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Research on Optimization of Cathodic Protection System for Long Distance Pipeline Based on Numerical Simulation

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***Abstract*—The cathodic protection method is the most commonly used method for pipeline anticorrosion. A current is applied to the protected structure, and the potential is negatively shifted to the protective potential range by cathodic polarization, thereby the electrochemical corrosion of the structure is suppressed. In the cathodic protection system, small distance between the anode and the structure, high resistivity of the environment medium , and high requirement for the protection current, may result in uneven potential distribution on the structure. Aiming at this problem, the parameter optimization of the system is studied by boundary element method (BEM), radial integration method (RIM) and particle swarm optimization (PSO) based on the cathodic protection potential distribution model. Then, establish a cathodic protection optimization model that meets the cathodic protection requirements and has a uniform potential distribution by optimizing the anode position and the current density.**

Keywords—long-distance pipeline; cathodic protection(CP); boundary element method(BEM); radial integration method(RIM); anode position; particle swarm algorithm(PSO)

I. INTRODUCTION

The cathodic protection method is divided into two types: sacrificial anode cathodic protection (SACP) and the impressed current cathodic protection (ICCP). The sacrificial anode method is simple and easy, does not require an external power supply, and rarely causes corrosion interference. It is widely used to protect small metal structures or structures in low soil resistivity environment, such as urban pipe network, small storage tanks, etc. The impressed current method uses a DC power source and an auxiliary anode to force the current flowing to the protected metal from the soil, so that the potential of the protected object is lower than the surrounding environment. The protection distance of a single impressed current cathodic protection station is generally dozens of kilometers. It is mainly used to protect large metal structures or structures in high soil resistivity environments, such as long-distance buried pipelines and large tank groups, etc [1].

In the cathodic protection system, the distribution of potential and current on the pipeline surface are important parameters in the pipeline design and maintenance process. It is not only an important object in daily monitoring work, but also a criterion for evaluating the effectiveness of the cathodic protection system [2].

In this paper, in view of the specific problems existing in the design and maintenance of cathodic protection, firstly, the existing techniques are used to study the potential distribution law of the pipeline surface under cathodic protection, and the main influencing factors of potential distribution are obtained. Secondly, the optimization of cathodic protection is studied by boundary element method (BEM), radial integration method (RIM) and particle swarm optimization (PSO). Based on the potential distribution model, the optimization of the auxiliary anode position and current density is carried out to establish a cathodic protection optimization model that the potential meets the requirements of cathodic protection and evenly distributed on the pipeline, which has a strong guiding significance for practical engineering implementation.

II. THE MATHEMATICAL MODEL OF CATHODIC PROTECTION SYSTEM

Due to limitation in soil resistivity and non-adjustable current for sacrificial anode cathodic protection (SACP), the impressed current cathodic protection (ICCP) method is usually adopted in the corrosion protection of long-distance pipelines[3]. The main structure of the impressed current cathodic protection (ICCP) system is shown in Fig 1.

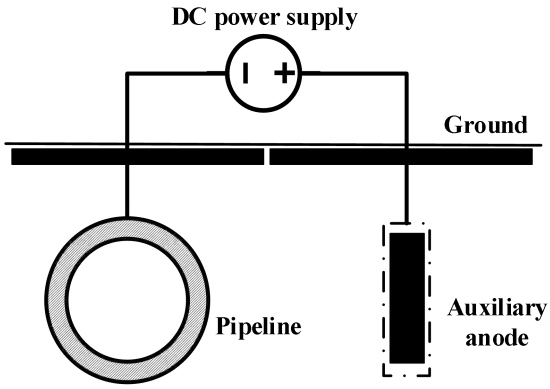

Fig. 1. The system composition diagram of ICCP.

A. Description Equation

At present, most of the studies on the potential distribution of cathodic protection system at home and abroad are based on the steady-state distribution model. The potential distribution can be described by the Laplace equation:

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \tag{1}$$

where u is the cathodic protection potential in the area Ω.

If the protected structure is in an active electric field, it can be expressed by the Poisson equation:

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{I_e}{\sigma}\delta\left(X - X_e\right) \tag{2}$$

where I_e is the output current for the anode point $X_e(x, y, z)$, σ is the electric conductivity, $\delta(\bullet)$ is a dirac function.

B. The Boundary Conditions

The boundary of the numerical model is Ω (the study area) and the total boundary Γ (Γ envelops Ω). The total boundary Γ includes the infinite ground surface boundary Γ_d, the outer surface boundary Γ_b of the pipeline, and the soil semi-infinite soil spherical crown boundary Γ_∞, as is shown in Fig 2.

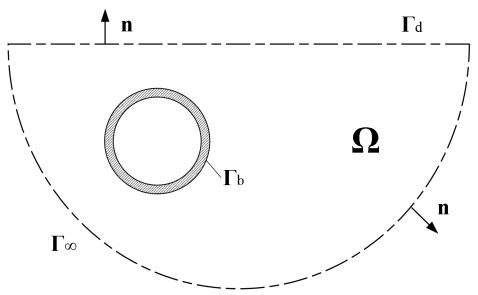

Fig. 2. The boundary situation diagram..

The boundary conditions are simplified as follows:

1. the ground surface boundary Γ_d is regarded as the insulation boundary;

2. the potential at infinity is assumed to be zero;

3. the boundary condition of the cathode is defined by a polarization function.

Therefore, the mathematical model and boundary conditions are as follows:

$$\begin{cases} \Omega: \ \nabla^2 u(\mathbf{x}) + Q(\mathbf{x}) = 0 \\ \Gamma_d : q = -\sigma \dfrac{\partial u}{\partial \mathbf{n}} = 0 \\ \Gamma_\infty : u = 0, q = -\sigma \dfrac{\partial u}{\partial \mathbf{n}} = 0 \\ \Gamma_b : q = f\left(u - u_{eq}\right) \end{cases} \tag{3}$$

III. Boundary Element Method

The boundary element method (BEM) is a numerical calculation method developed in the past 18 years. The engineering application of this method starts from elastic mechanics, and it's now applied to many fields such as fluid mechanics, electromagnetic engineering, civil engineering, etc [4].

A. Boundary Integral Equation

Multiply the weight function G on both the left and right sides of the control equation (3), and integrate it over the entire solution field.

Then the weighted residual expression of equation (3) is

$$\int_\Omega G\nabla^2 u d\Omega + \int_\Omega GQ d\Omega = 0 \tag{4}$$

For the first term domain integral in the above formula, after twice fractional integrations and using the Gaussian divergence theorem, it can be deduced that

$$\int_\Omega G\nabla^2 u d\Omega = \int_\Gamma G\frac{\partial u}{\partial \mathbf{n}} d\Gamma - \int_\Gamma u\frac{\partial G}{\partial \mathbf{n}} d\Gamma + \int_\Omega u\nabla^2 G d\Omega \tag{5}$$

where Γ is the boundary of the calculation domain Ω.

If the weight function G is taken as the basic solution of the problem, the domain integral in the above formula can be eliminated. The basic solution should satisfy the following equation

$$\nabla^2 G + \delta(x, p) = 0 \tag{6}$$

where $\delta(x, p)$ is the Dirac delta function, p is the source point and q is the field point in the area.

When p is a point inside the region Ω, according to the integral property of the Dirac function, there is

$$\int_\Omega u\nabla^2 G d\Omega = -u(p) \tag{7}$$

Substituting the above formula into (5) and substituting the result into (4), there is

$$u(p) = \int_\Gamma G\frac{\partial u}{\partial \mathbf{n}} d\Gamma - \int_\Gamma u\frac{\partial G}{\partial \mathbf{n}} d\Gamma + \int_\Omega GQ d\Omega \tag{8}$$

It is the integral equation for calculating the potential inside the study area.

When p is a point on the boundary, the boundary-domain integral equation of the source point on the boundary can be derived as:

$$cu(p) = \int_\Gamma G\frac{\partial u}{\partial \mathbf{n}} d\Gamma - \int_\Gamma u\frac{\partial G}{\partial \mathbf{n}} d\Gamma + \int_\Omega GQ d\Omega \tag{9}$$

where c is a constant associated to the p-point geometry, and for a smooth boundary, c=1/2.

After the formal solution G is determined, the conventional boundary element method can be used to discrete and solve equation (9).

B. Deduction of The Basic Solution

The governing equation for the three-dimensional potential problem is

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \tag{10}$$

The Laplace operator can be expressed as a spherical coordinate system,

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{1}{r}\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2} \quad (11)$$

If the solution medium is an infinite isotropic medium, according to the symmetry of the Dirac function, the basic solution can be considered as a function of the radial coordinate r. Only the radial distribution is considered, there is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial G}{\partial r}\right) = 0 \quad (12)$$

For the above equation, after two times integration in the radial direction r ,there is

$$G = -\frac{c_1}{r} + c_2 \quad (13)$$

where c2 is a constant solution and will not be considered. In order to obtain the unknown coefficient c1, consider the spherical region with the radius ε around the source point p, and integrate the equation (6), there is

$$\int_{\Omega_\varepsilon}[\nabla^2 G + \delta(q,p)]d\Omega_\varepsilon = \int_{\Omega_\varepsilon}\nabla^2 G d\Omega_\varepsilon + \int_{\Omega_\varepsilon}\delta(q,p)d\Omega_\varepsilon \quad (14)$$

Using the Gaussian divergence theorem, the 3D domain integral is transformed into boundary area integral, which is

$$\int_{\Omega_\varepsilon}\nabla^2 G d\Omega_\varepsilon = \int_{\Omega_\varepsilon}\frac{\partial}{\partial x_i}\frac{\partial G}{\partial x_i}d\Omega_\varepsilon = \int_{\Gamma_\varepsilon}\frac{\partial G}{\partial x_i}n_i d\Gamma_\varepsilon = \int_{\Gamma_\varepsilon}\frac{\partial G}{\partial \mathbf{n}}d\Gamma_\varepsilon \quad (15)$$

Since Ω_ε is a spherical area, there is,

$$\frac{\partial G}{\partial \mathbf{n}} = \frac{\partial G}{\partial r} \quad (16)$$
$$d\Gamma_\varepsilon = r^2\sin\theta d\theta d\phi$$

Take (13) into (15), there is

$$\int_{\Omega_\varepsilon}\nabla^2 G d\Omega_\varepsilon = \int_{\Gamma_\varepsilon}\frac{\partial}{\partial r}\left(-\frac{c_1}{r}\right)d\Gamma_\varepsilon = \int_0^{2\pi}\int_0^{\pi}\frac{c_1}{r^2}r^2\sin\theta d\theta d\phi = c_1 \bullet 4\pi \quad (17)$$

According to the property of the Dirac function, there is

$$\int_{\Omega_\varepsilon}\delta(q,p)d\Omega_\varepsilon = 1 \quad (18)$$

Take (17) and (18) into (14), there is

$$c_1 = -\frac{1}{4\pi} \quad (19)$$

Thus, the basic solution to the three-dimensional potential problem is

$$G = \frac{1}{4\pi r} \quad (20)$$

C. Transformation of Domain Integral to The Boundary By RIM

If the field source Q exists in the calculation domain, the integral equations (8) and (9) will contain domain integrals. In order to avoid calculating the domain integrals by dividing the computational domain into internal grids, use the radial integration method to transform the domain integrals into boundary integrals.

Using the radial integration formula [5], the domain integrals in the integral equations (4) and (5) caused by the field source term can be calculated by the following boundary integrals.

$$\int_\Omega GQd\Omega = \int_\Gamma \frac{F}{r^\alpha}\frac{\partial r}{\partial \mathbf{n}}d\Gamma \quad (21)$$

where F is the radial integral, and the formula is

$$F = \int_0^r GQr^\alpha dr \quad (22)$$

where α is 1 in the 2D problem and 2 in the 3D problem.

After the concrete forms of the basic solution G and the field source Q are given, the radial integral F can be obtained by the equation (22), and then the boundary integral can be calculated by the equation (21) according to the conventional boundary element integration method. For some simple functions, equation (21) can be directly integrated by the analytical method, and for some complex functions, the numerical integral formula can be used for calculation.

D. Discretization of Boundary Integral Equation

The first two terms in equation (8) are boundary integrals, and the boundary integral equation can be converted into algebraic equations by a conventional discrete solution. As shown in the Fig 3, the entire boundary is discretely divided into n units.

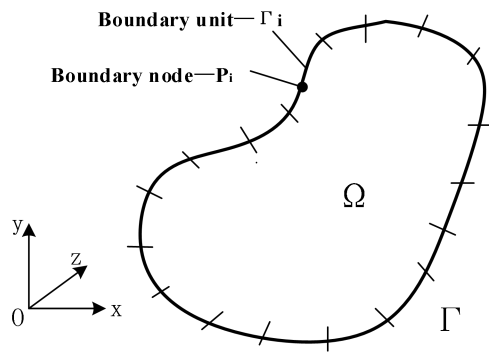

Fig. 3. The discrete boundary diagram..

For the node P_i, the boundary integral equation (8) can be discretized into:

$$c_i u_i(p) = \sum_{j=1}^{n}\int_{\Gamma_j} G\frac{\partial u}{\partial \mathbf{n}}d\Gamma_j - \sum_{j=1}^{n}\int_{\Gamma_j} u\frac{\partial G}{\partial \mathbf{n}}d\Gamma_j + \sum_{j=1}^{m}\int_{\Omega_j} GQd\Omega_j \quad (23)$$

In the formula (23),

$$q = -\frac{\partial u}{\partial \mathbf{n}}, q^* = \frac{\partial G}{\partial \mathbf{n}} \quad (24)$$

Then the boundary integral equation can be written as

$$c_i u_i(p) = \sum_{j=1}^{n}\int_{\Gamma_j} Gqd\Gamma_j - \sum_{j=1}^{n}\int_{\Gamma_j} uq^* d\Gamma_j + \sum_{j=1}^{m}\int_{\Omega_j} GQd\Omega_j \quad (25)$$

In the formula (25),

$$\begin{cases} \int_{\Gamma_j} Gqd\Gamma_j = [G_{ij}][q]^T \\ \int_{\Gamma_j} uq^* d\Gamma_j = [H_{ij}][u]^T \\ \int_{\Omega_j} GQd\Omega_j = B_j \end{cases} \tag{26}$$

Then, the formula (25) becomes

$$c_i u_i + \sum_{j=1}^{n} [H_{ij}][u]^T = \sum_{j=1}^{n} [G_{ij}][q]^T + \sum_{j=1}^{m} B_j \tag{27}$$

In the above formula,

$$H_{ij} = \begin{cases} \hat{H}_{ij} & (i \neq j) \\ \hat{H}_{ij} + c_i & (i = j) \end{cases} \tag{28}$$

Then, the formula (27) becomes

$$\sum_{j=1}^{n} [H_{ij}][u]^T - \sum_{j=1}^{n} [G_{ij}][q]^T = \sum_{j=1}^{m} B_j \tag{29}$$

IV. ANODE POSITION OPTIMIZATION MODEL

Through the theoretical research on the position of the auxiliary anode embedding, it is found that when the anode position is close to the buried pipeline, the protective current is obviously increased. Therefore, the reasonable anode layout should meet the potential distribution requirements and ensure a small current density. Therefore, a regional protection mathematical model for optimizing the auxiliary anode position and current density will be established. The influence of the buried depth on the current distribution will be considered when optimizing the auxiliary anode position [6].

By derivation, a linear equation (29) of anode position, current density and potential has been obtained. Accordingly, the potential of each node is not only affected by its own position, but also related to the anode position and current density. The purpose of the cathodic protection optimization design is to make all potentials of the pipeline surface reach requirements. For the protected structure, it must be guaranteed that its potential is lower than u_p, that is

$$u \leq u_p \tag{30}$$

where u_p is the cathodic protection potential, and its value is determined by the characteristics of the metal and the medium.

Therefore, the following cathodic protection optimization model is established:

$$W(Q_e, X_e) = \sum_{i=1}^{N} (u_i - u_p)^2 \tag{31}$$

where Q_e is the current for the anode; X_e is the position of the anode; N is the number of nodes.

For the convenience of calculation, the equation (30) is used as the constraint of the formula (31), and the compensation coefficient method is used to determine the objective function as:

$$\begin{cases} \min W^*(Q_e, X_e) = \left\{ W(Q_e, X_e) + k \sum_{i=1}^{N} \left[(-u_i + u_p) e^{(u_i - u_p)} \right]^2 \right\} \\ Q_e \geq 0 \\ X_e \leq R \end{cases} \tag{32}$$

where k is a penalty factor, usually a large positive number, u_i is potential value of the i-th node of the surface, R represents the limit value of source point position in the model.

V. PARTICLE SWARM OPTIMIZATION ALGORITHM

For an optimization problem: min f(X) , the specific process is as follows:

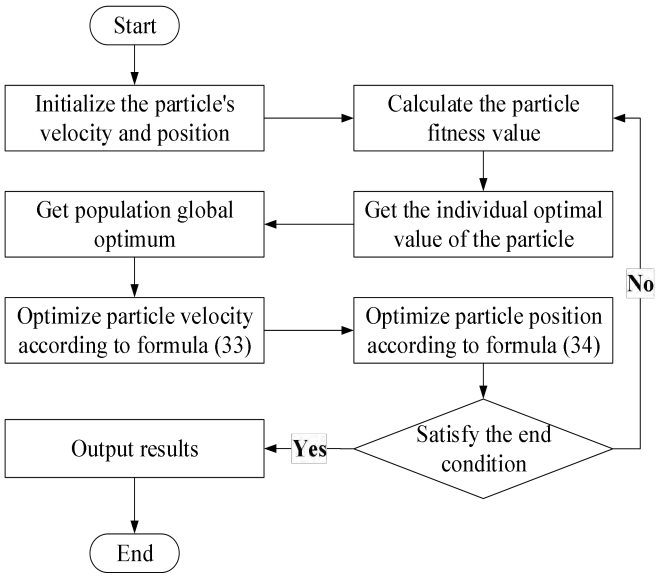

Fig. 4. The calculation flow chart for PSO.

1. Initialization: set the parameters (number of particles n, maximum iteration number, update the coefficients in the formula), randomly generate n initial particles $\{X_1, X_2, ..., X_n\}$;

2. Evaluate each particle and calculate the fitness value of each particle $\{f(X_1), f(X_2), ..., f(X_n)\}$;

3. Update the local optimal {Pbest1,Pbest2,...Pbestn} and global optimal Gbest for each particle;

4. Update each particle, that is to change X constantly:

$$V_i^k = w * V_i^{k-1} + c_1 * rand * \left(Pbest_i^k - X_i^{k-1}\right) + c_2 * rand * \left(Gbest_i^k - X_i^{k-1}\right) \tag{33}$$

$$X_i^k = X_i^{k-1} + V_i^k \tag{34}$$

where k represents the k-th iteration, i represents the i-th particle, and w, c1, and c2 are parameters.

5. If the maximum iteration number iteration is not reached, go to step 2 to continue iteration, otherwise output Gbest.

So it can be seen that the entire iteration is moving in the direction of Gbest.

VI. CASE STUDY

A. Model Description

The following is an example to verify the theory and calculation method. The diameter of the pipeline in the area is ϕ914*28, the buried depth is 2m, the distribution position and the length of the pipeline are as shown in the figure (Note: the coordinate system shown in the figure is established with the left end of the pipeline as the coordinate origin). Due to the large protection area, three sets of anodes are used, and the material is high silicon cast iron. It is necessary to select a reasonable protection potential value u_p in the calculation. It is not appropriate to set the -1.15V as the upper limit. If the -0.85V is selected, the pipe potential will be concentrated at around -0.85V, resulting in partial protection of the pipe. Therefore, -0.9V is selected as the calculation standard u_p. Assume that the soil is evenly distributed in the model, and the soil resistivity is $30\Omega \bullet m$.

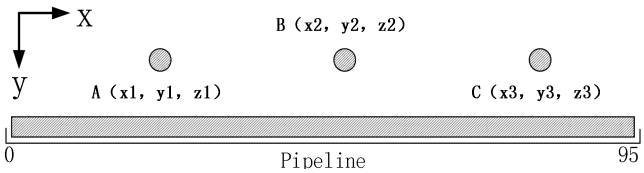

Fig.5. Distribution of the model.

B. Results and analysis

After 80 iterations, the best anode positions are A (6.0, 22.4, 1.4), B (45, 19, 1.4), C (62, 41.5, 2), and the current output density is 6 mA/m^2, 8 mA/m^2 and 4 mA/m^2. Because of the random property of PSO, the optimization results are different each time. In order to obtain better results, the optimization is repeated multiple times, and the optimal results are taken. After that, calculate the potential value of each node, as is shown in Table I, and potential distribution curve is shown in Fig 4.

TABLE I. THE POTENTIAL VALUE OF EACH NODE

Node	Potential	Node	Potential	Node	Potential
1	0.8537	18	1.0877	35	1.0298
2	0.8664	19	1.1060	36	09940
3	0.8710	20	1.1298	37	0.9781
4	0.8734	21	1.1590	38	0.9605
5	0.8840	22	1.1758	39	0.9489
6	0.8944	23	1.1868	40	0.9345
7	0.90622	24	1.1906	41	0.9206
8	0.9178	25	1.1976	42	0.9050
9	0.9290	26	1.1862	43	0.8964
10	0.9558	27	1.1780	44	0.8925
11	0.9497	28	1.1576	45	0.8854
12	0.9540	29	1.1458	44	0.8727
13	0.9665	30	1.1375	45	0.8680
14	0.9807	31	1.1298	46	0.8540
15	0.9920	32	1.1277	47	0.8557
16	1.0356	33	1.1193	50	0.8503
17	1.0758	34	1.0965		

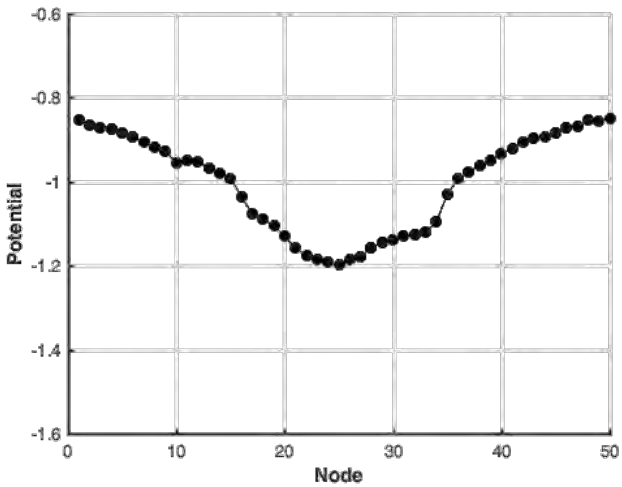

Fig.4. Distribution of the potential.

It can be seen in the Fig 4 that the potentials of the nodes in the pipeline are within the protection potential range, and both float within a small range of the selected reference -0.7V, indicating that the potential distribution is relatively uniform and achieves the purpose of optimization.

VII. CONCLUSION

The study has drawn a series of conclusions, mainly due to the following points.

1. The potential distribution of buried pipeline in cathodic protection can be described by equations according to the nature of the electrostatic field.
2. In this paper, the three-dimensional boundary element numerical solution is used to design the regional cathodic protection system, and the auxiliary anode depth is dissolved into the auxiliary anode position optimization process. The mathematical model is closer to the actual situation, and the calculation result is more accurate.

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