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Multi-objective Linear Programming for Supplier Selection and Order Allocation of Raw Material

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***Abstract*—The production process of a company is very dependent on the availability of raw materials. Raw materials will be available if there are suppliers who send raw materials continuously to the company. As we know that each supplier has difference in price, supply capacity, delivery time and quality of raw materials. Therefore, determining the right supplier is the important process in maintaining company production process. The objective of this study is to select the supplier and allocate orders to the selected supplier by developing the multi-objective linear programming (MOLP). The developed MOLP model will minimize 3 objective functions which include total cost, number of defective raw materials and total delivery time. Numerical examples is provided in this study for application the proposed model. The obtained results prove that the developed model has a better ability to select supplier and allocate the orders to the selected supplier.**

Keywords—multi-objective linear programming; supplier selection; order allocation;

I. INTRODUCTION

The supplier is required in the company's production management [1]. The selection of the right supplier is an important activity in the procurement activity [2]. It also can increase the company's competitive advantage [3]. Supplier selection and order allocation are the important process to maintain the availability of raw material. The most important criteria for supplier selection are price, quality and delivery time [4].

Many previous studies have discussed about supplier selection problem. The proposed method for solving the supplier selection problem can be classified into mathematical programming models and multi criteria decision making (MCDM) approach. MCDM approach used to solve supplier selection problem include Analytic Hierarchy Process [5], Analytic Network Process [6], Promethee [7], and Electre [8].. Mathematical programming models used in the proposed models include linear programming, mixed integer programming, and multi criteria optimization [9]. The model that optimized the allocation procurement to suppliers using the single objective mixed integer linier was developed by [10]. Reference [11] developed mixed integer linear programming (MILP) to minimize the total cost of supplier selection with price break. The MILP also was used to solve supplier selection and order quantity allocation by [12]. The mixed integer nonlinear programming model was constructed by [13] to select and property allocate orders to suppliers.

Reference [14] described the supplier selection problem and how much order quantity should be assigned to each. The two phase fuzzy multi objective Linear Programming (FMOLP) was developed by [15] to solve the supplier selection problem and order allocation under multi price level and multi product. The decision support system was developed by [16] for supplier selection and order allocation in stochastic, multi-stakeholder and multi-criteria environments.

Reference [17] developed integer linear programming (ILP) to determine the selected supplier and allocated the orders to the selected supplier. The developed ILP model had objective function to minimize the total cost. The considered costs in the objective function of this model are purchasing cost, ordering cost and holding cost. There were 8 constraints in the model which include supplier's capacity, order allocation, demand, inventory balance, safety stock, delivery time, binary, non negative and integer. The weakness of model would appear on the delivery time constraint when all alternative suppliers have less delivery time of product than the allowable delivery time of company. So there is no trade off for this constraint when the condition occurs.

This study will improve the weakness of previous model by eliminating the delivery time constraint. In this model will choose supplier which not only have delivery time that meets the company's requirement but also choose supplier which has the fastest delivery time. To achive this purpose, the delivery time constraint in the previous model will be changed to minimized objective functions. This improved model will have more than one objective function. The model which can accomadate the problem with many objective functions is multi-objective linear programming.

As we know, supplier is selected based on criteria of price in general. But sometimes, the price is not guarantee the product quality and delivery time. When the company prioritizes the low price criteria in choosing a supplier, the company actually gets low quality product and slow delivery times. But when the company prioritizes quality of product and fast delivery times, company must pay for products at higher prices. Regarding to conflicts between the three price, quality and delivery time criteria, we propose a multi-objective linear programming approach to solve a supplier selection and order problem allocation. According to [18], the supplier selection problem is include in MOLP problem.

II. RESEARCH METHODS

The first step begins with developing the MOLP model which includes model assumptions, model components, and model notations. The second stage is the verification of the model. The model verification process is carried out to ensure that the model is logically and mathematically consistent. The third stage is a model test. This stage aims to obtain a model solution by inputting data to the model and solving it with Lingo software.

A. The Model Assumptions

The MOLP model is built on assumptions as follows:

- Demand, supplier's capacity, price, cost and other parameter considered in the model are constant and known.
- Raw material shortage is not allowed from suppliers.
- Only one raw material is considered in model.
- Each supplier has finite capacity.

B. The Model Notations

The component of MOLP model consists of index set, parameters, variable, decision variables, objective functions, and constraint functions. The following notations are used to in the MOLP model.

- Index set

  The index set of this model are:
  i : Index of supplier, $i = 1, 2, \dots, m$
  j : Index of periods, $j = 1, 2, \dots, n$

- Parameters

  The following are the parameters used in this MOLP model.
  d_j : Demand of raw material at period *j*
  C_{ij} : Capacity of supplier i at period *j*
  p_i : Unit price of supplier i
  k : Ordering cost per order of raw material
  hj : Holding cost per unit of raw material at period *j*
  q_i : Defect rate of raw materials from supplier *i*
  SS_j : Safety stock at period *j*
  L_i : Delivery time of supplier i

- Variable

  The variable used inthis model is as follows:
  I_j : Number of inventory at period *j* (pcs)

- Decision Variables

  The developed MOLP model has two decision variables.
  X_{ij}: Number of raw material ordered from supplier i at period *j*

$$Y_{ij} = \begin{cases} 1, if\ an\ order\ is\ placed\ on\ supplier\ i \text{ at } period\ j \\ 0, \quad otherwise \end{cases}$$

C. Objective Functions

The developed MOLP model has three objective functions consisting of minimize total costs (Z_1), minimize number of defective raw materials (Z_2), and minimize total delivery time (Z_3).

- Minimize the total cost (Z_1)

  The total cost is sum of the purchasing cost, the ordering cost and the holding cost. The purchasing cost is obtained from raw material price multiplied by number of raw material ordered to the selected supplier. The ordering cost is calculated from total ordering cost of all selected suppliers. The holding cost is obtained from the holding cost per unit raw material multiplied by number of inventory.

$$Min\ Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} p_i X_{ij} + \mathrm{k}\sum_{i=1}^{m}\sum_{j=1}^{n} Y_{ij} + \sum_{j=1}^{n} h_j I_j \quad (1)$$

- Minimize number of defective raw materials

  The quality is one important of quality is one of the important criteria in supplier selection problems and is widely used in practical situations [19]. Raw materials quality in this model is represented by the number of defective products. The number of defective raw materials is obtained from sum of multiplication between defect rate of raw material from selected suppliers and the number of product ordered.

$$Min\ Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} q_i X_{ij} \quad (2)$$

- Minimize total delivery time

  The total delivery time is obtained from total delivery time of selected suppliers.

$$Min\ Z_3 = \sum_{i=1}^{m}\sum_{j=1}^{n} L_i Y_{ij} \quad (3)$$

Multiple objective functions in the MOLP model will be changed to a single objective function by adding weight to each objective function. Total weight equals one. Assuming that W_1 is the weight for the total cost minimization, W_2 is the weight for the number of defective raw materials minimization, and W_3 is the weight for the total delivery time minimization. The following single objective function is formed.

$$Min\ Z = W_1 \left[\sum_{i=1}^{m} \sum_{j=1}^{n} p_i X_{ij} + \mathrm{k} \sum_{i=1}^{m} \sum_{j=1}^{n} Y_{ij} + \sum_{j=1}^{n} h_j I_j \right] + W_2 \left[\sum_{i=1}^{m} \sum_{j=1}^{n} q_i X_{ij} \right] + W_3 \left[\sum_{i=1}^{m} \sum_{j=1}^{n} L_i Y_{ij} \right] \qquad (4)$$

D. The Constraint Functions

The constraints of MOLP model consists of supplier capacity, demand, order allocation, inventory balance, safety stock, nonnegative and binary.

- The capacity of supplier

$$X_{ij} \leq C_{ij} \qquad \forall i,j \qquad (5)$$

Supplier's capacity is extensively used in the literature of supplier selection problem. The number of raw material ordered to suppliers must consider the supply capacity of each supplier. The constraint in (5) ensuress that the number of raw material ordered to suppliers must not exceed the supply capacity of suppliers.

- Demand

$$\sum_{i=1}^{m} X_{ij} + I_j \geq d_j, \qquad \forall j \qquad (6)$$

The number of raw material ordered from supplier is also affected by the raw material inventory and demand. The constraint in (6) shows that the number of raw material ordered plus the inventory of raw materials must exceed the demand. This constraint ensures that demand at the each period must be met.

- Order allocation

$$X_{ij} \leq MC_{ij} Y_{ij} \qquad \forall i,j \qquad (7)$$

Order allocation is the determination of the number of raw material ordered to the supplier. Ordering the raw material to suppliers is done on selected supplier. The number of ordered raw material is less than the order allocation. *M* is the biggest positive number.

- Inventory balance

$$I_j = I_{j-1} + \sum_{i=1}^{m} X_{ij} - d_j\,, \qquad \forall j \qquad (8)$$

Inventory balance of raw material needs to be maintained to smooth the production process. The number of current inventory is affected by the number of inventory in the previous period, the number of ordered raw material, and demand.

- Safety stock

$$I_j \geq SS_{j,} \qquad \forall j \qquad (9)$$

The inventory quantity for the current period must be greater than the safety stock. Safety stock is used to prevent the shortages, defect and raw material damage during production process.

- Binary

$$Y_{ij} \in (0,1), \qquad \forall i,j \qquad (10)$$

The binary constraint shows that order allocation will be placed on to a supplier with a value of 1 and no order allocation to a supplier with a value of 0.

- Nonnegative and integer

$$X_{ij}, I_j \geq 0\ and\ integer \qquad \forall i,j \qquad (11)$$

The result obtained must be positive and integer.

The complete MOLP model is as follows:

$$Min\ Z = W_1 \left[\sum_{i=1}^{m} \sum_{j=1}^{n} p_i X_{ij} + \mathrm{k} \sum_{i=1}^{m} \sum_{j=1}^{n} Y_{ij} + \sum_{j=1}^{n} h_j I_j \right] + W_2 \left[\sum_{i=1}^{m} \sum_{j=1}^{n} q_i X_{ij} \right] + W_3 \left[\sum_{i=1}^{m} \sum_{j=1}^{n} L_i Y_{ij} \right]$$

Subject to:

$$X_{ij} \leq C_{ij} \qquad \forall i,j$$
$$\sum_{j=1}^{n} X_{ij} + I_j \geq d_j, \qquad \forall j$$
$$X_{ij} \leq MC_{ij} Y_{ij} \qquad \forall i,j$$
$$I_j = I_{j-1} + \sum_{i=1}^{m} X_{ij} - d_j \qquad \forall j$$
$$I_j \geq SS_{j,} \qquad \forall j$$
$$Y_{ij} \in (0,1), \qquad \forall i,j$$
$$X_{ij}, I_j \geq 0\ and\ integer, \qquad \forall i,j$$

III. RESULT AND DISCUSSION

This section will provide numerical examples for the application of this proposed model. There are eight raw material suppliers that will be chosen to fulfill the orders during four periods. Each supplier has difference in price, supply capacity, delivery time and defect rate of raw materials. Table I shows the demand of raw material for four periods. Table 2 summarize the supplier's parameters.

The results obtained in this model will be compared to the previous model. For comparison purposes, the same data in [13] is used. In addition, the value of $W_{1=}0.5$, $W_{2=}0.3$, $W_{3=}0.2$, and the defect rate of each supplier. The ordering cost is Rp. 5,000 /order. The holding cost is Rp.100/pcs/period. The initial raw material inventory is 3,200 Pcs and safety stock is set at 5% of demand per period.

TABLE I THE DEMAND OF RAW MATERIAL

Period	Demand (pcs)
1	37,224
2	32,668
3	59,032
4	39,764

TABLE II THE SUPPLIER'S PARAMETERS

Suppliers	Capacity (pcs)	Price (pcs)	Delivery Time (days)	Defect Rate (%)
1	22,050	6,800	2	1
2	18,624	6,700	3	1
3	10,500	5,900	2	4
4	6,900	6,650	4	3
5	9,000	5,950	1	4
6	2,225	5,300	1	5
7	8,000	6,600	4	3
8	1,298	6,700	2	1

Supplier parameters and other data are included in the MOLP model and solved by Lingo. The Lingo solver produce global optimum output with value of the objective function, Z = 523,503,000. Table III shows the result of selected suppliers and order allocation of the proposed model. While table IV is a summary of results from the previous model.

TABLE III THE SELECTED SUPPLIERS AND ORDER ALLOCATION OF MOLP MODEL

Suppliers	Order Allocation (pcs)			
	Period 1	Period 2	Period 3	Period 4
1	0	0	3,804	0
2	0	0	18,624	2,175
3	10,500	10,500	10,500	10,500
4	6,160	2,715	6,900	6,900
5	9,000	9,000	9,000	9,000
6	2,225	2,225	0	2,225
7	8,000	8,000	8,000	8,000
8	0	0	1,298	0
Safety stock	1,861	1,633	2,952	1,988

Based on the results in table 3 for the current model, it is found that there are five suppliers are selected to fulfill the company's order in the first period. They are supplier 3,4,5,6,7. Order allocation to the selected supplier are supplier 3 =10,500 pcs, supplier 4 = 6,160 pcs, supplier 5 = 9,000 pcs, supplier 6 = 2,225 pcs, and supplier 7 = 8,000 pcs. The total order allocation in the first period is 35,885 pcs. The safety stock in the first period is 1,861 pcs. The selected supplier to fullfill the orders in the second period are the same with the selected supplier in the first one. But the allocation of orders to supplier 2 is different. Order allocation of raw material to the supplier 2 in the second period is 2,715 Pcs. The total order for raw materials in second period is 32,440 pcs. The inventory in period 2, I_2 = 1,633 pcs.

There are seven selected suppliers in the third period for the current model. Only supplier 6 is not the selected supplier. Order allocation to each supplier are supplier 1 = 3,804 pcs, supplier 2 = 18,624 pcs, supplier 3 = 10,500 pcs, supplier 4 = 6,900 pcs, supplier 5 = 9,000 pcs, supplier 7 = 8,000 pcs, and supplier 8 is 1,298 pcs. The total order of raw material to all selected suppliers is 58,126 pcs. The final inventory in period 3 is 2,952 pcs.

In the fourth period for the current model, there are 6 selected suppliers to supply the raw materials. They are supplier 2, 3, 4, 5, 6, and 7. The order allocation to each supplier are supplier 2 = 2,175 pcs, supplier 3 = 10,500 pcs, supplier 4 = 6,900 pcs, supplier 5 = 9,000 pcs, supplier 6 = 2,225 pcs and supplier 7 = 8,000 pcs. The total order of raw materials in the fourth period is 38,800 pcs with an initial inventory of 2,952 pcs and the final inventory in period 4 is 1,988 pcs.

The conclusion that can be drawn from table III is that there are 4 suppliers that are always selected to meet the demand. They are supplier 3, 4, 5 and 7. It can be said that suppliers 3, 4, 5, and 7 are the main suppliers. While the suppliers 1, 2, 6, and 8 are not always selected. The result also shows that the supplier 3, 5, and 7 are the main suppliers with the order allocation according to their maximum capacity.

TABLE IV THE SELECTED SUPPLIERS AND ORDER ALLOCATION OF PREVIOUS MODEL [17]

Suppliers	Order Allocation (pcs)			
	Period 1	Period 2	Period 3	Period 4
1	0	0	0	0
2	0	0	18,624	2,175
3	10,500	10,500	10,500	10,500
4	6,160	6,519	6,900	6,900
5	9,000	9,000	9,000	9,000
6	2,225	2,225	2,225	2,225
7	8,000	8,000	8,000	8,000
8	0	0	1,298	0
Safety stock	1,861	5,437	2,952	1,988

Comparison of the results obtained from both models can be concluded that: (1) both models produce the same number and composition of suppliers for each period, (2) both models have the same allocation of orders to each supplier in the first and fourth periods, (3) there are differences in order allocations for periods two and three for both models, (4) the previous model produces 5 main suppliers while the current model produces 4 main suppliers. Related to the results of the comparison of the two models point 3, in period 3, the current model selects supplier 1 to supply raw materials to the company compared to the previous model that chose supplier 6. This is the optimal compromise solution of the model. Supplier 1 has a higher price but low defect rate and moderate delivery time compared to supplier 6. While supplier 6 has a lower price and delivery time but the defect rate is higher.

IV. CONCLUSION

Supplier selection and order allocation are the important process to maintain the availability of raw material. The objective of this study is to select the suppliers and allocate the order to the selected suppliers. The developed MOLP model proved able to be used to select the best supplier and allocate orders to the selected supplier. The model consider more comprehensive criteria of supplier selecting problem and also overcome the conflicts between criteria. So that the solution obtained is more optimal. For further research, the model can be integrated with pairwise comparison methods to determine the weight of the objective function. Model development can also be implemented by considering other relevant criteria.

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