The application of a type of BCH code optimization combination in LTE
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Abstract: In this paper, we propose a theory of optimization and combination of BCH code, which can generate a type of similar-orthogonal pseudo-random matrix with the encoding method of group counterchange. Compared with the Walsh code, the length of similar-orthogonal matrix is not limited. In addition, it can take any value. However, the rows and columns of orthogonal matrix are not the same. Both of them have the features of similar-orthogonal. The similar-orthogonal matrix is also used in LTE communication systems which can transmit with different baud rate. Proved through the simulation, the similar-orthogonal pseudo-random matrix will be able to restore various user signals completely.

INTRODUCTION

BCH code was proposed by Hocquenghem[1] in 1959. One year later, Bose and Chandhair[2] presented a kind of cycle code which can correct multiple random errors. The generator polynomial of BCH code became a Generator matrix with the encode method of group counterchange in the process of BCH coding. And in the generator matrix, there is a kind of matrix called P. The matrix will have similar-orthogonal properties after it is changed by rule. Consequently, the similar-orthogonal matrix can be used in multi-user transmission system of LTE.

THE GENERATION OF PMATRIX

Optimal combination of BCH codes

For a prime polynomial of BCH codes:

\[ f^m(x) = \sum_{i=0}^{m} c_i x^i \]  

(1)

\( x^i \) only indicates the coefficient (1 or 0), which represents the value of the \( c_i \). The value of x has no real meaning. The generator multinomial \( g(x) \) of BCH code can be formed either by a prime polynomial or by a combination of several factors of the polynomial.

For most primitive polynomial of BCH code, their Order ‘m’ and code length ‘n’ tend to be different. We put factor polynomials together whose order ‘m’ of generator polynomial is less than or equal to m, and the code length ‘n’ is \( 2^m - 1 \) or a factor of \( 2^m - 1 \). They were defined as prime polynomial of BCH code in one group. Several factor polynomials in same group can form a generator multinomial \( g(x) \) of BCH code:

\[ g(x) = F(f_1^m(x), f_2^m(x), \ldots, f_j^m(x)) \]  

(2)

The number of Prime polynomial which can compose a generator multinomial \( g(x) \) is uncertain. According to specific requirements we can select the appropriate prime polynomial and we will get some generator polynomials which meet the requirements [3].

The group transformation of BCH encoding

In (n,k) BCH code, the highest order of generator multinomial \( g(x) \) is m while its code length is \( n(n \leq 2^m - 1) \). In addition, its length of message is k whereas its grade of created multinomial \( g(x) \) is \( r=n-k \).
The encode method of group counterchange BCH code is applied in this formula. Firstly, according to the generator multinomial \( g(x) \) of BCH code, we can get a generator matrix \( G \) with the method of group counterchange\(^{[4]}\)\(^{[5]}\) whose size is \( k \times n \).

\[
g(x) \xrightarrow{\text{group counterchange}} G = \begin{pmatrix}
1 & 0 & L & 0 & g_{0,0} & g_{0,1} & L & g_{0,r-1} \\
0 & 1 & L & 0 & g_{1,0} & g_{1,1} & L & g_{1,r-1} \\
M & M & M & L & M & M & M & M
\end{pmatrix}
\]

The generator matrix \( G \) can be divided into two parts. First half of the matrix is a Unit matrix \( I_k \), whose size is \( k \times k \). The second half is \( P \) matrix, whose size is \( k \times r \).

\[
P = \begin{pmatrix}
g_{0,0} & g_{0,1} & L & g_{0,r-1} \\
g_{1,0} & g_{1,1} & L & g_{1,r-1} \\
M & M & L & M
\end{pmatrix}
\]

\[
G = (I_k, P)
\]

The research and simulations above demonstrate that row vectors or column vectors of \( P \) matrix are similar orthogonal, which indicates that \( P \) matrix is a class of pseudo-random orthogonal matrix.

THE SIMILAR ORTHOGONAL PROPERTIES OF PMATRIX

The similar orthogonal properties of \( P \) matrix

The \( P \) matrix is generated form generator matrix of BCH code after group counterchange. The number of "0" and "1" is approximately equal in each row vector of \( P \) matrix while the number of "0" and "1" is approximately equal in each column vector of \( P \) matrix. If we want to calculate the correlation between rows or columns, we should convert \( P \) matrix to a new matrix which only contains '+1' or '-1' as its symbol. Meanwhile we provided the "+1" instead of the binary number "1", with "-1" instead of the binary number "0". Finally the new matrix \( P' \) matrix is obtained.

The size of the \( P' \) matrix in Fig 1 turned into \( 64 \times 63 \) after group counterchange. As we can see in Fig 1, the self-correlation between same row vectors is remarkable in this matrix for most of mutual-correlation coefficient number between same rows is \( \rho(x, x) = 1 \). The figure also shows that the mutual-correlation between different rows is very weak since most of mutual-correlation coefficient number between different rows is less than 0.5. At the same time, the column of \( P' \) matrix
has the same characteristic with the row. In conclusion, the same columns have a good self-correlation while the mutual-correlation is weak between different column vectors.

**Optimization and combination of the similar-orthogonal properties of BCH codes**

The histogram of the similar-orthogonal matrix in Fig 1 demonstrates that the mutual-correlation coefficient between some rows or columns in the similar-orthogonal matrix is 0, which indicates the rows or columns are completely orthogonal. According to the optimization and combination of BCH codes, it is implied that polynomial compose of different elements from which you can get different generator polynomial so as to get different P matrix. The numbers of rows or columns are different in different P matrix. The characteristics of similar-orthogonal matrix are different. Therefore, according to the specific needs we can find similar-orthogonal polynomials or completely orthogonal rows or columns by optimization and combination.

**The comparison between similar-orthogonal pseudo-random matrix and Walsh matrix**

Walsh matrix is made by Hadmard through Hadmard transformation. Since Each row or column of Walsh matrix is completely orthogonal, it is often used for multi-user of LTE. Hardmard transformation is changed by $2^n$. Therefore the size of each Walsh matrix is $2^n \times 2^n$. So the length of coding sequence of each users in LTE multi-user transmission system is $2^n$. But as we know the size of similar-orthogonal pseudo-random matrix $P'$ is $k \times r$, which means the length of row in $P'$ matrix is decided by the order of generator polynomial $r$. The length of column in $P'$ matrix is decided by the message length $k$. Because different generator polynomial can get different BCH code, its message length $k$ and order of generator polynomial $r$ are different too. The length of message length $k$ and order of generator polynomial $r$ can take any value. They are not limited by $2^n$.

**THE APPLICATION OF SIMILAR-ORTHOGONAL PSEUDO-RANDOM MATRIX IN LTE**

**Wide band Code Division Multiple Access (LTE)**

In LTE communication system, when multiple users need to transfer voice, video, data and other digital information, they need different baud rate. Consequently, we need different carrier frequency to modulate binary digital signal of different users. And then we use the modulated user signal in code division multiplexing. At the same time, we should use orthogonal coding sequences to encode users’ information. Finally, we changed multiple-use information into one user information and sent it away. At the receiving end, we can demodulate various quarters User signal information from a mixed signal with orthogonal coding sequences. And then we demodulate various quarters User signal information with different carrier frequency. Finally, we can get binary digital signal which is same to the sending end. The detailed diagram shows in Fig 2.
The application of similar-orthogonal pseudo-random matrix in LTE

From the previous study, we can see \( P' \) matrix which is generated by the generator multinomial \( g(x) \) after group counterchange is a similar-orthogonal pseudo-random matrix. The rows or columns of \( P' \) matrix are similar-orthogonal. From Fig 2, we can see that we should encode multiple users with Orthogonal coding sequences. So we use rows or columns of \( P' \) matrix as multiplexed coding sequence in LTE communication system.

CONCLUSION

We can get \( P \) matrix by the generator multinomial \( g(x) \) after group counterchange. Then we changed ‘0’ to ‘-1’ and ‘1’ to ‘1’ in \( P \) matrix. After that we can get a new matrix defined as \( P' \). \( P' \) matrix is a similar-orthogonal pseudo-random matrix. The rows or columns of \( P' \) matrix have a remarkable similar-orthogonal. In this paper, we calculate the cross-correlation coefficient between row vectors or column vectors in \( P' \) matrix. We select several row vectors or column vectors which are Complete orthogonal to use in LTE communication system as well. After simulation, we can see that these row vectors or column vectors which are complete orthogonal can restore multiple user signals. Because of the Optimization and combination of BCH codes, it is able to meet specific requirements.

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