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Numerical Simulation in Prey-Predator Model with a Stage-Structure for Prey

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***Abstract*—In the present paper, we studied a prey-predator model with stage-structure for prey. Prey populations which are divided into subpopulations, one is immature prey and the other is mature prey. We investigate the stability of prey-predator models as well as local analysis of four equilibrium obtained and analyzed how the dynamical behavior if the solution is stable or unstable. Numerical simulations are performed from the system around the equilibrium point to show dynamic behavior around that point with several different conditions andinitial values.**

Keywords; simulation; immature; mature; dynamics

I. Introduction

Mathematical models have been used to better understand phenomena in ecology, especially predator-prey interactions. The dynamic behavior of each species depends on many factors, such as mortality rates, environmental conditions, and stage structures [2]. A mathematical ecological model is one of the typical ways to introduce dynamic relationships between predators and their prey. The importance of age structures in the dynamics of interacting species has been recognized for a long time [3]. The strage-structure population model has a much simpler way to simulate diversity as well as several interaction models and predation patterns that show real-world phenomena.

Motivated by Jana, Ghorai, and Kar [5] who carried out the research on the dynamics of prey-predator with stage-structure for the predator. Then Wang [12] did the same in analyzing prey-predator models with stage-structure for predators. There are several results of the research on the dynamics of prey-predator and how predator predation patterns. The predator responds to changes of the prey interaction or so-called the functional responses have been commonly used in ecological models with a variety of different form. There are several well-known the functional responses in the prey-predator interactions which refer to as Holling type I, type II, type III and type IV [4], Monod-Haldane, Beddington De-Angelis type and ratio dependence type the functional responses. How predators respond to changes in prey availability is an issue of particular importance [4]. Abadi and Savitri [1] have studied ecological systems consisting of one prey and two predators, with predators applying different types Holling the functional responses. A literary work discusses prey-predator models relationships between prey and predator with stage-structures using various the functional responses. They used Holling type II [8] and Beddington De-Angelis [9] to investigate prey-predator models with stage-structure for prey or predators.

Dubei [2] developed a general mathematical Gauss model consisting of two prey and one predator populations with Holling type II the functional responses. Upadhyay and Raw [12] examined the dynamical behavior of a three species food chain system with Holling type IV of the predator response. Kar and Chaudury [8] describe a two-prey one-predator model based on Lotka-Volterra dynamics which are affected not only by harvesting but also by the presence of a predator.

In this paper, we present the dynamical behavior of stage-structured for prey in prey-predator interactions with Holling response function. Our objective of this paper is to investigate the prey-predator model which consists of a prey population which is divided into two subpopulations, one is immature prey and other is mature prey. Based on theories ecology, we have chosen Holling type I the functional responses for immature prey and Holling type II response function for mature prey.

The dynamical analysis of the prey-predator model is investigated from the point of view of local stability. Exact solution of this system are rarely in access and usually complicated. Numerical method is often the method of choice. In the last study, several numerical simulations were carried out for illustrating the results.

II. Mathematical Model

A. Basic Model

Jana, Ghorai and Kar [5] proposed a Lotka Volterra predator-prey model with Holling type I functional responses for weak prey and Holling type II for strong prey. Functional responses describe the relationship between an individual rate of consumption and food density. It assumes prey and predator populations grow logistically and are limited by the carrying capacity of the environment.

$$\frac{dx_1}{dt} = r_1 x_1\left(1-\frac{x_1}{K_1}\right) - \beta_1 x_1 y$$
$$\frac{dx_2}{dt} = r_2 x_2\left(1-\frac{x_2}{K_2}\right) - \frac{\beta_2 x_2 y}{1+x_2} \qquad (1)$$

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$$\frac{dy}{dt} = \alpha_1\beta_1 x_1 y + \frac{\alpha_2\beta_2 x_2 y}{1+x_2} - \mu y$$

Where r_1, r_2, α_1, α_2, β_1, β_2, K_1, $K_2 > 0$. The parameters respectively are the environmental protection for every population. Parameters r_1 and r_2 are the intrinsic growth of weak prey and intrinsic growth of strong prey.

Falconi and Claudio [3] proposed dynamic behavior in predator-prey models which consist of two prey involving stage-structure and one predator with a defense group. The population parameters of young prey to adult prey are expressed in β. The response function of the young prey population density depends on the defense mechanism and combines the predator rate of the adult prey population to the young prey populations.

$$\begin{aligned}\frac{dx_1}{dt} &= rx_1\left(1-\frac{x_1}{K}\right) - \beta x_1 - \frac{\beta_1 x_1 y}{1+m_1 x_1^2 + n_1 x_2}\\ \frac{dx_2}{dt} &= \beta x_1 - \frac{\beta_2 x_2 y}{1+m_2 x_1 + n_2 x_2} - \mu_1 x_2 \qquad (2)\\ \frac{dy}{dt} &= \frac{\alpha_1\beta_1 x_1 y}{1+m_1 x_1^2 + n_1 x_2} + \frac{\alpha_2\beta_2 x_2 y}{1+m_2 x_1 + n_2 x_2} - \mu_2 y\end{aligned}$$

In system (2), it is assumed that the predator population has the same ability to feed on each individual prey. Predator population y feeds on each individual prey x_1 and x_2, both according to the Beddington De-Angelis functional responses. Prey species go through two life stages in different stages. Kar and Batabyan [8] studied the following strengthen type prey-predator model with stage-structure prey population.

B. Assumptions Prey-Predator Model

Before we introduce the mathematical model, let us describe the basic assumptions that we made to formulate it.

In this paper, we construction from the system (1) and system (2). In this work, we have presented the dynamical behavior of a three species prey-predator system. The prey population is divided into two stage, namely immature and mature whose denoted are $x_1(t)$ and $x_1(t)$ respectively. The predator, whose population density is denoted by $y(t)$. Where y is a predator for immature prey x_1 and mature prey population x_2. It is assumed that the immature prey and the mature prey are consumed by a predator. In the absence of the predators, the immature prey population density grows according to logistic law with an intrinsic growth rate r positive and carrying capacity K positive.

The biological system concerned with the effect of the functional response and stage-structure on the dynamics of the prey-predator model. In prey-predator relationship assumed that predator used difference functional responses to feed on each individual prey.

III. Result and Discussion

Therefore a new model can be represented mathematically by the following system of differential equations:

$$\begin{aligned}\frac{dx_1}{dt} &= rx_1\left(1-\frac{x_1}{K}\right) - \beta x_1 - \alpha x_1 y\\ \frac{dx_2}{dt} &= \beta x_1 - \frac{\varepsilon x_2 y}{1+mx_2} - \mu_1 x_2 \qquad (3)\\ \frac{dy}{dt} &= \frac{\gamma\varepsilon x_2 y}{1+mx_2} - \mu_2 y\end{aligned}$$

The system has eight parameters, the parameters $r, K, \beta, \varepsilon, \gamma, \alpha, \mu_1, \mu_2$, and m are positive. The constant K is the carrying capacity of habitat where the immature evolves. Immature individuals become mature with a rate β. The impact of the immature prey density on the intrinsic reproductive rate r is given by the factor $1-\frac{x_1}{K}$.

The functional response describes a typical predator interaction with prey. For this case we using $\frac{\varepsilon x_2 y}{1+mx_2}$ is referred to Michaelis-Menten as a functional response of Holling type II, for predator growth which consists of a fixed mortality rate and birth rate. The parameter ε and m are the exact parameters of the response function, where ε represent the effects of the capture level and m represents the handling time. Ecologically, when low densities of the predator will consume prey at a rate is entirely limited by handling time. The high-density predation rate is the opposite of the handling time if the predator needs half an hour to handle (digest, swallow, and capture). The natural mortality rates for mature prey and predator are denoted by μ_1 dan μ_2, respectively. The term saturating interaction ensures that the solutions of the system remain limited to a wider range of parameter value, for realistic models of long-term interactions.

A. Model Construction

For ease of computations and simplicity the system, non-dimensionalize of the model represented by the system (3) is done so as to reduce the number of some parameters as following scaling.

Let $\delta = \varepsilon\gamma$, and then system (3) become:

$$\begin{aligned}\frac{dx_1}{dt} &= x_1\left(r - \frac{rx_1}{K} - \beta - \alpha y\right) = x_1 f(x,y)\\ \frac{dx_2}{dt} &= \beta x_1 - \frac{\varepsilon x_2 y}{1+mx_2} - \mu_1 x_2 = g(x,y) \qquad (4)\\ \frac{dy}{dt} &= y\left(\frac{\delta x_2}{1+mx_2} - \mu_2\right) = yh(x,y)\end{aligned}$$

Our aim is to study the dynamics behavior of the predation parameters.

B. Existence

In this section, the local stability analysis of all possible equilibrium point of the system (4) are investigated. By the definition of equilibria, the equilibria of system (4) is satisfied when

$$\frac{dx_1}{dt} = \frac{dx_2}{dt} = \frac{dy}{dt} = 0$$

There are four equilibrium points of the system (4).
The stability and existence for each are as follows:

i. The equilibrium point $E_0 = (0,0,0)$ always exists.
ii. The equilibrium of immature prey extinction $E_1 = \left(0, \frac{\mu_2}{-m\mu_2+\delta}, \frac{\mu_1\delta}{(-m\mu_2+\delta)\varepsilon}\right)$
iii. The equilibrium of predator extinction $E_2 = \left(-\frac{k(\beta-r)}{r}, -\frac{\beta k(\beta-r)}{\mu_1 r}, 0\right)$

Immature and mature prey can survives without any predation by predator.

iv. The system (4) has a unique coexistence equilibrium $E_3 = (x_1{}^*, x_2{}^*, y^*)$
Where $E_3 = \left(-\frac{k\mu_2(B)}{A}, \frac{\mu_2}{-m\mu_2+\delta}, -\frac{\delta(C)}{A}\right)$

C. The Local Stability of The Equilibria

The system (4) is a nonlinear autonomous. The stability of the nonlinear autonomous can be obtained by linearising the system (4) with the Jacobian matrix is given by:

$$J(x_1^*, x_2^*, y^*) = \begin{bmatrix} r\left(1-\frac{x_{1,}}{k}\right)-\frac{rx_{1,}}{k}-\beta-y\alpha\beta & 0 & -\beta\alpha x_{1,} \\ \beta & -\frac{\varepsilon x_{2,}}{mx_{2,}+1}+\frac{\varepsilon x_{2,}ym}{(mx_{2,}+1)^2}-\mu_1 & -\frac{\varepsilon y}{mx_{2,}+1} \\ 0 & \frac{\delta y}{mx_{2,}+1}-\frac{\delta ym}{(mx_{2,}+1)^2} & \frac{\delta y}{mx_{2,}+1}-\mu_2 \end{bmatrix}$$

The characteristic equation is given by $\left|J(x_{1,}{}^*, x_{2,}{}^*, y^*) - \lambda I\right| = 0$ where $(x_{1,}{}^*, x_{2,}{}^*, y^*)$ is the equilibrium of (4).

The local dynamical behavior of equilibrium point is investigated where the result is obtained by computing the eigenvalue of the matrix from each the equilibrium point.

- Equilibrium point $E_0 = (0,0,0)$

$$|J(0,0,0) - \lambda I| = 0$$
$$\begin{vmatrix} (r-\beta)-\lambda & 0 & 0 \\ \beta & -\mu_1-\lambda & 0 \\ 0 & 0 & -\mu_2-\lambda \end{vmatrix} = 0$$

The eigenvalues of this matrix are $\lambda_1 = r - \beta$, $\lambda_2 = -\mu_1$ and $\lambda_3 = -\mu_2$. Because all the parameters on (4) are all positive, then λ_2, λ_3 have to be negative. The equilibrium point E_0 is locally stable provided the following condition is satisfied if $r < \beta$ and if $\lambda_1 = r > \beta, E_0$ is unstable.

- Equilibrium point $E_1 = \left(0, \frac{\mu_2}{-m\mu_2+\delta}, \frac{\mu_1\delta}{(-m\mu_2+\delta)\varepsilon}\right)$

The three eigenvalue of this matrix are

$$\lambda_1 = -\frac{-\beta\varepsilon m\mu_2+\varepsilon m\mu_2 r+\beta\delta\varepsilon-\delta\varepsilon r-\delta\beta\alpha\mu_1}{(-m\mu_2+\delta)\varepsilon},$$

$$\lambda_2 = \frac{1}{2}\frac{-\mu_2\mu_1 m+\sqrt{m^2\mu_1{}^2\mu_2{}^2-4\delta m\mu_1\mu_2{}^2+4\mu_1\mu_2\delta^2}}{\delta} \text{ and}$$

$$\lambda_3 = -\frac{1}{2}\frac{\mu_2\mu_1 m+\sqrt{m^2\mu_1{}^2\mu_2{}^2-4\delta m\mu_1\mu_2{}^2+4\mu_1\mu_2\delta^2}}{\delta}$$

Because all of the parameter positive, then λ_1, λ_3 should be negative. As λ_2 positive. Therefore, the system (4) is always unstable around E_1 which is, in fact, a saddle-node. If $\lambda_1 > 0$ a saddle node with initial values for α a defference case.

- Equilibrium point $E_2 = \left(-\frac{k(\beta-r)}{r}, -\frac{\beta k(\beta-r)}{\mu_1 r}, 0\right)$

The eigenvalues are $\lambda_1 = \beta - r$, $\lambda_2 = -\mu_1$ and $\lambda_3 = -\frac{-\beta^2 km\mu_2+\beta km\mu_2 r+\beta^2\delta k-\beta\delta kr+r\mu_1\mu_2}{\beta^2 km-\beta kmr-\mu_1 r}$.

Because all of the parameters in (4) are all positive, then λ_2, λ_3 should be negative. If $\lambda_1 = \beta > r$, then E_2 is unstable. The equilibrium point E_2 is locally stable provided the following condition is satisfied $\beta < r$

- Non zero Equilibrium point $E_3 = (x_1{}^*, x_2{}^*, y^*)$

Where $x_1{}^*, x_2{}^*, y^*$ are obtained by solving very complicated. From the ecological point of view, this equilibrium is an important and one of the interesting analysis in mathematical ecology that in this case all of three population will coexistence in the ecosystem.

We need a further analysis for the existence of E_0 is asymptotically stable under certain condition while E_1 is unstable. E_2 and E_3 are also asymptotically stable on some condition. The behavior of the solution is closely related to the initial value of parameters in those three population. We also confirm the analysis by simulating the value of β on some condition.

IV. Numerical Simulation

In this section, we only performed some of the key findings using numerical simulation. It gives a touch of completeness to the analytical findings and observed the effect on the dynamics of the system (4). In our numerical analysis, we used the software Matlab to obtain the steady-state curves.

A. Simulation of the dynamic behavior

The numerical study presented here show the dynamics of the prey-predator model around the positive interior steady-state. Model is simulated by assuming initial value and parameters as follows table I.

TABLE I. Initial Value of Parameters

Parameter	Value
r	1.5
α	0.7
β	1.15
ε	1.47
δ	0.8
m	0.1
μ_1	0.05
μ_2	0.7
k	1.4

The first simulation, we represent the dynamic behavior between the predation parameters and the environment's capacity. By considering the different value of parameter ε, because is contained in functional responses that the formed the main components of prey-predator models. We varied the value of the effects of capturing rate parameter ε, and observed the effect on the dynamics of the system (4).

1) The dynamics behavior of parameter predation ε, if condition $\varepsilon > K$.

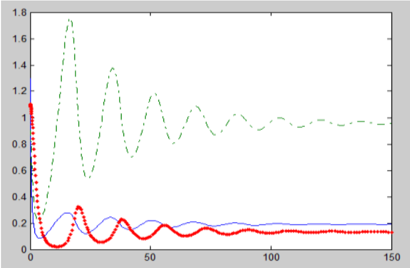

Fig. 1. The dynamical behavior at $\varepsilon = 1.47$

Therefore, for the above choice of parameter predation $\varepsilon = 1.47$. Figure 1 shows that the axial equilibrium the system (4) is locally stable. It means that complete extinction of life in that ecosystem. All the population of the system (4) converges to the positive equilibrium. The populations approach their equilibrium values in finite time

2) The dynamical behavior of parameter predation ε, this condition if $\varepsilon < K$.

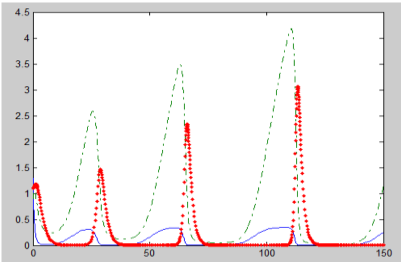

Fig. 2. The dynamical behavior at $\varepsilon = 0.9$

The different value of parameter predation ε, if $\varepsilon = 0.9$, it means $\varepsilon < K$. The dynamical behavior show unstable for three population.

3) By considering different value of parameter predation ε, if $\varepsilon = 0.47$

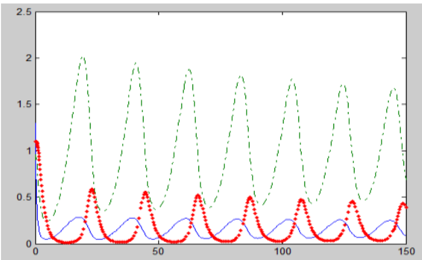

Fig. 3. The dynamical behavior at $\varepsilon = 0.47$

These condition on figure 2 and figure 3, no population will be survived ecologically. Figure 3 represents the instability of the coexistence equilibrium point. That all are unstable and this condition naturally is not a good sign for living species.

B. Numerical Simulation

Further by numerical solution of the system and numerical simulations, we reveal more dynamic behaviors of the model in one of four equilibrium point with parametric value.

Again, we also consider another set of parametric values such as follows table II.

TABLE II. SET OF PARAMETRIC VALUES

Parameter	Value
r	1.1
α	1.2
β	1.15
m	0.05
δ	0.8
ε	1.47
μ_1	0.1
μ_2	1.32
k	1.5

By considering defferent value of parameter α, because is a component interaction between the predator and immature prey.

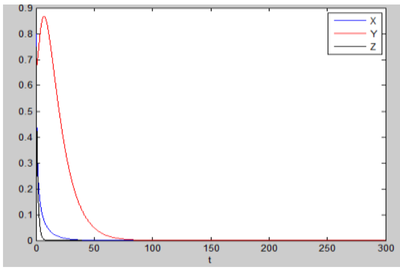

Fig. 4. Extinction of three population

The corresponding phase portrait for a different initial choice of the parameter in table II, it shows that the axial equilibrium the system (4) is locally asymptotically stable. This is also shown in Figure 4 graphically.

1) Simulation in the Equilibrium E_0

a) The values of parameters are chooses to satisfy the stability in non-periodic solutions. The other parameters are fixed.

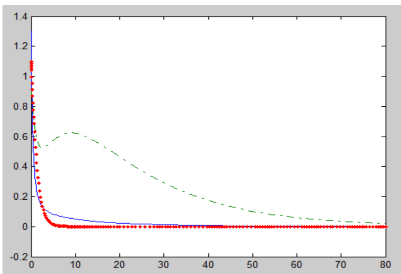

Fig. 5. The dynamical behavior of Eo at $\beta < r$

It shows that the axial equilibrium is stable for this condition.

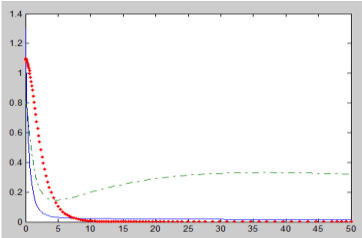

Fig. 6. The dynamical behavior of Eo at $r = \beta$

The system (4) around equilibrium Eo with an initial value of $\beta = 1.1$. Variation of population with r = β for initial point [1.5 1.7 2.2].

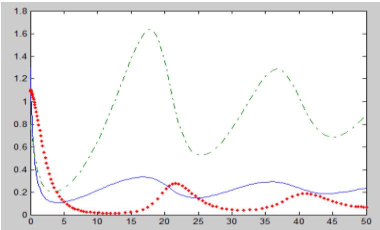

Fig. 7. The dynamical behavior of Eo at $r < \beta$

Figure 7 shows that the axial equilibrium is unstable for this condition at $r < \beta$.

2) Equilibrium point of the system around E_0

Now, talking the same set of parametric values we draw dynamical behavior for the equilibrium point E_0.

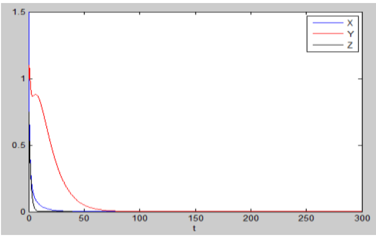

Fig. 8. Time series of stability system (4)

From this figure, it is observed that all three populations are going to extinction with respect to time. If at any moment for any ecosystem trivial equilibrium point become stable it means that complete extinction of life in that ecosystem. The trivial equilibrium point always exists but never stable ecologically as well as mathematically.

The stable and unstable equilibrium point has been drawn in figure 9 until figure 12.

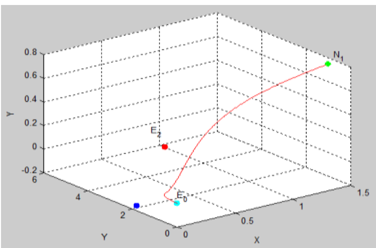

Fig. 9. Equilibrium point system (4) with N1

We also consider another set of initial value such as N1, N2, N3 and N4. Where initial value of N1 = [1.1 0.5 2.75]; N2 = [2.5 2.2 3.03]; N3 = [1.2 1.4 1.1]; and N4 = [0.4 0.8 2.1];

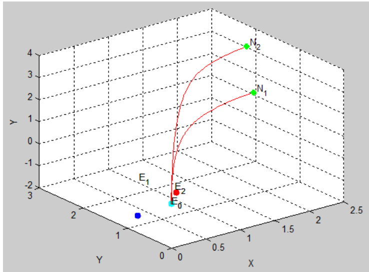

Fig. 10. Dynamical behavior with initial value N1 and N2 around E_0

Figure 10 shows the phase portraits with initial value of N1 and N2, where N1=[1.7 0.8 2.9], and N2 = [2.5 2.2 3.03].

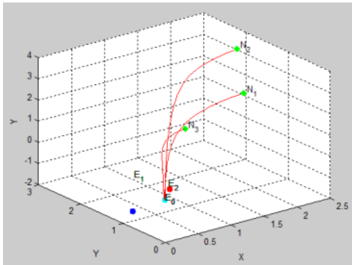

Fig. 11. Initial Value for dynamical behavior with N1, N2 and N3

With parameter value in table II, figure 11 clearly indicates that phase portraits of the system around equilibrium point E_0.

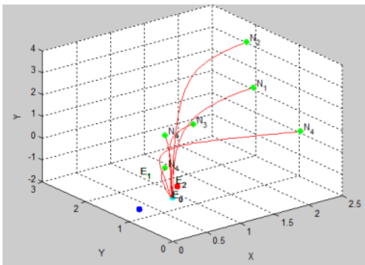

Fig. 12. Phase portraits of system (4) around E_0

Figure 12 Phase portraits of the system (4), corresponding to the different initial value of N4. Where initial value N4 =[0.4 0.8 1.3] and[2.4 0.8 0.3].

CONCLUSION

In the present paper a prey-predator model of interactions three population with a stage-structure for prey. Numerical simulation of the prey-predator model with a stage-structure for prey is investigated, which shows phase portraits of the system (4) around equilibrium point E_0. By a computational, we established some new phase portraits such as the existence of stable or unstable equilibrium point under a suitable value of the parameter in the prey-predator model with a stage-structure for prey.

REFERENCES

[1] Abadi and D. Savitri, “Stability analysis of three species food chain with competition stability analysis of three species food chain with competition,” AIP Conference Proceeding, BioMath, vol. 1651, p. 114, March 2015.

[2] B. Dubai, B.D. Deka, A. Patra, “Stability and Hopf-bifurcation in a general Gauss type two-prey and one-predator system,” Appl. Mathematical Modelling, vol. 40, pp. 5793-5818, June 2016.

[3] M. Falconi, M. Huenchucona, C. Vidal, “Stability and global dynamic of stage-structured predator-prey model with group defense mechanism of the prey,” Applied Mathematics and Computation, vol. 270, pp. 47-61, November 2015.

[4] H. Gao, H. Wei and Zhai, “Functional used in biological models and their influence on simulations,” Indian J. of Marine Sci., vol. 29, pp. 230-237, 2000.

[5] S. Jana, A. Ghorai, S. Guria and T.K. Kar, “Global dynamics of a predator, weaker prey and stronger prey system,” Applied Mathematics and Computation, vol. 250, pp. 235–248, January 2015.

[6] Jarvis and Antony, “Predicting population sizes and priority conservation areas for 10 endemic Namibian bird species,” Biological Conservation, vol. 88, pp. 221-231, April 1999.

[7] T.K. Kar and K.S. Chaudhury, “Harvesting in a two-prey one-predator fishery,” ANZIAM J., vol. 45, pp. 443-456, January 2004.

[8] T.K. Kar and A. Batabyal, “Persistence and stability of a two prey one predator system,” International J. of Eng., Sci. and Technology, vol. 2(2), pp. 174–190, 2010.

[9] S. Khajanchi, “Dynamic behavior of a Beddington-DeAngelis type stage structure predator-prey model,” Applied Mathematics and Computation, vol. 244, pp. 344-360, October 2014.

[10] V. Madhusudanan, and S. Vijaya, “Dynamical Behaviour in Two Prey-Predator System with Persistence,” vol. 16, pp. 20–37, 2016.

[11] P. Panja, S. Mondal and J. Chattopadhyay, “Dynamical Effects of Anti-predator Behaviour of Adult Prey in a Predator-Prey Model with Ratio-dependent Functional Response,” Asian Journal of Mathematics and Physics, vol. 1, pp. 19-32, February 2017.

[12] R.K. Upadhyay, S.N. Raw, “Complex dynamics of a three species food chain odel with Holling type IV functional Response,” Nonlin. Anal, vol. 16(3), pp. 353-374, September 2011.

[13] W. Wang, and L. Chen, A predator prey system with stage structure for predator, Comput. Math. Appl., vol. 33 (8), pp. 83-91, September 1997.