Analysis of Reliable Life of Inclined Strut Structure for Conveyor Based on CAE Simulation

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Abstract: The finite element simulation model for inclined strut of a conveyor is built by CAE tool, then the design of experiment and response surface method are introduced to fit response surface model for fatigue life of inclined strut. On this basis, considering the effect of random factors such as dimension, load and material performance etc., the reliable life prediction problem can be solved by using function measurement approach, and then the reliable life and sensitivity degree can be gotten directly. Finally, the structure of inclined strut is improved by using sensitivity degree, the reliable life of new structure is longer than before.

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Introduction

Along with the development of belt conveyors towards large capacity and high belt speed, their loading conditions become increasingly severe. Especially for some conveyors with large inclination, the problem of their service lives become more and more outstanding [1,2]. Fatigue failure of such structures as the inclined strut, roller, gear and axle is the most common failure mode for conveyor components because of the large influence dispersivity for random fluctuation of component life along with factors in engineering. Therefore, it is usually required to conduct reliability calculation when predicting component life.

Existing analysis of conveyor life mainly centers on median life without considering life dispersivity but regarding life as a determined value [3-5]. In terms of reliable life analysis of conveyor, it is generally assumed that life is submitted to Weibull distribution and lognormal distribution and distribution parameters are calculated to determine reliable life by using probability distribution function [6-7]. Or repeated reliability calculations are conducted with given life to obtain reliable life through substitute calculation [8].

Computer simulation technology has been extensively applied in conveyor structure analysis, which will help to reduce product cost and development cycle. Simulation analysis of reliable life of key structures for conveyor will facilitate the prediction and analysis of structure durability. However, repeated life calculation will be required during reliable life analysis with large calculation amount and poor convergence. For this reason, a response surface model for life of key structure of conveyor can be built based on simulation results through experiment design and response surface method, and then reliable life and sensitivity of key structure of conveyor can be analyzed with reliable life analysis method to guide engineering design.

Simulation Analysis Method for Reliable Life of Structure

Conduct statistics on structure test, simulation and other life data, determine probability distribution of life and then based on the probability density function, obtain the life, i.e. reliable life $N_R$ upon
the given reliability $R$ as shown in Fig.1.

Fig.1 Schematic diagram of reliable life and life probability density

According to structural reliability theory, structure fatigue reliability is expressed as the probability when the life $N(x)$ is greater than the given life $N_R$, i.e.:

$$ R = P\{N(x) \geq N_R\} = 1 - \int_{N(x) \leq N_R} f(x) dx = 1 - F_N(N_R) $$

Wherein: $f(x)$ is the joint probability density function of random variable $x$ and $F_N(N_R)$ is the cumulative distribution function of life $N(x)$.

If $F_N^{-1}(1 - R)$ is the inverse distribution function of life, then the reliable life is:

$$ N_R = F_N^{-1}(1 - R) = F_N^{-1}(\phi(-\beta)) $$

Wherein: $\beta$ is reliability index, i.e. $R = \phi(\beta)$. Commonly used reliable life analysis methods in engineering include probability distribution method, iteration method, Monte Carlo simulation method and probability function measurement approach, etc [9].

With regard to fatigue life analysis of structure, finite element software is generally used to analyze stress amplitude of structure first and then life is analyzed with stress-life method, strain-life method and fracture mechanics method, etc. ANSYS Workbench, MSC-Fatigue and other CAE simulation tools cover fatigue life analysis function. In this paper, CAE simulation and reliable life analysis theory are combined to analyze reliable life of structure and its sensitivity with main process as shown in Fig.2. First, it is required to determine major random variables of the structure, take them as parameters and achieve parametric simulation with CAE tools. Then, experiment design and response surface method are selected and used for repeated simulation and fitting of response surface function for life, and reliable life analysis is carried out with Monte Carlo simulation method and probability function measurement approach to obtain reliable life and sensitivity results.
Simulation of Fatigue Life of Inclined Strut

Major design parameters of inclined strut component
Inclined strut, as an important supporting member on conveyor, will bear great push-pull cyclic load and may easily suffer fatigue crack during falling and transportation of materials. Its structure diagram is as shown in Fig.3.

![Fig.3 Schematic diagram of inclined strut structure and design parameters](image)

Such strut structure is provided with 7 design parameters and their standard deviations are listed in Table 1 in unit mm. For uniform expression, \((r_1, r_2, r_3, l, z_1, z_2, h)^T\) is denoted by vector \(x\), i.e. \((x_1, x_2, x_3, x_4, x_5, x_6, x_7)^T\).

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Average</th>
<th>Standard Deviation</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_1)</td>
<td>14</td>
<td>0.4</td>
<td>Aperture</td>
</tr>
<tr>
<td>(r_2)</td>
<td>80</td>
<td>2.4</td>
<td>Head chamfering</td>
</tr>
<tr>
<td>(r_3)</td>
<td>10</td>
<td>0.3</td>
<td>Groove chamfering</td>
</tr>
<tr>
<td>(l)</td>
<td>108</td>
<td>3.2</td>
<td>Groove length</td>
</tr>
<tr>
<td>(z_1)</td>
<td>21</td>
<td>0.6</td>
<td>Inclined strut width</td>
</tr>
<tr>
<td>(z_2)</td>
<td>15</td>
<td>0.45</td>
<td>Groove width</td>
</tr>
<tr>
<td>(h)</td>
<td>20</td>
<td>0.6</td>
<td>Groove depth</td>
</tr>
</tbody>
</table>

Simulation analysis of fatigue life of inclined strut
Input load in ANSYS software, build parametric model of finite element simulation for inclined strut and obtain stress field of the inclined strut as shown in Fig.2 with the maximum stress being located at the transition part between head and body.

![Fig.4 Analysis results for stress of inclined strut](image)
Utilize Fatigue module in ANSYS Workbench software, input cyclic load of uniform amplitude pulse, select Goodman average stress correction and build stress-fatigue life analysis model. Stress-fatigue life analysis is mainly based on the material stress-life ($S$-$N$) curve and its theoretical model is:

$$\lg N = a + b \cdot \lg S$$  \hspace{1cm} (3)

Wherein: $S$ is stress amplitude; $a$ and $b$ are material constants. Inclined strut material is 42GrMo and its $S$-$N$ curve is expressed as multilinear in Fatigue module as shown in Fig.5.

![Fig.5 Material S-N curve in ANSYS fatigue module](image)

**Modeling of Response Surface for Fatigue Life of Inclined Strut**

**Building of response surface model for fatigue life of inclined strut**

Considering that Fatigue module will be called repeatedly for life calculation in subsequent reliability optimization design, response surface model is adopted to act as simulation calculation to help providing convergence for simulation efficiency and optimization calculation. Quadratic polynomial is used to build response surface model for logarithmic life:

$$\lg N = C_0 + \sum_{i=1}^{7} C_i x_i + \sum_{i=1}^{7} \sum_{j=i}^{7} C_{ij} x_i x_j + \sum_{k=1}^{7} C_k x_k^2$$  \hspace{1cm} (4)

Wherein: $C_0$, $C_i$, $C_{ij}$ and $C_k$ ($i = 1 \ldots 7$; $j = i \ldots 7$; $k = 1 \ldots 7$) are undetermined coefficients with a total of $2n + 1 + n (n + 1) / 2 = 36$.

To confirm undetermined coefficients in the model, surface central composite design is adopted to generate 143 groups of calculation samples. Input into Fatigue module, conduct 143 times of calculation and the quadratic response surface model for logarithmic life $N$ of the strut will be fitted. Coefficients are as listed in Table 2.
Table 2 Coefficients of response surface model for fatigue life of strut

<table>
<thead>
<tr>
<th>Item</th>
<th>Logarithmic Life lg(N)</th>
<th>Item</th>
<th>Logarithmic Life lg(N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant term</td>
<td>13.6786</td>
<td>$r_1 \times z_1$</td>
<td>0.014</td>
</tr>
<tr>
<td>$r_1$</td>
<td>-0.14191</td>
<td>$r_1 \times z_2$</td>
<td>-0.011</td>
</tr>
<tr>
<td>$r_2$</td>
<td>$1.7 \times 10^{-4}$</td>
<td>$r_1 \times h$</td>
<td>$-1.12 \times 10^{-3}$</td>
</tr>
<tr>
<td>$r_3$</td>
<td>0.051</td>
<td>$r_2 \times r_3$</td>
<td>$-3.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>$x_1$</td>
<td>0.116</td>
<td>$r_2 \times x_1$</td>
<td>$-3.02 \times 10^{-6}$</td>
</tr>
<tr>
<td>$z_1$</td>
<td>0.152</td>
<td>$r_2 \times z_1$</td>
<td>$-5.6 \times 10^{-5}$</td>
</tr>
<tr>
<td>$z_2$</td>
<td>-0.386</td>
<td>$r_2 \times z_2$</td>
<td>$-3.19 \times 10^{-6}$</td>
</tr>
<tr>
<td>$H$</td>
<td>-0.133</td>
<td>$r_2 \times h$</td>
<td>$5.56 \times 10^{-5}$</td>
</tr>
<tr>
<td>$r_1^2$</td>
<td>$2.2 \times 10^{-4}$</td>
<td>$r_1 \times x_1$</td>
<td>$-1.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>$r_2^2$</td>
<td>$2.49 \times 10^{-4}$</td>
<td>$r_3 \times z_1$</td>
<td>$-6.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>$r_3^2$</td>
<td>$-2.7 \times 10^{-4}$</td>
<td>$r_3 \times z_2$</td>
<td>$-3.48 \times 10^{-3}$</td>
</tr>
<tr>
<td>$x_1^2$</td>
<td>$8.3 \times 10^{-4}$</td>
<td>$r_3 \times h$</td>
<td>$9 \times 10^{-4}$</td>
</tr>
<tr>
<td>$z_1^2$</td>
<td>-0.011</td>
<td>$x_1 \times z_1$</td>
<td>$3.04 \times 10^{-3}$</td>
</tr>
<tr>
<td>$z_2^2$</td>
<td>-0.016</td>
<td>$x_1 \times z_2$</td>
<td>$-2.9 \times 10^{-4}$</td>
</tr>
<tr>
<td>$h^2$</td>
<td>$6.47 \times 10^{-3}$</td>
<td>$x_1 \times h$</td>
<td>$1.03 \times 10^{-5}$</td>
</tr>
<tr>
<td>$r_1 \times r_2$</td>
<td>$-5.53 \times 10^{-5}$</td>
<td>$z_1 \times z_2$</td>
<td>0.047</td>
</tr>
<tr>
<td>$r_1 \times r_3$</td>
<td>$1.16 \times 10^{-3}$</td>
<td>$z_1 \times h$</td>
<td>$1.13 \times 10^{-3}$</td>
</tr>
<tr>
<td>$r_1 \times x_1$</td>
<td>$-2.5 \times 10^{-4}$</td>
<td>$z_2 \times h$</td>
<td>$-0.012$</td>
</tr>
</tbody>
</table>

Test of response surface model

Commonly used model accuracy test indexes include: the maximum absolute value error, relative average absolute value error, root-mean-square error and certainty coefficient [10]. Among them, the certainty coefficient is extensively applied in engineering, i.e.:

$$R^2 = 1 - \frac{\sum_{i=1}^{n_s} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n_s} (y_i - \bar{y}_i)^2}$$ (5)

Wherein: $y_i$ is response value in sample; $\hat{y}_i$ is response value obtained from calculation with approximation model; $\bar{y}_i$ is average of response value of all samples; and $n_s$ is sample size. The closer $R^2$ obtained from the test is to 1.0, the smaller the proportion of mean square error in population variance is and the higher the model accuracy will be.

After calculation based on formula (3), $R^2$ in the response surface model for logarithmic life is 0.9952, being close to 1.0, and thus the response surface model for logarithmic life fitted here has a relatively high accuracy.

Analysis of Reliable Life of Inclined Strut

Random fluctuation of the structure size, load, material property, environment and operating conditions will often result in random fluctuation of the life of mechanical structure [11-12]. Under such circumstance, analysis of the reliable life of structure will have more engineering significance.
Since existing engineering data is limited, structure size variation will be mainly considered in this paper to calculate the reliable life of inclined strut. In the future, randomness of load, material and environment can also be considered in this method.

Convert formula (2) into independent standard normal space representation by utilizing the reliable life analysis method \[12\] based on probability function measurement, respectively convert \(x\) and \(N(x)\) into \(u\) and \(Nu(u)\), and represent the calculation of fatigue reliable life in the following mathematical optimization problem, i.e. making sure \(u^*\) meets the following condition:

\[
\min Nu(u) \\
s.t. \|u\| = \beta
\]

Write out the iterative formula for solving the optimization problem formula (6) with advanced mean value (AMV) method

\[
u^{k+1} = -\beta \frac{\nabla_u Nu(u^k, v)}{\|\nabla_u Nu(u^k, v)\|}
\]

Wherein: \(\nabla_u Nu(u^k)\) is the gradient vector of life function at \(u^k\) and \(k\) stands for the number of iterations. Initial value of iteration can be the original point or design point of standard normal space. When \(\|u^{k+1} - u^k\|\) is less than the allowable error \(\varepsilon\), the optimal solution \(u^* = u^{k+1}\) can be obtained. Substitute \(u^*\) into the life function and the reliable life can be obtained, i.e. \(N_r = Nu(u^*)\).

Respectively calculate the life corresponding to each reliability among 0.5, 0.6, ..., 0.95, ..., 0.99 and 0.999 as shown in Fig.6.

![Fig.6 Analysis results for reliable life of inclined strut](image)

Since the life of inclined strut is only 7340 cycles when the reliability is 0.99 (i.e. 1% probability of failure), sensitivity analysis results for reliable life of inclined strut will be calculated to improve structure design. Besides, the sensitivity to design parameter average and standard deviation is also analyzed. Fig.7 shows the histogram for the sensitivity of reliable life to each design parameter average, indicating that the average of strut width \(z_1\) and groove width \(z_2\) has greater impact on the life of inclined strut. Fig.8 shows the histogram for the sensitivity of reliable life to standard deviation of each design parameter, indicating that the greater the standard deviation of parameter is, the shorter the reliable life will be, and the standard deviation of strut width \(z_1\) and groove width \(z_2\) has greater impact on the life of inclined strut.
According to sensitivity analysis results, appropriate increase of the average of strut width $z_1$ and decrease of its standard deviation will help to improve the fatigue reliable life of inclined strut. If $z_1$ average is 22mm and the standard deviation is 0.8, life of the inclined strut is 23635 cycles when the reliability is 0.99 with an increase of 2.2 times comparing with the life before improvement.

**Conclusions**

(1) Simulation analysis methods for reliable life of structure are given based on CAE simulation tools; taking inclined strut structure of a conveyor as the object, a life model is built with finite element model, experiment design and response surface method to solve reliable life and sensitivity based on the function measurement method and determine the reliable life of inclined strut, as well as such key factors as the strut width $z_1$ and groove width $z_2$.

(2) With improvement measures for inclined strut design proposed based on the analysis results, the life is improved by 2.2 times after calculation and verification. Therefore, research in this paper has great engineering application value.

**References**

[3] Zhang Chunzhi, Dynamics Modeling of Scraper Conveyers and Research on Fatigue Reliability of Chain Rings [D], China University of Mining and Technology (Beijing), 2012.