Mathematics Teaching from "Three Dimensional Objectives" to "Core Accomplishment"
—Taking "Functional Extremum and Derivative" as an Example

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Abstract—The main goal of mathematics teaching was to cultivate students' core quality of mathematics, but it was difficult for teachers to change from three-dimensional goal to core goal of mathematics in actual classroom teaching. This article analyzed the mathematics teaching goals under three-dimensional goals and core literacy, gave the integration of teaching goals under three-dimensional goals and core literacy, and gave the teaching design under core literacy with the example of "functional extremum and derivative". In the course of teaching, students' core knowledge of mathematics can be better realized.

Keywords—Three-dimensional targets; Core literacy; The core quality of mathematics; Teaching Design

I. INTRODUCTION

In order to make up for the deficiency of double-base teaching, the Ministry of Education put forward the educational concept of "three-dimensional goal" in the Outline of Basic Education Curriculum Reform (Trial Implementation) promulgated in 2001, which is the beginning of a new round of basic education curriculum reform in China. In March 2014, the Ministry of Education first expounded the connotation of "core literacy" in "Opinions on Fully Deepening the Curriculum Reform and Implementing the Fundamental Task of Lide and Tree People". With the attention and exploration of scholars from all walks of life, combined with the characteristics of mathematics, the core qualities of mathematics that students should possess are given. "Three-dimensional goal" and "core literacy" are proposed to train students in an all-round way, so that students can grow into talents with different personal qualities and meet the social requirements. "Three-dimensional goal" is the basis of "core literacy", which is produced to meet the requirements of the times, create broad development space for students, and supplement the insufficiency of "three-dimensional goal".

From "three-dimensional objectives" to "core literacy" reflects the basic direction of basic education reform in China [1,2].

II. CHARACTERISTICS OF MATHEMATICS TEACHING OBJECTIVES UNDER "THREE-DIMENSIONAL OBJECTIVES"

A. "Three-Dimensional Objectives" and Its Significance

In the Outline of Basic Education Curriculum Reform (Trial Implementation) promulgated in 2001, it is clearly pointed out that the curriculum should reflect the basic requirements of the state for students at different stages in knowledge and skills, process and methods, emotional attitudes and values, stipulate the nature, objectives, content framework of each course, and put forward teaching and evaluation suggestions. Therefore, based on Bloom's taxonomy of educational objectives and combined with the actual situation of China's development, three-dimensional goals emerged as the times require. On the basis of paying attention to the basic theoretical knowledge imparting, paying attention to the teaching process and students' learning process experience, so that students can be promoted in theory and practice after the end of the course, and further stimulate students' thirst for knowledge, forming values in line with the mainstream of society [3].

The application of the three-dimensional goal of mathematics promotes the development of students from all aspects. It is hoped that in the process of mathematics classroom teaching, teachers can firmly grasp the basic theoretical knowledge of mathematics, form mathematical thoughts and solve practical problems, and shape students' correct outlook on life, values and abilities through their teaching design and guidance. Life attitude promotes the continuous improvement of mathematical literacy and provides all-round development talents for the society. Because of the increasing interaction between teachers and students in the classroom, the classroom has gradually become
relaxed and full of vitality, stimulated students' interest in mathematics learning, and can integrate theory with practice, so that students can understand the application of mathematics. At the same time, it has created a more harmonious and equal relationship between teachers and students. Teachers can really enter students' hearts and think from different angles.

B. Limitations of Mathematics Teaching Objectives under "Three-dimensional Objectives"

The emergence of "three-dimensional goal" is a qualitative change for the traditional double-base teaching mode, but in mathematics classroom teaching, there are also various problems of different sizes. First of all, most front-line teachers cannot clearly understand the true meaning of three-dimensional goals. When designing mathematics teaching objectives, process and method objectives, emotional attitudes and values goals adopt a fixed template, and fail to give appropriate process and emotions combined with mathematics classroom content. Furthermore, compared with the two-base teaching, the three-dimensional goal of mathematics has not been substantially changed. It still pays attention to the results of basic knowledge, which limits the diversity and flexibility of classroom forms. Secondly, education at this stage emphasizes the teaching concept of people-oriented, but in the design of three-dimensional objectives of mathematics, due to the pressure of entering school, most mathematics teachers still take knowledge and skills as the premise, ignoring the most basic learning conditions, leading to the emergence of a large number of "high marks and low abilities" students. Finally, in the three-dimensional objectives of mathematics, the theoretical basis and connotation of the three objectives are different, representing the various qualities that a comprehensive development student should possess. However, as a mathematics teacher, how to implement the three-dimensional goals has not given a practical way and direction.

The great development of mathematics since the seventeenth century owes much to analytic geometry. It can be said that differential calculus and integral calculus cannot be imagined without the advance development of analytic geometry. The creation of plane analytic geometry as a branch of mathematics is attributed to French mathematicians Descartes and Fermat. Its basic idea is to use coordinate method to unify the spatial figures (points, lines and surfaces) reflecting the same motion law with the quantitative relations (coordinates and the equations they satisfy), thus reducing the geometric problems to algebraic problems. As a branch of analytic geometry, plane analytic geometry is indispensable for research and discussion.

III. CHARACTERISTICS OF MATHEMATICS TEACHING OBJECTIVES UNDER "CORE LITERACY"

A. The Proposal and Connotation of "Core Literacy" and "Mathematics Core Literacy"

"Core literacy" first appeared abroad. Different countries and organizations have different understandings and applications of core literacy. On the whole, all countries believe that core literacy is a dynamic process of development, in order to cope with life-long development and social development and its various problems in the process of development. In March 2014, China issued "Opinions on Fully Deepening Curriculum Reform and Implementing the Fundamental Task of Lide-Shuren" and put forward that "core literacy" is the essential character and key ability that students should possess to meet the needs of lifelong development and social development. On the basis of core literacy, the curriculum reform direction of "subject core literacy system" is given. This indicates that the reform of basic courses in China has entered a new stage. The proposition of core literacy has also injected new vitality into mathematics teaching, and the goal of mathematics teaching can better reflect the times.

Mathematics core literacy refers to the key abilities and thinking qualities of people who have the basic characteristics of mathematics and meet the needs of personal life-long development and social development. Core accomplishment is an important part of mathematics teaching goal. By designing mathematics teaching from four aspects of "situation and problem", "knowledge and skill", "thinking and expression" and "communication and reflection", students' core accomplishment of mathematics can be improved, and students can observe the world with mathematical eyes and use mathematics. Thinking analyses the world, expresses the world in mathematical language, firmly grasps the basic knowledge, and forms critical thinking.

B. Characteristics of Mathematics Teaching under "Core Literacy"

In the "core literacy" throughout the mathematics classroom, mathematics teaching is no longer simply knowledge imparting, but gradually transformed into a student-centered, teacher-led teaching model. In a real sense, it realizes the value of "people-oriented", realizes that students are the masters of the classroom, and realizes the autonomy of students' development.

Mathematics teaching under the concept of "core literacy" is to enable students to master basic theoretical knowledge proficiently, exercise students' ability of independent learning, develop students' innovative ideas, enhance teaching effect and quality through hierarchical questioning, and stimulate students' interest in learning. Specifically, teachers become the guides of classroom teaching, and students are the real masters of the classroom. Teachers create problem scenarios, create a relaxed learning atmosphere for students, and guide students to think positively.

On the basis of students' cognitive structure and level, questions are asked from easy to difficult, so that students' thinking is deepened, which expands students' thinking space and ensures efficient classroom teaching. Let teachers' teaching and students' learning combine organically, realize the double optimization of subject dominance, and realize the win-win situation of mathematics classroom teaching.

IV. INTEGRATING THREE-DIMENSIONAL OBJECTIVES WITH MATHEMATICAL CORE LITERACY

Core literacy is the inheritance and integration of three-dimensional goals, and the transcendence of three-dimensional
goals. The key competence proposed by core literacy is the integration and promotion of knowledge and skill goals and process and method goals in three-dimensional goals. The essential character is a high description of emotional attitude and value goals. Specifically, the three-dimensional goal of mathematics is not the end of mathematics education, but the basis of the formation of mathematics core literacy. It is the concretization and precision of mathematics core literacy, and the way to explore mathematics core literacy in depth.

In mathematics classroom teaching, the common expectation of teachers and students is to master theoretical knowledge proficiently, and to transform knowledge into students' inner quality and spiritual wealth. Literacy is the synthesis of quality and education, knowledge and skills are the basis of the formation of literacy. The formation of mathematics core literacy is the result of a lot of learning. Mathematics teaching activities are the way to improve students' core literacy on the basis of knowledge and skills and through the teaching of mathematics knowledge. But the core literacy of mathematics is not only a simple accumulation of mathematical knowledge. Only through students' independent learning and teachers' effective guidance, can knowledge gradually evolve into the core literacy required by everyone in modern society.

To cultivate students' core literacy, specifically, is to cultivate students' ability to face difficulties in future life, and internalize this ability into their own literacy. The goal of process and method is an important guarantee for the formation of core literacy. In the process of mathematics classroom teaching, teachers pay attention to the improvement of students' abilities, and through long-term accumulation and adaptation, make abilities finally become literacy, so as to achieve the ultimate goal of cultivating core literacy.

Literacy is the manifestation of human essential characteristics and points out the direction for human development. Core literacy is the most important component of literacy, is the integration and transcendence of individual emotions, attitudes and values, and is the indispensable character that an individual should possess. In the teaching of mathematics with three-dimensional objectives, the teaching of emotion, attitude and values is often neglected. Under the core literacy, students' growth needs to learn the content of the situation, teachers need to inspire students' wisdom to achieve the ultimate goal of cultivating core literacy.

V. Teaching Design of Functional Extremum and Derivative Based on Core Literacy

"Function Extremum and Derivative" is the second section of the application of derivatives in the first chapter, the third section of the first chapter of the elective 2-2 of Senior Mathematics Education Edition A. The Extremum and Derivative of Function is learned by students after they have learned the monotony and derivative of function. It has the ability to study function by using derivative. It also lays the foundation of knowledge and method for the Maximum (Minimum) Value and Derivative of Function and plays a connecting role between the preceding and the following. This lesson plays a very important role in this unit and even in the whole mathematics learning. The following table shows the differences and connections between the three-dimensional goal and the teaching goal under the core literacy of mathematics.

The following is the specific teaching design of the course "Function Extremum and Derivative":

The teaching process of this lesson is composed of six teaching links: introduction of review, exploration of new lessons, application examples, feedback exercises, summary, assignment and so on.

A. Reviewing old knowledge and introducing topics

Teacher: Let's recall, what are the steps to find the monotonicity of a function by its derivative?

Student: I summarized the steps to find the monotony of the function. (The student answers, the teacher shows the idea of solving each step.)

Teacher: Ask students to draw pictures of functions. What conclusions can be drawn from the pictures?

Student: (Teacher's Guide Answer) The function is the largest and the function is the smallest.

Teacher: Can we give a new name to the function values at these two points?

Student: Students discuss and come up with various creative names.

Teacher: The students think very well, so the teacher gave it a new name: we say a maximum value of a function; a minimum value of a function.

[Processing method] Two questions are completed through question presentation.

[Designing Intention] By reviewing knowledge, students' cognitive conflict, curiosity and thirst for knowledge are aroused, and problems are further explored. The definition of function extremum is introduced through specific function image, and the main content of this lesson is put forward to point out the topic of this lesson. To cultivate students' cognitive ability from concrete to abstract and from special to general.

B. Cooperative inquiry and concepts

Teacher: By introducing nature, we give the definition of function extremum: Generally, let a function be defined near a point, if there are all points nearby, we say that it is a maximum value of a function, which is recorded; if there are all points nearby, we say that it is a minimum value of a function, which is recorded. Maximum and minimum are called extremum.

Teacher: Using the definition of function extremum and the analysis of function image, the following questions are
explored: Is the function extremum concerned with global or local properties?

Students communicate with each other and express their ideas.

Teacher: Can a function have multiple minimum or maximum points? Must the minimum be smaller than the maximum?

Students draw their own pictures first, and then discuss in groups. Select group representatives to summarize the results of the discussion. Teachers timely affirm and praise the results of the discussion.

Teacher: If so, must it be the extreme point of the function? Can students sum up the necessary and sufficient conditions for the extreme point of a function? Under the guidance of teachers, students communicate, expand their thinking and draw conclusions.

Teacher: Then guess what the extreme point of the function will be at the end of the interval?

[Processing methods] Lead students to analyze and explore, carry out group cooperation and discussion, and show in groups after solving problems.

[Design Intention] By exploring the problem, we can further understand the definition of extreme value and its relationship with derivatives. The combination of numbers and types highlights the intuition and reduces the theoretical nature. The above figure guides students to find the relationship between the function extreme point and the derivative. This is conducive to the cultivation of students' thinking integrity. And we can summarize the methods of finding and judging the extremum points of derivative function.

1. If it's on the left and right side of the neighborhood, it's the maximum value; (The left plus right minus is the maximum)
2. If it is on the left and right side of the neighborhood, then it is the minimum; (The left negative right positive is the minimum)
3. Application of knowledge and deepening of new knowledge
4. Knowledge application, consolidation and improvement
5. Classroom summary, reflection and improvement

[Requirements] Guiding students to review and summarize the learning content of this lesson, to exchange learning experience, to build their own knowledge network, and to improve their summary and generalization ability. Specific contents: 1 definition: definition of extremum; 2 key points: (1) function value of derivative at mechanism point is 0; (2) derivative must be different sign on both sides of extremum point; (1) determine the definition field of F (x) and find the root; (2) find the root; (3) check the sign of the value around the root of equation in list lattice, left positive and right negative poles. For large values, there is a positive minimum on the left.

6. Assignment of work and sublimation of ability.

VI. CONCLUSION

Through discussion, it is found that integrating the three-dimensional objectives under the core literacy can not only enable students to master knowledge proficiently, but also cultivate students' essential character and key abilities in a true sense. Therefore, in mathematics teaching activities, teachers should pay attention to the teaching design under the core quality of mathematics, and promote students to develop in a better direction.

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