

Effects of frequency parameter due to elastic wave in monoclinic elastic medium

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The propagation of elastic waves has many applications especially in geophysics and engineering. They are used in the detection of notches and faults in different types of materials such as in railway tracks, buried landmines, seismic activities, and exploration of valuable materials such as minerals, crystals, natural gas, etc. When the seismic signal passes through different discontinuities inside the earth, the phenomena of reflection and transmission takes place. In this paper, the phenomenon of reflection and transmission of elastic waves due to incident qSV-wave at a corrugated interface between two different monoclinic elastic half-spaces are discussed. There exist an irregularly reflected and transmitted waves due to undulated nature of the interface, besides the regularly reflected and transmitted ones. Using Rayleigh's method of approximation, the expressions for the reflection and transmission coefficients of the regular and irregular waves are obtained. It is observed that these coefficients are functions of angle of incidence and elastic constants and they are greatly influenced by the frequency and corrugation parameter. For a special type of interface, $x_3 = d \cos px_2$ we have computed these coefficients and discussed the effects of frequency parameter.

Keywords: qSV and qP-waves, monoclinic medium, reflection and transmission coefficients, corrugated interface, phase velocity.

INTRODUCTION

Elastic wave propagation is a very interesting topic that deals with numerous problems in various fields such as earthquake engineering, geophysics, signal processing, echography and metallurgy. They are useful in detection of notches and faults in different types of materials such as in railway tracks, buried land-mines, etc. The technique of seismic wave propagation is a tool for investigating the internal structure of the earth and also used for exploration of valuable materials such as minerals, crystals, fluids (oils, water) etc. beneath the earth surface. When the seismic signal passes through different discontinuities inside the earth, the phenomena of reflection and transmission takes place. These signals carry lots of information about the earth's structures and many such problems are in open literatures, i.e. Chattopadhyay and Choudhury (1995), Chattopadhyay *et al.* (1997), Dowaikh and Ogden (1991), Sheriff and Geldart

(1995), Singh (1999), Singh and Khurana (2002), Udias (1999) and others.

Singh and Khurana (2001) investigated the reflection and transmission of P and SV-waves at the interface between two monoclinic elastic half-spaces. Singh and Tomar (2007) investigated the problem of qP-waves at a corrugated interface between two dissimilar monoclinic elastic half-spaces and obtained the reflection and transmission coefficients of the irregular waves using Rayleigh's technique. Kaur and Tomar (2004) investigated the problem of reflection and transmission of shear wave incident upon a corrugated interface between two monoclinic elastic half-spaces with the help of Rayleigh's technique. But, there are many experimental evidences that the interfaces between layers are not perfectly plane, but they are of irregular in nature. These irregular natures of the interfaces affect in the reflection and transmission of elastic waves. Thus, it is important to take into account the problems related with the effects

of irregular interfaces.

It was Rayleigh (1907) who first discussed the problems on reflection and transmission phenomena of light/sound waves from an irregular boundary surface. In his method, the function defining the corrugated interface in the expressions of boundary conditions is expanded in Fourier series and the unknown coefficients are determined up to any given order of approximation in terms of small parameter characteristics of magnitude and slope of corrugated interface. Later, Rayleigh's technique was applied to various other fields to study the reflection and transmission phenomena of waves at irregular boundary surface. Several papers on scattering of elastic waves from boundary surfaces have been appeared based on Rayleigh's method and other techniques such as Abubakar (1962), Asano (1966), Dunkin and Eringen (1991), Gupta (1978), Kuo and Nafe (1962), Levy and Deresiewicz (1967), Rice (1951), Singh and Tomar (2007,2008) and Tomar and Kaur (2003).

BASIC EQUATIONS

The constitutive relation in a homogeneous anisotropic elastic material of monoclinic type with x_2x_3 -plane as the plane of symmetry are given by Singh and Khurana (2001)

$$\begin{aligned} \sigma_{11} &= c_{11}\epsilon_{11} + c_{12}\epsilon_{22} + c_{13}\epsilon_{33} + 2c_{14}\epsilon_{23}, & \sigma_{22} &= c_{12}\epsilon_{11} + c_{22}\epsilon_{22} + c_{23}\epsilon_{33} + 2c_{24}\epsilon_{23}, \\ \sigma_{33} &= c_{13}\epsilon_{11} + c_{23}\epsilon_{22} + c_{33}\epsilon_{33} + 2c_{34}\epsilon_{23}, & \sigma_{23} &= c_{14}\epsilon_{11} + c_{24}\epsilon_{22} + c_{34}\epsilon_{33} + 2c_{44}\epsilon_{23}, \\ \sigma_{12} &= c_{55}\epsilon_{12} + c_{56}\epsilon_{13} + c_{57}\epsilon_{23}, & \sigma_{13} &= c_{56}\epsilon_{12} + c_{66}\epsilon_{13} + c_{67}\epsilon_{23}, \\ \sigma_{21} &= c_{55}\epsilon_{12} + c_{56}\epsilon_{13} + c_{57}\epsilon_{23}, & \sigma_{31} &= c_{56}\epsilon_{12} + c_{66}\epsilon_{13} + c_{67}\epsilon_{23}, \\ \sigma_{32} &= c_{56}\epsilon_{12} + c_{66}\epsilon_{13} + c_{67}\epsilon_{23} \end{aligned} \quad (2.1)$$

where $\epsilon_i = (\epsilon_1, \epsilon_2, \epsilon_3)$ are components of displacement, σ_{ij} are stress tensors, c_{ij} ($i, j = 1, 2, 3, \dots, 6$) are elastic constants and ϵ_{ij} is the strain tensor given by

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

The equation of motion in such anisotropic materials without body forces are given by

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j}, \quad (i, j = 1, 2, 3)$$

(2.2)

where ρ is density of the medium.

The equations of motion in terms of displacements components may be written as

$$2c_{24}\frac{\partial^2 u_1}{\partial x_2^2} + c_{23}\frac{\partial^2 u_1}{\partial x_3^2} + c_{44}\frac{\partial^2 u_1}{\partial x_2 \partial x_3} = \rho \frac{\partial^2 u_1}{\partial t^2}, \quad (2.3)$$

$$2c_{34}\frac{\partial^2 u_2}{\partial x_2^2} + c_{23}\frac{\partial^2 u_2}{\partial x_3^2} + c_{44}\frac{\partial^2 u_2}{\partial x_2 \partial x_3} = \rho \frac{\partial^2 u_2}{\partial t^2}, \quad (2.4)$$

The solution of these equations may be taken in the form

$$u_i = \exp[ik(x_2 \cos \theta + x_3 \sin \theta - ct)] \quad (2.5)$$

where c is the phase velocity, k the wavenumber, $\theta = (0, \theta_2, \theta_3)$ is the unit propagation vector, $\mathbf{u} = (u_1, u_2, u_3)$ is the unit displacement vector.

Using these expressions of u_2 and u_3 into Eqs. (2.3) and (2.4), we have

$$-2c_{24}\cos^2 \theta + c_{23}\sin^2 \theta = 0, \quad c_{24}\cos^2 \theta + c_{23}\sin^2 \theta = 0, \quad (2.6)$$

where

$$\begin{aligned} c_{24} &= c_{22}\frac{\partial^2}{\partial x_2^2} + c_{44}\frac{\partial^2}{\partial x_3^2} + 2c_{24}\frac{\partial^2}{\partial x_2 \partial x_3}, & c_{23} &= c_{24}\frac{\partial^2}{\partial x_2^2} + c_{34}\frac{\partial^2}{\partial x_3^2} + 2c_{23}\frac{\partial^2}{\partial x_2 \partial x_3}, \\ c_{23} &= c_{44}\frac{\partial^2}{\partial x_2^2} + c_{33}\frac{\partial^2}{\partial x_3^2} + 2c_{34}\frac{\partial^2}{\partial x_2 \partial x_3}. \end{aligned} \quad (2.7)$$

Using Eq. (2.6), we get the phase velocity as

$$c_{2,1}^2 = \frac{c_{22} \pm \sqrt{c_{22}^2 - 4c_{24}^2}}{2\rho}, \quad (2.8)$$

where $+$ sign is for $-$ wave and $-$ sign is for $+$ wave.

Problem formulation

Consider the Cartesian coordinates with x_1 and x_2 -axis lying horizontal and x_3 -axis as vertical with positive direction pointing downward. Suppose two dissimilar homogeneous monoclinic half-spaces, M and M' , $M = \{x_2; x_3 : x_2 \in (-\infty, \zeta), x_3 \in \zeta, \infty\}$ and $M' = \{x_2; x_3 : x_2 \in (-\infty, \zeta), x_3 \in (-\infty, \zeta)\}$ are separated by $x_3 = \zeta(x_2)$, which is a periodic function of y independent of x whose mean value is zero. We will denote all elastic constants, stress tensors and displacement components in medium, M without prime and those of M' with primes. The geometry of the problem is shown by Figure 1. The Fourier series expansion of $\zeta(x_2)$ is given as

$$\zeta(x_2) = \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x_2}{2} + b_n \sin \frac{n\pi x_2}{2} \right), \quad (3.1)$$

where a_n and b_n are the coefficients of series expansion of order n , p is the wavenumber and $\zeta = \overline{-1}$.

Introduce constants α ; β ; and γ as

$$\alpha_{\pm 1} = \frac{\alpha}{2}, \quad \beta_{\pm 1} = \frac{\beta}{2}, \quad (\alpha = 2, 3, 4, \dots)$$

so that

$$\zeta(x_2) = d \cos \frac{\alpha x_2}{2} + \sin \frac{\beta x_2}{2}. \quad (3.2)$$

If the interface is represented by only one cosine term, i.e. $\zeta(x_2) = d \cos(p x_2)$, then the wavelength of corrugation is $2/p$ and d is the amplitude of corrugation.

We shall now discuss the reflection and transmission of elastic waves due to incident plane qSV-wave at the corrugated interface, $x_3 = \zeta(x_2)$. Suppose a plane qSV-wave propagating in the half-space, M with an angle θ_0 and amplitude constant A_0 be incident at the corrugated interface. This incident wave give rises regularly and irregularly reflected and transmitted qSV and qP-waves [2].

The full structures of reflected and transmitted waves are given by:

(for the half-space, M)

$$\infty_{=1} \{ \exp \left(\frac{2}{\omega} \left(\frac{0}{2} - \frac{3}{\omega} \cos \theta \right) \right) + \exp \left(\frac{2}{\omega} \left(\frac{0}{2} - \frac{3}{\omega} \cos \theta \right) \right) \}, \quad (3.3)$$

$$\infty_{=1} \{ \exp \left(\frac{2}{\omega} \left(\frac{0}{2} - \frac{3}{\omega} \cos \theta \right) \right) + \exp \left(\frac{2}{\omega} \left(\frac{0}{2} - \frac{3}{\omega} \cos \theta \right) \right) \}, \quad (3.4)$$

where (A, D) are amplitude constants of the regularly reflected qSV-wave at angle θ , \pm are amplitude constants of the irregularly reflected qSV-waves at angles \pm , (B, E) are amplitude constants of the regularly reflected qP-wave at angle θ , \pm are amplitude constants of the irregularly reflected qP-waves at angles \pm and the expressions of $\frac{0}{\omega}$, $\frac{3}{\omega}$, $\frac{2}{\omega}$, $\frac{1}{\omega}$ are given by

$$\frac{0}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}, \quad \frac{3}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}, \quad \frac{2}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}, \quad \frac{1}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2} \quad \text{and} \quad \frac{1}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}.$$

These amplitude constants satisfy the following relations [21]

$$\frac{0}{\omega} = \frac{0}{\omega}, \quad \frac{3}{\omega} = \frac{3}{\omega}, \quad \frac{2}{\omega} = \frac{2}{\omega}, \quad \frac{1}{\omega} = \frac{1}{\omega} \quad (3.5)$$

where

$$\frac{0}{\omega} = \frac{0}{\omega}, \quad \frac{3}{\omega} = \frac{3}{\omega}, \quad \frac{2}{\omega} = \frac{2}{\omega}, \quad \frac{1}{\omega} = \frac{1}{\omega} \quad (3.6)$$

(for the half-space, M')

$$\infty_{=1} \{ \exp \left(\frac{2}{\omega} \left(\frac{0}{2} - \frac{3}{\omega} \cos \theta \right) \right) + \exp \left(\frac{2}{\omega} \left(\frac{0}{2} - \frac{3}{\omega} \cos \theta \right) \right) \}, \quad (3.6)$$

$$\infty_{=1} \{ \exp \left(\frac{2}{\omega} \left(\frac{0}{2} - \frac{3}{\omega} \cos \theta \right) \right) + \exp \left(\frac{2}{\omega} \left(\frac{0}{2} - \frac{3}{\omega} \cos \theta \right) \right) \}, \quad (3.7)$$

where (G, I) are amplitude constants of the regularly transmitted qSV-wave at angle θ , \pm are amplitude constants of the irregularly transmitted qSV-waves at angles \pm , (H, J) are amplitude constants of the regularly transmitted qP-wave at angle θ , \pm are amplitude constants of the irregularly transmitted qP-waves at angles \pm and the expressions of $\frac{0}{\omega}$, $\frac{3}{\omega}$, $\frac{2}{\omega}$, $\frac{1}{\omega}$ are given by

$$\frac{0}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}, \quad \frac{3}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}, \quad \frac{2}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}, \quad \frac{1}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2} \quad \text{and} \quad \frac{1}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}.$$

These amplitude constants also satisfy the following relations

$$\frac{0}{\omega} = \frac{0}{\omega}, \quad \frac{3}{\omega} = \frac{3}{\omega}, \quad \frac{2}{\omega} = \frac{2}{\omega}, \quad \frac{1}{\omega} = \frac{1}{\omega} \quad (3.8)$$

where

$$\frac{0}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}, \quad \frac{3}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}, \quad \frac{2}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}, \quad \frac{1}{\omega} = \frac{2 \sin \theta + 3 \cos \theta}{2}.$$

The Snell's law which gives the relation of angle of incident, reflected as well as transmitted waves is given by Asano (1966)

$$\frac{0}{\omega} = \frac{\sin \theta}{2} = \frac{\sin \theta}{1} = \frac{\sin \theta}{2} = \frac{\sin \theta}{1} = \frac{1}{2}, \quad (3.9)$$

where $\frac{0}{\omega}$ is apparent velocity.

Moreover, Spectrum theorem gives the relations between the angle of regular wave and those of irregular waves Abubakar (1962)

$$\sin \theta = \frac{1}{2}, \quad \sin \theta = \frac{1}{2}, \quad \sin \theta = \frac{1}{2}, \quad \sin \theta = \frac{1}{2} \quad (3.10)$$

Boundary condition

The component of displacements and tractions (normal and shear) are continuous at the corrugated interface. Mathematically, these conditions at $\theta = \theta$ may be written as

$$\frac{0}{\omega} = \frac{0}{\omega}, \quad \frac{3}{\omega} = \frac{3}{\omega}, \quad \frac{2}{\omega} = \frac{2}{\omega}, \quad \frac{1}{\omega} = \frac{1}{\omega} \quad (4.1)$$

$$\frac{0}{\omega} = \frac{0}{\omega}, \quad \frac{3}{\omega} = \frac{3}{\omega}, \quad \frac{2}{\omega} = \frac{2}{\omega}, \quad \frac{1}{\omega} = \frac{1}{\omega} \quad (4.2)$$

$$\frac{0}{\omega} = \frac{0}{\omega}, \quad \frac{3}{\omega} = \frac{3}{\omega}, \quad \frac{2}{\omega} = \frac{2}{\omega}, \quad \frac{1}{\omega} = \frac{1}{\omega} \quad (4.3)$$

where $\frac{0}{\omega}$ is the derivative of $\frac{0}{\omega}$ with respect to θ .

Inserting Eq. (2.1) into (4.2) and (4.3), we get

$$\{ (\frac{0}{\omega} - \frac{3}{\omega})' + 24(1 - \frac{2}{\omega}) \} \frac{2}{\omega} + \{ (\frac{3}{\omega} - \frac{2}{\omega})' + 34(1 - \frac{2}{\omega}) \} \frac{3}{\omega} + \{ \frac{3}{\omega} - \frac{2}{\omega} \} \frac{2}{\omega} = \{ (\frac{0}{\omega} - \frac{3}{\omega})' + 24(1 - \frac{2}{\omega}) \} \frac{2}{\omega} + \{ (\frac{3}{\omega} - \frac{2}{\omega})' + 34(1 - \frac{2}{\omega}) \} \frac{3}{\omega} + \{ \frac{3}{\omega} - \frac{2}{\omega} \} \frac{2}{\omega} \quad (4.4)$$

$$\{ \frac{0}{\omega} + \frac{2}{\omega} - 2 \frac{3}{\omega} \} \frac{2}{\omega} + \{ \frac{3}{\omega} + \frac{2}{\omega} - 2 \frac{3}{\omega} \} \frac{3}{\omega} + \{ \frac{3}{\omega} - \frac{2}{\omega} \} \frac{2}{\omega} = \{ \frac{0}{\omega} + \frac{2}{\omega} - 2 \frac{3}{\omega} \} \frac{2}{\omega} + \{ \frac{3}{\omega} + \frac{2}{\omega} - 2 \frac{3}{\omega} \} \frac{3}{\omega} + \{ \frac{3}{\omega} - \frac{2}{\omega} \} \frac{2}{\omega}$$

$$\left. \begin{matrix} ' \\ 24 \end{matrix} \right\}^2 - 2 \left. \begin{matrix} ' \\ 44 \end{matrix} \right\} \left(-\frac{2}{3} + \frac{3}{2} \right).$$

(4.5)

Using Eqs.(3.3), (3.4)), (3.6), (3.7), (3.9) and (3.10) into (4.1), (4.4) and (4.5), we get

[illegible]

$$\begin{aligned}
& 23 + 22 \quad '2 - 2 \quad 24 \quad ' [\quad 0 \quad 0 \quad i\zeta \quad 0 + \quad 0 \quad -i\zeta \quad + \\
& 0 \quad -i\zeta \quad + \quad \infty_{=1} \{ \quad 0 \pm \quad \pm \mp m \quad 2-i\zeta \quad \pm + \quad 0 \pm \\
& \pm \mp m \quad 2-i\zeta \quad \pm \} \} + \quad 33 + 23 \quad '2 - \\
& 2 \quad 34 \quad ' - \quad 0 \quad 0 \quad i\zeta \quad 0 + \quad -i\zeta \quad + \quad -i\zeta \quad + \\
& \infty_{=1} \{ \quad \pm \pm \mp m \quad 2-i\zeta \quad \pm + \quad \pm \pm \mp m \quad 2-i\zeta \quad \pm \} \} + \\
& [\quad 34 + \quad 24 \quad '2 - 2 \quad 44 \quad '] [- \quad 0 \quad 0 \quad i\zeta \quad 0 + \quad -i\zeta \quad + \\
& -i\zeta \quad + \quad 0 \quad 0 \quad i\zeta \quad 0 + \quad 0 \quad -i\zeta \quad + \quad 0 \quad -i\zeta \quad + \\
& \infty_{=1} \quad \pm \pm \mp m \quad 2-i\zeta \quad \pm + \quad \pm \pm \mp m \quad 2-i\zeta \quad \pm + \quad 0 \pm \\
& \pm \mp m \quad 2-i\zeta \quad \pm + \quad 0 \pm \quad \pm \mp m \quad 2-i\zeta \quad \pm = \\
& '23 + '22 \quad '2 - 2 \quad '24 \quad ' [\quad 0 \quad i\zeta \quad + \quad 0 \quad i\zeta \quad + \\
& \infty_{=1} \{ \quad 0 \pm \quad \pm \mp m \quad 2+i\zeta \quad \pm + \quad 0 \pm \\
& \pm \mp m \quad 2+i\zeta \quad \pm \} \} - \quad '33 + '23 \quad '2 - \\
& 2 \quad '34 \quad ' \quad i\zeta \quad + \quad i\zeta \quad + \quad \infty_{=1} \{ \quad \pm \pm \mp m \quad 2+i\zeta \quad \pm +
\end{aligned}$$

[illegible]

where

$$\begin{array}{rcl} 0 & = & \frac{\sin 0}{0}, \quad 0 = \frac{\cos 0}{0}, \quad = \\ \frac{\cos}{2}, \quad \pm & = & \frac{\cos}{2}, \quad = \frac{\cos}{1}, \\ \pm & = & \frac{\cos}{1}, \quad = \frac{\cos}{2}, \quad \pm = \frac{co}{2}, \\ = \frac{\cos}{1}, \quad \pm & = & \frac{co}{1} \end{array}$$

Solution of first order approximation

Assuming the amplitude of the corrugated interface is very small so that neglecting higher powers of ζ , we have

$$\zeta^0 = 1 \pm i\zeta^1, \quad \zeta^{\pm 1} = 1 \pm i\zeta^2, \text{ etc.}$$

(5.1)

Using Eqs. (3.5), (3.8) and (5.1) into (4.6)-(4.9) and collecting terms independent of ϵ and ϵ^2 , we obtain a set of equations

$$= \quad , \quad (5.2)$$

where

$$= \begin{array}{cccc} 1 & 1 & -1 & -1 \\ & 10 & -20 & -30 \\ 1 & 2 & -3 & -4 \\ & 1 & 2 & -3 \\ -1 & & & \end{array}, \quad = \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array},$$

$$\begin{aligned}
0 &= \begin{array}{c} 0 \quad 24 + \quad 44 \quad 0 - (\quad 0 \quad 44 + \quad 34) \quad 0, \\ 44 \quad 0 + (\quad 44 + \quad 34) \quad , \end{array} & 1 &= \begin{array}{c} 24 + \\ ' \end{array} \\
2 &= \begin{array}{c} 10 \quad 24 + \quad 44 \quad 0 + (\quad 10 \quad 44 + \quad 34) \quad , \\ 44 \quad 0 - (\quad 20 \quad 44 + \quad 34) \quad , \end{array} & 3 &= \begin{array}{c} 20 \quad 24 + \\ ' \end{array} \\
4 &= \begin{array}{c} 30 \quad 24 + \quad 44 \quad 0 - (\quad 30 \quad 44 + \quad 34) \quad , \\ 34 \quad 0 - (\quad 0 \quad 34 + \quad 33) \quad 0, \end{array} & 0 &= \begin{array}{c} 0 \quad 23 + \\ ' \end{array} \\
1 &= \begin{array}{c} 23 + \quad 34 \quad 0 + (\quad 34 + \quad 33) \quad , \\ 2 &= \begin{array}{c} 10 \quad 23 + \quad 34 \quad 0 + (\quad 10 \quad 34 + \quad 33) \\ , 3 &= \begin{array}{c} 20 \quad 23 + \quad 34 \quad 0 - (\quad 20 \quad 34 + \quad 33) \quad , \\ 34 \quad 0 - (\quad 30 \quad 34 + \quad 33) \quad . \end{array} \end{array} & 4 &= \begin{array}{c} 30 \quad 23 + \\ ' \end{array}
\end{aligned}$$

On solving Eq. (5.2), we get the ratios of the amplitude constants corresponding to the vertical components of the displacement as

$$\frac{\quad}{0} = \frac{\quad}{\quad}, \quad \frac{\quad}{0} = \frac{\quad}{\quad}, \quad \frac{\quad}{0} = \frac{\quad}{\quad}, \quad \frac{\quad}{0} = \frac{\quad}{\quad} \quad (5.3)$$

where the values of α , β , γ and δ are obtained by replacing first, second, third and fourth column of \mathbf{A} with

where the values of α_1 , α_2 , α_3 and α_4 are obtained by replacing first, second, third and fourth column of \mathbf{A} with column matrix \mathbf{T} respectively. The ratios of the amplitude constants corresponding to horizontal components of displacement is obtained with the help of Eqs. (3.5), (3.8) and (5.3) as

$$\frac{30}{0}, \quad \frac{-}{0} = \frac{-}{0}, \quad \frac{-}{0} = \frac{10}{0}, \quad \frac{-}{0} = \frac{20}{0}, \quad \frac{-}{0} = \quad (5.4)$$

The reflection and transmission coefficients of reflected and transmitted qSV and qP-waves for the incident qSV-wave are given by

$$= \frac{\overline{1 + \frac{2}{10}}}{1 + \frac{2}{10}} = \frac{\overline{1 + \frac{2}{10}}}{1 + \frac{2}{10}} \quad (5.5)$$

$$= \frac{\overline{1 + \frac{2}{20}}}{1 + \frac{2}{0}}, \quad = \frac{\overline{1 + \frac{2}{30}}}{1 + \frac{2}{0}}, \quad (5.6)$$

where $\frac{1}{1 + \frac{2}{0}}$ is the amplitude of incident qSV-wave. We come to know that these coefficients depend on elastic constants and angle of incidence.

Next, comparing coefficients of τ^2 on the both the sides of those equations, we get

$$\overline{+} \quad \overline{+} \quad \underline{\quad} \quad \overline{+}$$

(5.7)

where

$$\mp = \begin{array}{cccc} 1 & 1 & -1 & -1 \\ \pm & \pm & \pm & \pm \\ 1 & 2 & 3 & \\ \mp & \mp & \mp & \mp \\ 5 & 6 & 7 & 8 \\ \mp & \mp & \mp & \mp \\ 5 & 6 & 7 & 8 \end{array}, \quad \mp =$$

[illegible]

$$\begin{aligned}
& \frac{23}{44} - \frac{22}{30} \pm \frac{0}{33} - \frac{24}{23} \pm \frac{34}{34} - \frac{24}{34} - \\
& \frac{24}{0} + \frac{44}{0} \mp \frac{24}{5} = - \frac{44}{44} \pm + \\
& \frac{24}{0} \pm \pm + \frac{34}{34} \pm + \frac{44}{0} \pm , \quad \frac{24}{6} = \\
& - \frac{44}{44} \pm + \frac{24}{0} + \frac{1}{1} + \frac{34}{34} \pm + \frac{44}{0} \pm , \\
& \frac{24}{7} = \frac{44}{44} \pm - \frac{24}{24} \pm \frac{0}{2} + \frac{34}{34} \pm - \frac{44}{44} \pm \\
& , \quad \frac{24}{8} = \frac{44}{44} \pm - \frac{24}{24} \pm \frac{0}{3} + \frac{34}{34} \pm - \\
& \frac{44}{44} \pm , \quad \frac{0}{0} = \frac{23}{23} \pm \frac{0}{0} \pm \frac{2}{24} \pm \frac{0}{34} \pm \frac{0}{0} \pm \\
& \frac{2}{44} \pm \frac{0}{0} - \frac{33}{33} \pm \frac{2}{24} \pm \frac{34}{0} \pm \frac{0}{34} \pm \frac{0}{0} \pm \\
& \frac{2}{44} \pm \frac{0}{0} , \\
& \frac{24}{1} = - \frac{23}{23} \pm \frac{0}{0} \pm \frac{2}{24} \pm \frac{0}{34} \pm \frac{2}{24} \pm \frac{2}{44} - \\
& \frac{33}{33} \pm \frac{2}{24} \pm \frac{34}{0} \pm \frac{2}{44} \pm \frac{0}{0} \mp , \quad \frac{24}{2} = \\
& - \frac{23}{23} \pm \frac{0}{0} \pm \frac{2}{24} \pm \frac{0}{34} \pm \frac{2}{44} \pm \frac{2}{44} \pm \frac{10}{10} \mp \\
& \frac{33}{33} \pm \frac{2}{24} \pm \frac{34}{0} \pm \frac{2}{44} \pm \frac{0}{0} \mp , \quad \frac{24}{3} = \\
& , \frac{23}{23} \pm \frac{0}{0} \pm \frac{2}{24} \pm \frac{0}{34} \pm \frac{2}{44} \pm \frac{2}{44} \pm \frac{20}{20} - \\
& \frac{33}{33} \pm \frac{2}{24} \pm \frac{2}{34} \pm \frac{0}{0} \pm \frac{2}{44} \pm \frac{0}{0} \mp , \quad \frac{24}{4} = \\
& , \frac{23}{23} \pm \frac{0}{0} \pm \frac{2}{24} \pm \frac{0}{34} \pm \frac{2}{44} \pm \frac{2}{44} \pm \frac{30}{30} - \\
& \frac{33}{33} \pm \frac{2}{24} \pm \frac{2}{34} \pm \frac{0}{0} \pm \frac{2}{44} \pm \frac{0}{0} \mp , \quad \frac{24}{5} = \\
& - \frac{23}{23} \pm \frac{0}{0} \pm \pm + \frac{34}{34} \pm \pm + \frac{33}{33} \pm \pm + \frac{34}{34} \pm \pm \\
& , \quad \frac{24}{6} = - \frac{23}{23} \pm \pm + \frac{34}{34} \pm \pm + \frac{33}{33} \pm \pm + \\
& \frac{34}{34} \pm , \\
& \frac{24}{7} = - \frac{23}{23} \pm \pm + \frac{34}{34} \pm \pm + \frac{33}{33} \pm - \\
& \frac{34}{34} \pm , \\
& \frac{24}{8} = - \frac{23}{23} \pm \pm + \frac{34}{34} \pm \pm + \frac{33}{33} \pm - \\
& \frac{34}{34} \pm .
\end{aligned}$$

Solving Eq. (5.7), we get the ratios of the amplitude constants of irregular waves corresponding to the vertical components of the displacement.

$$\frac{\pm}{0} = \frac{\pm}{\pm}, \quad \frac{\pm}{0} = \frac{\pm}{\pm}, \quad \frac{\pm}{0} = \frac{\pm}{\pm}, \quad \frac{\pm}{0} = \frac{\pm}{\pm}, \quad (5.8)$$

where the values of \pm , \pm , \pm and \pm are obtained by replacing first, second, third and fourth column of \pm with column matrix T^\mp respectively. The ratios of the amplitude constants of irregular waves corresponding to horizontal components of displacement are obtained with the help of Eqs. (3.5), (3.8) and (5.8) as

$$\begin{aligned} \frac{\pm}{0} &= \frac{\pm}{0} \frac{\pm}{\pm}, & \frac{\pm}{0} &= \frac{\pm}{0} \frac{\pm}{\pm}, & \frac{\pm}{0} &= \frac{\pm}{0} \frac{\pm}{\pm}, & \frac{\pm}{0} &= \\ \frac{\pm}{0} \frac{\pm}{\pm}. & & & & & & & \end{aligned} \quad (5.9)$$

The reflection and transmission coefficients for irregularly reflected and transmitted qSV and qP-waves are

$$\begin{aligned}
\pm &= \frac{\overline{1+\frac{\pm^2}{2}}}{1+\frac{\pm^2}{2}} \frac{\pm}{\pm}, & \pm &= \frac{\overline{1+\frac{\pm^2}{1}}}{1+\frac{\pm^2}{1}} \frac{\pm}{\pm}, \\
\pm &= \frac{\overline{1+\frac{\pm^2}{2}}}{1+\frac{\pm^2}{2}} \frac{\pm}{\pm}, & \pm &= \frac{\overline{1+\frac{\pm^2}{3}}}{1+\frac{\pm^2}{3}} \frac{\pm}{\pm}.
\end{aligned}
\tag{5.10}$$

We come to know from Equation (5.10) that the coefficients corresponding to the irregularly reflected and

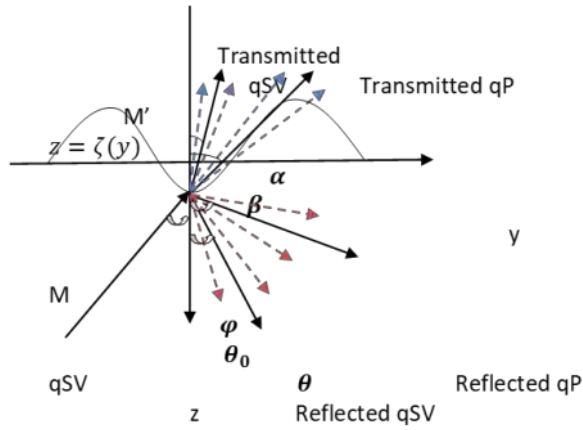


Figure 1: Geometry of the problem.

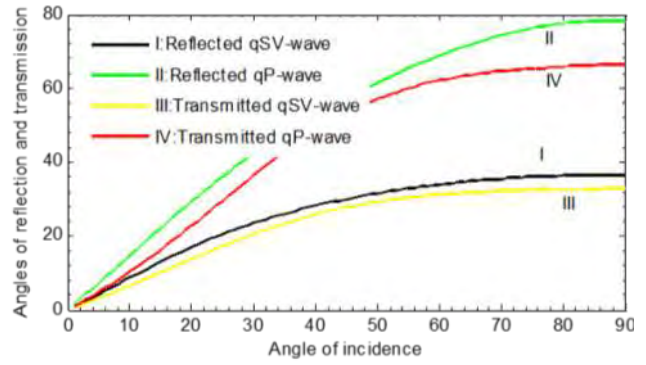


Figure 2: Variation of the angle of reflection and transmission coefficients with transmission with the angle of incidence.

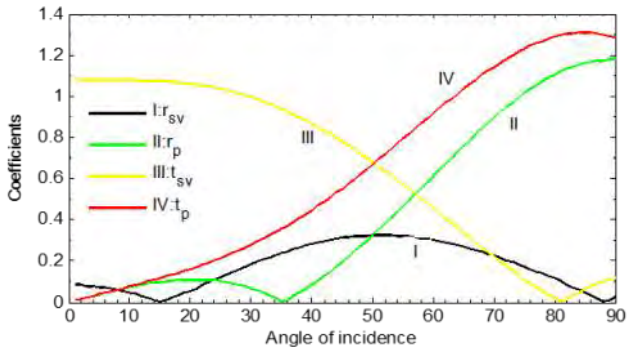


Figure 3: Variation of reflection and angle of incidence (θ_0).

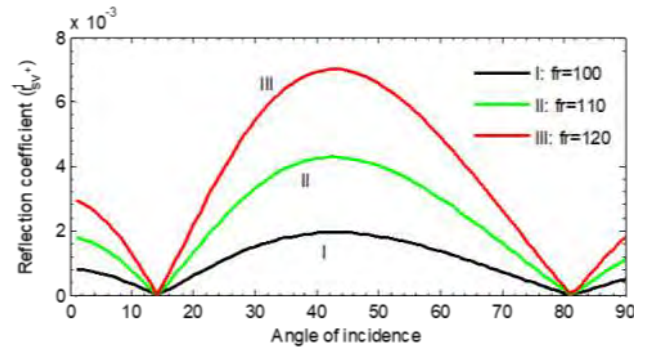


Figure 4: Variation of r_{sv}^{1+} with θ_0 for different values of ω/pc_0 .

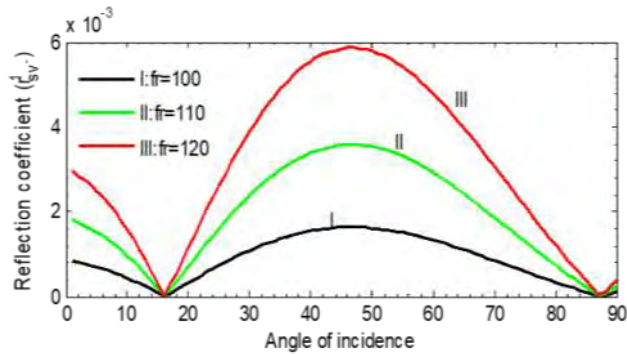


Figure 5: Variation of r_{sv}^{1-} with θ_0 for different values of ω/pc_0 .

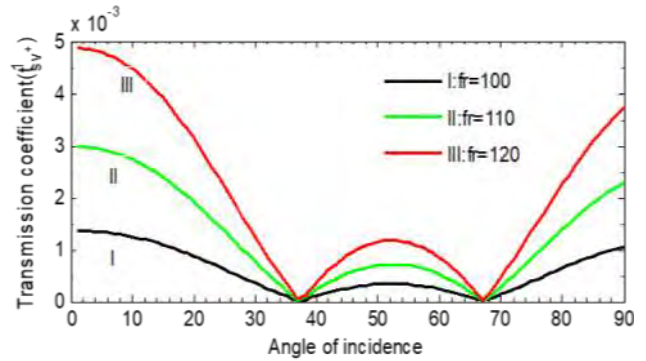


Figure 6: Variation of t_{sv}^{1+} with θ_0 for different values of ω/pc_0 .

We come to know from Equation (5.10) that the coefficients corresponding to the irregularly reflected and transmitted qSV and qP-waves are functions of the elastic constants, angle of incidence, corrugation and frequency parameters.

Special case:

When the interface is represented by $z = d \cos \frac{2\pi x}{\lambda}$ with d as the amplitude of corrugation. In this case

$$- = \frac{0}{2} \neq 1 = 1. \quad (6.1)$$

Thus, using these values, the reflection and transmission coefficients for the first order approximation of the corrugation are given by

$$\begin{aligned} 1_{\pm} &= \frac{1 + \frac{\pm^2}{1}}{1 + \frac{\pm^2}{0}} - \frac{\frac{\pm}{1}}{\pm}, \\ \frac{1 + \frac{\pm^2}{1}}{1 + \frac{\pm^2}{0}} - \frac{\frac{\pm}{1}}{\pm}, \\ 1_{\pm} &= \frac{1 + \frac{\pm^2}{21}}{1 + \frac{\pm^2}{0}} - \frac{\frac{\pm}{21}}{\pm}, \\ \frac{1 + \frac{\pm^2}{31}}{1 + \frac{\pm^2}{0}} - \frac{\frac{\pm}{31}}{\pm}, \end{aligned} \quad (6.2)$$

where values of $\frac{\pm}{1}$, $\frac{\pm}{11}$, $\frac{\pm}{21}$, $\frac{\pm}{31}$, $\frac{\pm}{}$, $\frac{\pm}{}$, $\frac{\pm}{}$ and $\frac{\pm}{}$ are obtained from Eq.(5.10) by using Eq.(6.1). We will compute these coefficients for a particular model.

Results and Discussion

We will compute the angles of reflected and transmitted waves through Snell's law given by Eq. (3.9) in which the apparent velocity v_a is related with dimensionless velocity by $v_a = v/c$. Let us find out the angles of reflected qSV and qP-waves in the half-space, M. Eq. (2.8) may be written as

$$4 - \quad + \quad ^2 + \quad - \quad ^2 = 0 \quad (8.1)$$

where

$$\begin{aligned} &= \frac{2}{2 \ 44}, & &= \frac{2}{2 \ \underline{44}}, & &= \frac{2}{2 \ 44}, \\ & & &= \frac{44}{44}, & &= \frac{\quad}{44} \end{aligned}$$

There are two roots of β^2 corresponding to qSV and qP-waves for a given $\beta = \frac{3}{2}$, and for a given value of β , there are two roots of p corresponding to the angles of reflected qSV and qP-waves [24]. Substituting the values of β , β^2 , and p into Eq.(8.1), we get

$$x_0^4 + x_1^3 + x_2^2 + x_3 + x_4 = 0$$

where

$$\begin{aligned}
0 &= {}_{33} - \frac{2}{34}, \quad 1 = 2({}_{24} {}_{33} - {}_{23} {}_{34}), \quad 2 \\
&\quad - 1 + {}_{33}^2, \quad 1 + {}_{22} {}_{33} + 2 {}_{24} {}_{34} - (1 + {}_{23})^2 \\
3 &= 2 {}_{22} {}_{34} - {}_{23} {}_{24} - {}_{24} + {}_{34}^2, \quad 4 \\
&\quad = {}_4 - 1 + {}_{22}^2 + {}_{22} {}_{24} \\
\text{We transform this equation with } &= \frac{1}{3} = \frac{2}{3} \text{ so that} \\
&{}_4^4 + {}_3^3 + {}_2^2 + {}_1 + {}_0 = 0
\end{aligned}
\tag{8.3}$$

We transform this equation with $\frac{1}{3} = \frac{2}{3}$ so that

This equation has two positive roots, i.e., the smaller positive root (α_1) and the larger positive root (α_2) which represent the directions of reflected qSV and qP-waves respectively. Thus, $\alpha_1 = \alpha_1^{-1}$ & $\alpha_2 = \alpha_2^{-1}$. Similarly, in the half-space M', the angles of transmitted qSV and qP-waves are obtained as $\alpha_1' = \alpha_1^{-1}(\alpha_1')$ & $\alpha_2' = \alpha_2^{-1}(\alpha_2')$.

For the numerical computation, the following relevant values of elastic constants are taken Singh and Tomar (2007)

(for half-space, M-Lithium tantalate)

$$\begin{aligned} 24 &= 0.11 \cdot 10^{11} \quad -2, & 23 &= 0.80 \cdot 10^{11} \quad -2, & 34 &= 0, & 44 \\ & & &= 0.94 \cdot 10^{11} \quad -2, & & & \\ 33 &= 2.75 \cdot 10^{11} \quad -2, & 22 &= 2.33 \cdot 10^{11} \quad -2, & & & \\ & & &= 7400 \quad -3. \end{aligned}$$

(for half-space, M'-Lithium neobate)\\

$\gamma'_{24} = -0.09 \cdot 10^{11} \text{ }^{-2}$, $\gamma'_{23} = 0.75 \cdot 10^{11} \text{ }^{-2}$, γ'_{34}
 $= 0$, $\gamma'_{44} = 1.06 \cdot 10^{11} \text{ }^{-2}$, $\gamma'_{22} = 2.03 \cdot 10^{11} \text{ }^{-2}$, $\gamma'_{33} = 2.45 \cdot 10^{11} \text{ }^{-2}$, and frequency parameter, $(\text{fr} = 4700 \text{ }^{-3}) = 100$

4700 $\times 10^{-3}$, and frequency parameter, $(fr = \omega_0) = 100$ and corrugation parameter ($cor = pd$) = 0.0001 are taken unless and otherwise mentioned. Figure 2 represents the variation of angles of reflected and transmitted qSV and qP-waves with angle of incidence. Here all angles, $(\theta_r, \theta_t, \theta_{tr})$ of the regularly reflected and transmitted waves increase with the increase of angle of incidence (θ_0) . It is observed that the angles of reflection and transmission for qSV-waves are less than that of the qP-waves.

Curve I in Figure 3 shows that η starts from certain value which decreases to zero at $\theta_0 = 15^\circ$ and then increases upto $\theta_0 = 51^\circ$ with the increase of θ_0 . Thereafter, it decreases, touching zero value at $\theta_0 = 87^\circ$. In the same figure, Curve II shows that η is parabolic in the region $0 \leq \theta_0 \leq 35^\circ$ and then, it increases with the increase of θ_0 . Curve III shows the decreasing nature of η upto $\theta_0 = 81^\circ$ and then increases with the increase of θ_0 , while Curve IV shows that η increases with the increase of θ_0 .

Effect of frequency parameters

We are interested to see the effect of frequency parameters on the reflection and transmission coefficients. In Figure 4, the reflection coefficient, r_+ corresponding to the irregularly reflected qSV-wave starts from certain value and decreases to zero at $\theta_0 = 14^\circ$ making a parabolic region in $14^\circ \leq \theta_0 \leq 81^\circ$, which then increases with the increase of θ_0 . It is observed that r_+ increases with the increase of frequency (fr) parameters.

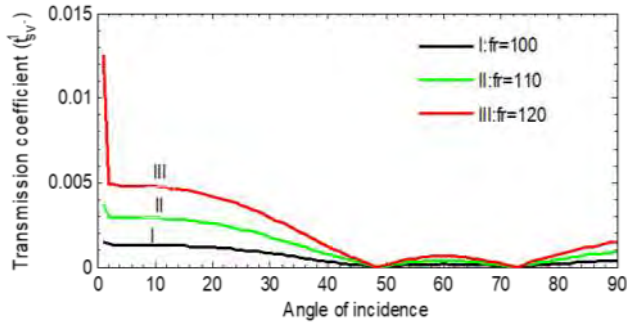


Figure 7: Variation of t_{sv}^{1-} with θ_0 for different values of ω/pc_0 .

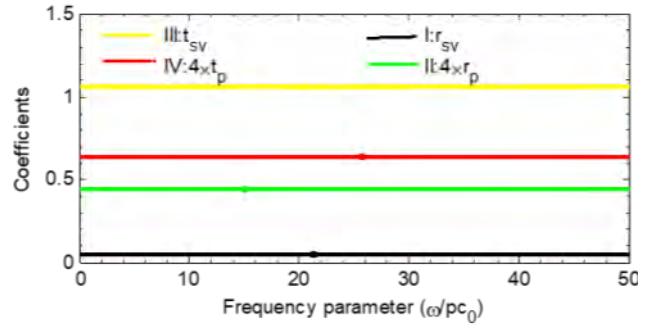


Figure 8: Variation of reflection and transmission coefficients of the regular qSV & qP-waves with ω/pc_0 .

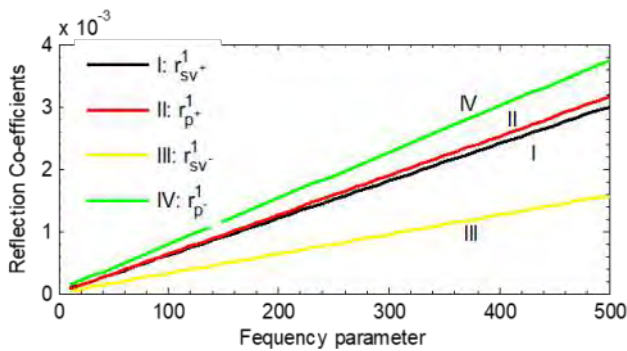


Figure 9: Variation of reflection coefficients of the irregularly reflected qSV & qP-waves with ω/pc_0 .

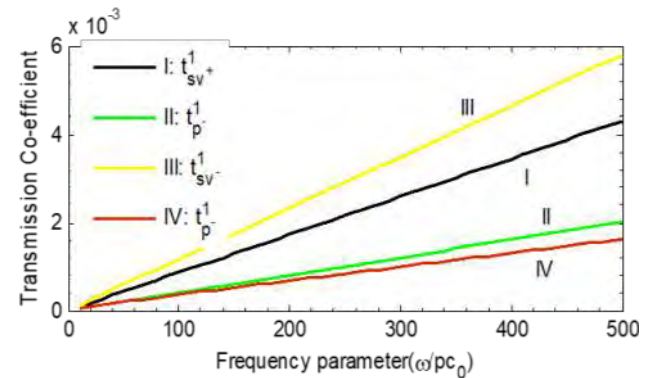


Figure 10: Variation of transmission coefficients of the irregularly transmitted qSV & qP-waves with ω/pc_0 .

We are interested to see the effect of frequency parameters on the reflection and transmission coefficients. In Figure 4, the reflection coefficient, r_{sv}^{1+} corresponding to the irregularly reflected qSV-wave starts from certain value and decreases to zero at $\theta_0 = 14^\circ$ making a parabolic region in $14^\circ \leq \theta_0 \leq 81^\circ$, which then increases with the increase of θ_0 . It is observed that r_{sv}^{1+} increases with the increase of frequency (fr) parameters. We have observed similar nature of r_{sv}^{1-} in Figure 5.

The coefficient, r_{p}^{1+} in Figure 6 decreases initially making a parabolic region in $37^\circ \leq \theta_0 \leq 67^\circ$ and then increases with the increase of θ_0 . It is observed that the values of r_{p}^{1+} increase with the increase of frequency parameter. Similar natures of r_{sv}^{1-} are observed in Figure 7.

We have seen from Figure 8 that the coefficients corresponding to the regularly reflected and transmitted waves are independent of frequency parameters. In Figures 9 and 10, the coefficients corresponding to the irregularly reflected and transmitted waves are linearly proportional to the frequency parameters, but at different rates.

CONCLUSION

The problem of incident qSV-wave at a corrugated interface between two dissimilar monoclinic elastic half-spaces has been investigated. We have obtained the reflection and transmission coefficients for the first order of approximation corresponding to regularly and irregularly reflected and transmitted qSV and qP-waves with the help of Rayleigh's method of approximation. These coefficients are computed numerically for a specific model and the effect of frequency parameters on these coefficients are discussed. We may conclude with the following remarks:

(i) All coefficients corresponding to regular waves are functions of angle of incidence and elastic constants, while those of irregular waves are found to be functions of angle of incidence, elastic constants and frequency parameters.

(ii) Theoretically and numerically, the reflection and transmission coefficients of the regular waves are independent of frequency parameters.

(iii) The coefficients corresponding to irregular waves

are found to be linearly proportional to frequency parameters.

(iv) It is found that the values of coefficients corresponding to irregular waves increase with increase of frequency parameter.

(v) The values of coefficients corresponding to irregular waves are found to be small.

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