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Develop Students' thinking Ability In Math Class

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Abstract. Developing students' thinking ability is always an extremely important subject in mathematics teaching and research. To study it, both a lot of teaching practice and education psychology as the scientific basis. Therefore, teachers should make clear the essence and meaning of "thinking ability", its occurrence, development and conditions on the basis of actual teaching and using the scientific principles elaborated by education psychology, so as to consciously take measures to promote the development of students' thinking ability in actual teaching.

The essence and meaning of thinking

Psychology points out that "the mind is the human brain's general reflection of reality". That is to say, people use the existing perceptual materials to analyze, synthesize, abstract and summarize in their minds, so as to achieve a certain understanding of the essence and internal laws of things, and reasoning and solving accordingly. The level of thinking required in the whole process of mathematics teaching (abstract degree, summary degree) is higher and higher with the deepening of the teaching material. It also shows the high abstractness of the science of "mathematics".

For example, the concept of "elementary function" seems easy to understand, but it is so vague to beginners of calculus that it is extremely difficult to study it deeply. We can't understand what we feel right away. We can only feel it more deeply if we understand it. The "understanding" should be carried out through thinking, but it cannot be realized only by perception (feeling and perception). In the study of mathematics, thinking also has special significance, because the field of perception can reach is limited, while the field of thought can reach is infinite. When people first learn to add in math, they can add in real objects (or fingers), but when they learn to add in Numbers, they can't replace the rule of addition with the rule of addition. In this case, they have to resort to thinking (which is related to the rule of addition).

For example, it is difficult for a student who has just learned to add fractions to immediately judge the results of the following series of Numbers:

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^{n-1}}+\cdots \tag{1}$$

Note: this string of Numbers will only increase more, unlimited to add; When you learn the geometric series, you know that the sum of the first n terms is $\frac{1-(\frac{1}{2})^n}{1-\frac{1}{2}}$, and the sum of the series is not going to be infinitely larger and the sum is not going to be greater than 2; We know that the sum of the learning limit is 2.

Through teacher's explanation and students' thinking activities, students can understand the above results. However, it is difficult for the student to understand the result of the following sequence of Numbers before learning the infinite series.

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n}+\cdots \tag{2}$$

This number line increases more and more; Add without limit, and the sum can be larger than anything!

Therefore, education psychology points out, there is no generalization, or only perceptual generalization (a kind of low-level generalization) but no rational generalization, and knowledge cannot be realized. The "generalization" here is the essence of thinking, which is of great significance for learning mathematics well. And this kind of generalization ability (a kind of psychological characteristic) should be cultivated through teaching.

The intuition of teaching material and summary

How to develop students' thinking ability? Cannot leave the textbook the research.

One of the main tasks of classroom teaching is to enable students to master the knowledge that human beings have efficiently. Clearly, this learning process is not the same as a mathematician's exploration of new knowledge. First of all, this difference is reflected in time. The time required by mathematicians to produce new knowledge and to understand an objective law cannot be compared with the time required by students to master the previous experience under the guidance of teachers and textbooks. Second, it is shown in order. The teaching material enables students to master the order of mathematical knowledge, which is different from the order of discovering mathematical knowledge by predecessors. For example, for some reason, textbooks always ask students to study the index first and then the logarithm, but history is just the opposite -- people study the exponent based on the logarithm.

Students master mathematical knowledge through textbooks and the former production of mathematical knowledge although there are various differences but common thinking development trend is consistent. That is to say, the process from concrete to abstract, from special to general, from perceptual to rational is consistent. We can't ignore this kind of thinking consistency just because the textbook can help students master the experience of predecessors efficiently. Otherwise. On the basis of books, students do not have access to the things marked by the words in the book, nor do they have access to the activities reflected in the students' minds. Therefore, it is impossible for the mathematical knowledge to flow from the books into the students' minds.

In order to be consistent with previous experience in thinking development, we must study the two cognitive links of intuition and generalization.

The intuitive content of the textbook

As has been said before, the essence of thinking lies in generalizations, and generalizations must have generalizations of material, otherwise thinking cannot occur and develop. Therefore, when the teacher prepares the lesson to delve into the teaching material, must pay attention to the research teaching material's direct content first.

From the perspective of cognitive process, intuition is the process of forming perceptual knowledge in students' mind under the action of things. Although intuition can only form perceptual knowledge, it is the starting point of thinking and the beginning of transforming from perceptual knowledge to rational knowledge. In the absence of this starting point and beginning, students tend to stay on empty concepts and principles. For example, students learn the concept of imaginary Numbers without exposing them to the quantities expressed in imaginary Numbers. I learned various concepts of calculus without exposing students to various applied problems of calculus. Then the student cannot carry on the positive thinking, but can only transfer the thought burden to the memory. (memorization, hard back), resulting in learning on the lack of understanding. Of course, what we are talking about here is not necessarily the physical object itself (physical intuition), but also models, charts, slides, movies (called mock-visual intuition). Here, to let students "touch" is not necessarily to let students touch with their hands and eyes, but also to let students restore and establish the image of things in their minds through the description of concrete examples (visual language). This is done through perception and imagination. This intuition is linguistic intuition.

Mathematical induction is a very abstract mathematical principle, and it is more necessary to make use of the image intuition. Some textbooks recommend using a line of vertical dominoes, pushing down the first one with your hand, then the second one, then the third one, then the second one So all the dominoes are pulled down, and to make sure all the dominoes are down, you have to have two broad strokes; The first one must be inverted, and when one is inverted, the next one must be able to be reversed. For example, we can use language to describe the scene of the dominoes being knocked down one by one, so that the student can restore to his mind the image he used to play this kind of domino game, which is called "memory representation". If the student has not played this kind of domino game in the past (this kind of thing image has not been perceived in the past) but through the teacher vivid language description, the student also may depend on the imagination to establish the thing image in the mind, this is called "imagination appearance".

Language intuition has many advantages. It is not limited by time, place and equipment. However, the image of things restored and established in the mind through language, whether memory representation or imagination representation, reflects the integrity, stability, freshness and correctness of reality, which is often inferior to perception. Moreover, in the process of imagination, there is often confusion between similar things. Because of this limitation of linguistic intuition, it is necessary to match physical intuition, especially mock-image intuition, when possible (this is the basis for illustration in textbooks).

Overview of the textbook

Generalization is the process from the transformation of perceptual knowledge to the formation of rational knowledge. It is the process of reflecting things more correctly and completely, thus leading to the deep understanding of students. This is the main task of the math textbook, but also we study a point of the textbook. In this regard, I think the following points should be noted:

Distinguish essential features from common features

In the process of generalization, students often confuse the common features and essential features of a kind of things, so that they regard common features as essential features.

For example, students are learning that even functions an image symmetry (about the y axis) properties, mistakenly take the essential features of the axisymmetric character as the even functions, and in this matter, I'm in the lead the students to find "function" is both odd function and even function when found, and all of these, teaching material is impossible to narrative in detail, must requires teachers to delve into teaching material can be found.

Generally speaking, the textbook can only sum up a kind of things positively, but asks the student to be able to truly grasp its essential characteristic, must study it again from the reverse even from the side. Taking the periodic function as an example, the textbook always starts from the trigonometric function, and generalizes that the essential feature of periodic function is the existence of a non-zero constant T, which makes the following equation true for any number x in the domain:

$$f(x+T) = f(x) \tag{3}$$

For example, some other functions: $y = x+1$， $y = \sin|x| \cdots\cdots$ In order for the students to really grasp the periodicity of the function, sometimes we are required to study it from the side. Through the study of the front, the back and the side, students' understanding of the concept of periodic function must be profound.

The level of generalization should be constantly improved

Generally speaking, with the growth of students' age and the deepening of the textbook content, the general level of students should be gradually improved, which teaches students from perceptual generalization (spontaneous, low-level) to rational generalization (advanced). "One to one correspondence" is a problem that seems easy but difficult to understand. On the basis of single value correspondence, I have listed several examples of "one to one correspondence" (summary materials). Ask students to generalize. It turns out that this correspondence can be summarized as: if the set does not go to set B, it is a single value corresponding to set B. On the contrary, set B to set A is also A single value corresponding, so it is said that from set A to set B constitutes A one-to-one correspondence ".

On this basis, the introduction of the system function, basic elementary function, the concept of composite function, thus generalize the concept of elementary function. And use the above concepts to gradually master the skills and skills of analysis function composition, which lays a foundation for subsequent learning.

Generalize the material to be perfect

The material that the textbook provides is often positive, standard, so the essential feature of the object and non-essential feature are sometimes not easy to distinguish. For example, when we generalize the concept of continuity of a function, we first generalize the continuity of a function at a certain point x, which is defined as $\lim_{\Delta x \to 0}[f(x+\Delta x)-f(x)]=0$ (the function $f(x)$ is defined in some neighborhood of x_0) .A careful analysis of the definition includes three conditions that must be met, one is indispensable; The second is to derive the continuity in the open interval, which also includes three conditions that must be satisfied and cannot be done without one. Finally, the problem of continuity of elementary function is summarized. In addition to using the standard mathematical model, function teaching should be carefully analyzed (variable materials). This is necessary for students to form general appearances. Otherwise, there will be unclear generalizations, which is often an important factor for students to master knowledge.

Pay attention to the teaching of concept and application ability

According to education psychology, concept is the "cell" that constitutes abstract logical thinking, which is the first essential element for abstract logical thinking, while mathematical concept is the reflection of human brain on the relationship between quantity and form in real things. Therefore, it is consistent to emphasize the concept and principle in mathematics teaching and to develop students' thinking ability.

We are in the era of attaching importance to the cultivation of innovation ability. At present, the application of knowledge in teaching materials to solve practical problems is heavier than in the past. The guiding idea for solving this problem should be to make sure that there are general and special relationships in mathematics. And this general knowledge is the most basic concept and principle in mathematics, and when they are learned, they can be used to draw inferences with simplicity. On this basis, the whole mathematical knowledge can be learned by solving some practical problems. In my opinion, instead of spending a lot of time doing mechanical exercises, it is better to put more efforts on mathematical concepts and applications, so that students' thinking ability and innovation ability can be developed normally, reflecting the dialectical relationship between imparting knowledge and developing intelligence.

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