\alpha_0} \cdot L_0^{\beta_0}} \right), \]

\[ \Theta_0 = 0, \]

\[ \Theta_t = \Delta \Theta_t + \Delta \Theta_t + \ldots + \Delta \Theta_t = \Theta_t + \Delta \Theta_t + \ldots + \Delta \Theta_t = \lambda. \]

The modified PF (2) is an universal and under certain conditions it is transformed, respectively, either into a static Cobb-Douglas PF [11] or to a dynamic Tinbergen PF [5]:

\[ Y_t = A \cdot K_t^{\alpha} \cdot L_t^{\beta}, \quad \Theta_0 = 0; \]

\[ Y_t = A \cdot K_t^{\alpha} \cdot L_t^{\beta} \cdot e^{\Delta \theta_t}, \quad \Theta_0 = 0. \]

If we differentiate (2), we obtain:

\[ \frac{\Delta Y_t}{Y_{t-1}} = \alpha_0 \cdot \frac{\Delta K_t}{K_{t-1}} + \beta_0 \cdot \frac{\Delta L_t}{L_{t-1}} + \Delta \Theta_t. \]

On other hand, from equations in the logarithms (2) and in the cumulative data (9) we will have following expressions, considering (5) and (6):

\[ \ln Y_t = \ln A_0 + \alpha_0 \ln K_t + \beta_0 \ln L_t + \Theta_t; \]
\[
y(1,t) = \alpha_0 k(1,t) + \beta_0 l(1,t) + \Theta_t, \tag{11}
\]
\[
y(1,t) = \sum_{i=1}^{t} \frac{\partial y}{\partial Y_{t-1}} = \sum_{i=1}^{t} \frac{\partial K_i}{\partial K_{t-1}} + l(1,t) = \sum_{i=1}^{t} \frac{\partial L_i}{\partial L_{t-1}} \tag{11'}
\]

Excluding the value \( \Theta_t \) from (10) and (11), we have:
\[
\ln(Y_{t0}^{(0)}) = \ln(A_0) + \gamma_0 \ln(K_{t0}^{(0)}) + \beta_0 \ln(L_{t0}^{(0)}); \tag{12}
\]
\[
Y_{t0}^{(0)} = Y_t e^{-\gamma_0 t} + K_{t0} e^{-\gamma_0 t} + L_{t0} e^{-\gamma_0 t} - \text{modified variables at time } t \text{ relatively to the base year } t=0.
\]

So, the evaluation \( A_0, \alpha_0, \beta_0 \) on the basis of the least squares method is possible only after transformations done (12).

In practical calculations, we can use instead of \( \Theta_t \) the value of \( \Theta_t' \) from (11):
\[
\Theta_t' = y(1,t) - (\alpha_0 k(1,t) + \beta_0 l(1,t)). \tag{13}
\]

To compare values \( \Delta \Theta_t' \) and \( \lambda \) we can calculate of value \( \Delta \Theta_t' \):
\[
\Delta \Theta_t' = \frac{\sum_{i=1}^{t} \delta \Theta_t'}{n}. \tag{14}
\]

To calculate the proportion of unaccounted factors into the cumulative increase rate of production for \( t \) years we use the equation (11).

When divided both parts of (11) by \( y(1,t) \) we have
\[
l = \alpha_0 \frac{k(1,t)}{y(1,t)} + \beta_0 \frac{l(1,t)}{y(1,t)} + \frac{\partial k}{y(1,t)} \tag{15}
\]

The first and second items in (15) characterize the contribution of accounted factors \( K \) and \( L \), and the third item describes the contribution of unaccounted factors to the cumulative increase rate of output for \( t \) years.

From (15) we have that the contribution of accounted and unaccounted factors related as follows
\[
\frac{\partial k}{y(1,t)} = l \left( \alpha_0 \frac{k(1,t)}{y(1,t)} + \beta_0 \frac{l(1,t)}{y(1,t)} \right). \tag{16}
\]

The contribution of factors to the growth rates for the Tinbergen’s PF (8) as known has the form analogous to (9):
\[
\frac{\partial k}{y(1,t)} = \alpha_0 \frac{\Delta K_t}{K_{t-1}} + \beta_0 \frac{\Delta L_t}{L_{t-1}} + \lambda. \tag{16'}
\]

Naturally, the smaller cost of capital and labor, the greater the proportion of unaccounted factors into the cumulative increase rate of output. When the value \( \frac{\partial k}{y(1,t)} \) increases, it corresponds to an intensification of production. And conversely, when the value \( \frac{\partial k}{y(1,t)} \) decreases, it corresponds to an extensification of production.

In the analysis the estimation is more interest not an average annual of factors contribution to the output increase, obtained after division (11) by \( y(1,t) \), and direct variable of factors contribution to the output increase:

Since the function parameters (2) correspond to the parameters \( A_0, \alpha_0, \beta_0 \) of function (1), the elasticity coefficients \( \alpha_0, \beta_0 \) of base year \( t=0 \) satisfy to identities:
\[
\alpha + \beta = \chi. \tag{17}
\]

With respect to these correlations we have from (9):
\[
\frac{Y_t - Y_{t-1}}{Y_{t-1}} = \alpha_0 \frac{\Delta K_t}{K_{t-1}} + \beta_0 \frac{\Delta L_t}{L_{t-1}} + \frac{\partial k}{y(1,t)} = \chi \tag{17'}
\]

When multiplied both parts of the obtained correlation by \( Y_{t-1} \) we obtain:
\[
Y_t - Y_{t-1} = \alpha_0 \Delta K_t + \beta_0 \Delta L_t + a_0 \cdot (f_{t-1}^{(0)} - 1) \cdot \Delta K_t + b_0 \cdot (f_{t-1}^{(0)} - 1) \cdot \Delta L_t + \Delta \Theta_t \tag{17''}
\]

Indicators \( f_{t-1}^{(0)}, p_{t-1}^{(0)} \) show the increase rates of average efficiencies of factors \( K \) and \( L \) in regard to the efficiency of the base year 0, or base indices of capital productivity and labor productivity respectively.

Having summed up (17) we have:
\[
Y_t - Y_0 = \alpha_0 \cdot (K_t - K_0) + \beta_0 \cdot (L_t - L_0) + \chi \tag{17'''}
\]

Let’s derive following correlation:
\[
\sum_{i=1}^{t} (f_{i-1}^{(0)} - 1) \cdot \Delta K_t = \sum_{i=1}^{t} (f_{i-1}^{(0)} - 1) \cdot \Delta K_t - \sum_{i=1}^{t} \Delta K_t = \chi \tag{17''''}
\]

Also we can obtain:
\[
\sum_{i=1}^{t} (p_{i-1}^{(0)} - 1) \cdot \Delta L_t = \sum_{i=1}^{t} (p_{i-1}^{(0)} - 1) \cdot \Delta L_t - \sum_{i=1}^{t} \Delta L_t = \chi \tag{17''''}
\]

The efficiency of labor and capital engaged in production during different years, which are changing relatively of efficiency of base year \( t=0 \), is reflected in values \( K_{t,0} \) and \( L_{t,0} \).
by the aid of $f_{t-1}^{(0)}$ and $p_{t-1}^{(0)}$. Adding (20) and (22) to (19) we have:

$$Y_t - Y_0 = a_0(K_t - K_0) + b_0(L_t - L_0) + a_0(K_{t,0} - K_t) +$$

$$+ b_0 \cdot (L_{t,0} - L_t) + \sum_{i=1}^{t} \Delta \Theta_i \cdot Y_{t-i};$$  

(23)

$$Y_t - Y_0 = a_0(K_t - K_0) + b_0(L_t - L_0) + (a_0^{(0)} - a_0)K_t +$$

$$+ (b_0^{(0)} - b_0) \cdot L_t + \sum_{i=1}^{t} \Delta \Theta_i \cdot Y_{t-i};$$  

(24)

$$a_t^{(0)} = a_0 \cdot \frac{K_0}{K_t}, \quad b_t^{(0)} = b_0 \cdot \frac{L_0}{L_t};$$  

(25)

$$a_0^{(0)} = a_0, \quad b_0^{(0)} = b_0.$$  

(25)

The values $a_t^{(0)}$ and $b_t^{(0)}$ grow with the increase of efficiency and reduce with the decrease of these, because $f_{t-1}^{(0)}$ and $p_{t-1}^{(0)}$ in the first case are more than 1, and in the second case they are less than 1.

We can derive following correlations:

$$\left(a_t^{(0)} - a_0\right) \cdot K_t = \left(a_0^{(0)} - a_0\right) \cdot K_0 + \left(a_t^{(0)} - a_0\right) \cdot (K_t - K_0);$$

$$\left(b_t^{(0)} - b_0\right) \cdot L_t = \left(b_0^{(0)} - b_0\right) \cdot L_0 + \left(b_t^{(0)} - b_0\right) \cdot (L_t - L_0).$$

After adding these correlations in (24) we have:

$$Y_t - Y_0 = a_0(K_t - K_0) + b_0(L_t - L_0) + \left(a_t^{(0)} - a_0\right)K_t +$$

$$+ \left(b_t^{(0)} - b_0\right) \cdot L_t + \left(a_t^{(0)} - a_0\right) \cdot (K_t - K_0) +$$

$$+ \left(b_t^{(0)} - b_0\right) \cdot (L_t - L_0) + \sum_{i=1}^{t} \Delta \Theta_i \cdot Y_{t-i}.$$  

(26)

After dividing obtained (26) by $(Y_t - Y_0)$ we have the formula:

$$I = a_0 \left(\frac{K_t - K_0}{Y_t - Y_0}\right) + b_0 \left(\frac{L_t - L_0}{Y_t - Y_0}\right) + \left(\frac{a_t^{(0)} - a_0}{Y_t - Y_0}\right) K_0 +$$

$$+ \left(\frac{b_t^{(0)} - b_0}{Y_t - Y_0}\right) L_0 + \sum_{i=1}^{t} \Delta \Theta_i \cdot Y_{t-i}.$$  

(27)

According to this formula (27) the first two items $E_K$ and $E_L$ show the contribution of factors $K$ and $L$ into the increase of output at the expense of their extensive growth, the third and the fourth items $I_0(a)$ and $I_0(b)$ at the expense of their efficiencies change. The fifth and the sixth items $I_0(a,K)$ and $I_0(b,L)$ are the indivisible residuals obtained at the expense of changing efficiency and factors growth. The last item $I_{(un)}$ shows the contribution of unaccounted factors. The sum of the third and the fifth items as well as the fourth and the sixth items we will define as $E_K$ and $I_L$ respectively:

$$I_K = \left(a_t^{(0)} - a_0\right) \cdot \left(\frac{K_t - K_0}{Y_t - Y_0}\right) +$$

$$= I_K(a) + I_K(a, K).$$  

(28)

$$I_L = \left(b_t^{(0)} - b_0\right) \cdot \left(\frac{L_t - L_0}{Y_t - Y_0}\right) =$$

$$= I_L(b) + I_L(b, L).$$  

(29)

$$I_{(un)} = \frac{\sum_{i=1}^{t} \Delta \Theta_i \cdot Y_{t-i}}{Y_t - Y_0} = 1\left(E_K + E_L\right) + I_K + I_L.$$  

(30)

The values $E_K$ and $E_L$ characterize the intensive contribution of the factors $K$ and $L$ caused by both qualitative changes $K$ and $L$ and their quantitative growth. Hence, if the first two items of the obtained correlation determine $E_t$ contribution of extensive factors, so the other items $I_K, I_L, I_{(un)}$ determine the contribution of all intensive factors:

$$E_t = E_K + E_L = a_0 \left(\frac{K_t - K_0}{Y_t - Y_0}\right) + b_0 \left(\frac{L_t - L_0}{Y_t - Y_0}\right),$$  

(31)

$$I = I_K + I_L + I_{(un)} = 1 - E_t.$$  

(32)

For example, if during all period the capital and labor productivities are invariable ($f_{t-1}^{(0)} = 1$), then contribution of accounted factors $K$ and $L$ at the expense of change of their efficiencies is equivalent to 0, because in this case $a_t^{(0)} = a_0, b_t^{(0)} = b_0, K_t = K_0, L_t = L_0$.

### III. EXPERIMENTAL ESTIMATING OF THE CONTRIBUTION OF INTENSIVE FACTORS TO ECONOMIC GROWTH OF THE USA

For the experimental evaluation of the contribution of intensive factors to the US economic growth in 1950-1979, and 1990-2003, we use traditional and modified PF (8) and (2).

In 1950 – 1979 as a final result of production of the USA economy we chose $Y_t$, Gross National Product (GDP) at constant 1972 prices (in billions dollars), and as factors of production – $K_t$, Net Stock of Fixed Nonresidential Private Capital at Constant 1972 Prices (in billions of dollars) (with the account of capacity utilization rates in Manufacturing) and $L_t$, Hours Worked by persons Engaged in production (in billions hours).

In 1990 – 2003 as a final result of production we chose $Y_t$, Gross National Product (GDP) at constant 2000 prices (in billions dollars), and as factors of production – $K_t$, Real Net Stock of Equipment and software (Billions of chained (2000) dollars; yearend estimates) and $L_t$, Hours Worked by persons Engaged in production (in billions hours).

In 1990-2003 as a final result of production we chose $Y_t$,
The obtained estimates of $\lambda$ and $\Delta \theta^*_t$ for the USA economy for the periods 1950-1979 and 1990-2003 are close respectively (Table 1, Fig. 1-2):

$$\lambda = 0.014; \quad \Delta \theta^*_t = 0.015; \quad \lambda = 0.011; \quad \Delta \theta^*_t = 0.009.$$ 

![Fig. 1. Dynamics of the values $\theta^*$ and $\lambda \cdot t$ of the USA economy in 1950-1979](image1)

![Fig. 2. Dynamics of the values $\theta^*$ and $\lambda \cdot t$ of the USA economy in 1990-2003](image2)

Therefore, Tinbergen’s PF can be used for modeling USA’s economic growth for the periods 1950-1979 and 1990-2003.
IV. CONCLUSION

The rates of US economic growth in 1950-1979 were higher than in 1990-2003. Thus, the rates of economic growth of the US due to technical progress in 1950-1979 amounted to an average of 1.5% per year, and in 1990-2003 - 0.9%.

Economic growth of the USA in 1950 - 1979 years was predominantly intensive as for the almost entire period the aggregate contribution of intensive factors to the increase of GNP exceeded 50%. Thus, the contribution of \( t \) in 1950 - 1960, 1950 - 1970 and 1950 - 1979's was 57.16%, 52.08 and 48.05%, respectively.

References


