Simulation-Based Estimation of Microwave Radiation Attenuation in the Cloud Environment

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Abstract — The paper presents a mathematical model describing formation of the spectra of particles in a convective cloud. A method for calculating absorption and scattering of monochrome microwave radiation in the cloud environment is described. The cloud model describes drop coagulation, accretion, drop freezing, drop and crystal sedimentation and transportation by air flows. Explicit microphysical methods based on solving kinetic equations for functions of distribution of drops and ice particles by their masses is applied. To study microwave radiation extinction in the cloud environment, absorption and scattering coefficients are calculated for given wavelengths and sizes of drop and ice particle. Functions of distribution of drops and crystals by their sizes help determine absorption and scattering of electromagnetic radiation in the cloudy environment with regard to actual spectra at different stages of cloud formation.

Keywords — cloud model, microstructural parameters, absorption and scattering, microwave radiation, extinction assessment, cloud particles.

I. INTRODUCTION

As is known, radio engineering and electronic optical systems depend on meteorological parameters of the atmosphere, haze, fogs and clouds. The study on conditions of distribution of electromagnetic waves of the IR cloud environment is a crucial task. In a number of theoretical studies, the issues of refraction of electromagnetic waves were solved for various aerosols and cloud particles. A variety of types and properties of dispersed media, relatively short lifetime and strong spatial-temporal heterogeneity make it difficult to obtain reliable quantitative estimates.

Physical properties of dispersed environments (clouds) depend on the size of particles, their concentration, and phase state. These issues are dealt with in many researches [1-4]. The influence of dispersed environments on electromagnetic radiation is discussed in [5-14]. In order to study various applied issues of electromagnetic radiation in cloud environments, data on the microstructure of probed cloudiness is required. To obtain such data, measurement data (probe, aircraft, etc.) and simulation results for detailed microphysical cloud models considering functions of distribution of liquid and solid particles by size or mass are used [1, 2, 4].

The present article aims to develop a mathematical model describing formation of spectra of cloud particles, and to obtain data on absorption and scattering properties of real aerosol formations during microwave radiation propagation.

In nature, the shape and type of clouds depend on thermodynamic atmospheric conditions which depend on climatic features of a particular region. The article analyzes cumulonimbus clouds which can reach the middle and upper tiers. Powerful clouds can reach the tropopause delivering a large number of small ice particles to the upper layers of the troposphere.

II. METHODS AND MATERIALS

A. Model description

The purpose statement for the mathematical model of a convective cloud involves the following equations of thermodynamics, microphysics and electrical statistics [1]:

\[
\frac{\partial u}{\partial t} + (\vec{V} \cdot \nabla) u = -\nabla \pi^\prime + \Delta^\prime u + \nu \nabla^2 u, \tag{1}
\]

\[
\frac{\partial \nu}{\partial t} + (\vec{V} \cdot \nabla) \nu = -\nabla \pi^\prime + \Delta^\prime \nu - \nu \nabla^2 \nu, \tag{2}
\]

\[
\frac{\partial w}{\partial t} + (\vec{V} \cdot \nabla) w = -\nabla \pi^\prime + \Delta^\prime w + g(\frac{\theta'}{\theta_0} + 0.61x' - Q_s), \tag{3}
\]

equation of continuity

\[
\frac{\partial u}{\partial x} + \frac{\partial \nu}{\partial y} + \frac{\partial w}{\partial z} = \alpha \nu, \tag{4}
\]
equation of thermodynamics

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\[ \frac{\partial \theta}{\partial t} + (\vec{V} \cdot \nabla) \theta = \frac{L_k \partial M_k}{c_p T} + \frac{L_c \partial M_c}{c_p T \delta t}, \]

\[ \frac{L_3 \partial M_3}{c_p T \delta t} + \Delta \theta \]

\[ \frac{\partial \omega}{\partial t} + (\vec{V} \cdot \nabla) \omega = -\frac{\partial M_k}{\partial t} \frac{\partial \omega}{\partial t} + \Delta \omega, \]

\[ \text{equations for calculating the function of distribution of drops, crystals and freezing fragments by their masses:} \]

\[ \frac{\partial f_1}{\partial t} + u \frac{\partial f_1}{\partial x} + v \frac{\partial f_1}{\partial y} = \left( \frac{\partial f_1}{\partial t} \right)_K + \left( \frac{\partial f_1}{\partial t} \right)_c, \]

\[ \begin{aligned} \frac{\partial f_2}{\partial t} + u \frac{\partial f_2}{\partial x} + v \frac{\partial f_2}{\partial y} &= \left( \frac{\partial f_2}{\partial t} \right) + \Delta f_2 + I_{1b} \\
\frac{\partial f_3}{\partial t} + u \frac{\partial f_3}{\partial x} + v \frac{\partial f_3}{\partial y} &= \left( \frac{\partial f_3}{\partial t} \right) + \Delta f_3, \end{aligned} \]

\[ \text{Poison equation for the electrostatic field} \]

\[ \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -\frac{\rho_e}{\epsilon_0}, \]

\[ \text{Initial conditions for (1)-(11) can be written as} \]

\[ \begin{aligned} u(\vec{r},0) &= u_0(\vec{r}), v(\vec{r},0) = v_0(\vec{r}), w(\vec{r},0) = w_0(\vec{r}), \\
\theta(\vec{r},0) &= \theta_0(\vec{r}), s(\vec{r},0) = s_0(\vec{r}), \\
f_1(\vec{r},0) &= f_1(\vec{r},0), f_2(\vec{r},0) = f_2(\vec{r},0), f_3(\vec{r},0) = f_3(\vec{r},0), \\
\rho_e(\vec{r},0) &= \rho_e(\vec{r},0). \end{aligned} \]

\[ \text{Boundary conditions:} \]

\[ \begin{aligned} u(\vec{r},1) &= u_0(\vec{r}), v(\vec{r},1) = v_0(\vec{r}), w(\vec{r},1) = w_0(\vec{r}), \\
\theta(\vec{r},1) &= \theta_0(\vec{r}), s(\vec{r},1) = s_0(\vec{r}), \\
f_1(\vec{r},1) &= f_1(\vec{r},1), f_2(\vec{r},1) = f_2(\vec{r},1), f_3(\vec{r},1) = f_3(\vec{r},1), \end{aligned} \]

\[ \text{The system of equations is used for the space-time domain} \]

\[ 0 \leq x \leq L_x, \quad 0 \leq y \leq L_y, \quad 0 \leq z \leq L_z, \quad 0 \leq m < \infty, \quad t > 0. \]

\[ \text{The following notation is used:} \]

\[ (\vec{r}, \vec{v}) = (u, v, w), \quad \sigma(\vec{r}) = \text{potential temperature}, \quad \Delta x = \frac{E_d}{\rho_0}, \quad \Delta x - \text{dimensionless pressure}; \quad \bar{v} = \text{average potential temperature}; \quad R = \text{gas constant}; \quad \bar{v}(\vec{r}) = \text{summary relation of liquid and solid phases in the cloud}; \quad \sigma(\vec{r}) = \text{parameter changes in specific pressure density in the cloud on height}; \quad \rho(\vec{r}) = \text{heating air capacity under constant pressure}; \quad L_x, L_y, L_z = \text{liquid condensation, sublimation, freezing heat}; \quad \pi(\vec{r}) = \text{parameters of dimensionless pressure, potential temperature and specific humidity from their standard values} \quad \pi_0(\vec{r}), \theta_0(\vec{r}), s_0(\vec{r}); \quad \frac{\delta M_k}{\delta t}, \frac{\delta M_c}{\delta t} = \text{changes in specific humidity due to vapor diffusion into drops and crystals}; \quad \frac{\delta M_k}{\delta t} = \text{mass of drop water freezing per unit time per unit volume of air}; \quad K(\vec{r}) = \text{turbulent diffusion coefficient}. \]
\( \rho_s(\bar{r}, t) \) - summary total electrical charge, \( \varepsilon_0 \) - dielectric constant in vacuum.

The following notation is used for the boundaries of the spatial area: \( \Omega, \Omega_L, \Omega_0, \Omega_r, \Omega_0, \Omega_r, \).

Integral differential equation used to describe cloud coagulation processes can be written as

\[
\left( \frac{\partial f}{\partial t} \right)_{\bar{r}} = -f_{\Omega}(\bar{r}, m, t) \int_0^m f_{\Omega}(\bar{r}, m', t) dm' + \int_\Omega \beta_{\Omega}(m, m') \cdot f_{\Omega}(\bar{r}, m', t) dm',
\]

(18)

where \( \beta_{\Omega}(m, m') = \pi (r(m) + r(m'))^2 \cdot (V_1(m) - V_2(m')) \cdot e_i(m, m') \); \( r(m) \) and \( r(m') \) - radii of colliding particles; \( V_1(m) \) and \( V_2(m') \) - their fall speed rates; \( e_i(m, m') \) - capture coefficient for drops.

Interaction of drops and crystals is calculated by formulas

\[
\left( \frac{\partial f}{\partial t} \right)_{cr} = -f_{\Omega}(\bar{r}, m, t) \int_0^m f_{\Omega}(\bar{r}, m', t) dm' + \\
+ \int_\Omega \beta_{cr}(m, m') \cdot f_{\Omega}(\bar{r}, m', t) dm' + \int_\Omega \beta_{cr}(m, m') \cdot f_{\Omega}(\bar{r}, m', t) dm' +
\]

(19)

where \( \beta_{cr}(m, m') = \pi (r(m) + r(m'))^2 \cdot (V_1(m) - V_2(m')) \cdot e_i(m, m'), e_c(m, m') \) - capture coefficient for drops and crystals. Collision of drops and crystals causes coagulation of drops.

Variables are transformed for solving numerical coagulation equations \( m = m_0 \exp[3 \cdot (1 - 1)/I_0] \) and distribution functions \( G(I)dl = mf(m)dm \). The values of function \( G(I) \) between groups is calculated using interpolation of function values in the nearest groups.

Changes in function \( f_i(\bar{r}, m, t) \) caused by drop fragmentation are calculated by formula

\[
\left( \frac{\partial f}{\partial t} \right)_{\Omega} = -P_{\Omega}(m) f_i(\bar{r}, m, t) + \int_\Omega Q(m, m')P_{\Omega}(m') \cdot f_i(\bar{r}, m', t) dm',
\]

(21)

where \( P_{\Omega}(m) \) is disintegration probability for a drop with mass \( m \). \( Q(m, m') \) per time unit. Formation of the drop with mass \( m \) is probable while disintegrating the drop with mass \( m' \). \( P_{\Omega}(m) \) is calculated by formula

\[
P_{\Omega}(m) = 2.94 \cdot 10^{-7} \exp(34 (\bar{r})).
\]

(22)

Function \( Q(m, m') \) is calculated by formula

\[
Q(m, m') = \frac{145.37 \cdot \bar{r}(m) \exp(-7 \bar{r}(m)/\bar{r}(m'))}{m - m'}.
\]

(23)

Changes in functions \( f_i(\bar{r}, m, t) \) and \( f_j(\bar{r}, m, t) \) due to drop crystallization are calculated using function \( R_{\Omega}(\bar{r}, m) \) - coagulation probability for the drop with mass \( m \) at point per time unit \( \bar{r} \):

\[
\left( \frac{\partial f_i}{\partial t} \right)_{\Omega} = -R_{\Omega}(\bar{r}, m) f_i(\bar{r}, m, t),
\]

(24)

\[
R_{\Omega}(\bar{r}, m) = k_{2m} \cdot \exp[2B_{m} \cdot (T_m - T_\infty(\bar{r}))],
\]

(25)

where \( k_{2m}, B_{m} \) - parameters; \( T_m(\bar{r}) \) — medial congelation temperature for drops with mass \( m \); \( T_\infty(\bar{r}) \) — air temperature at a specific point.

Formation of new drops in condensation nuclei considers term \( f_i(\bar{r}, m, t) \). The process can be described by formula

\[
I_1(\bar{r}, m, t) = a q_{i} - q_{i}(\bar{r}),
\]

(26)

where \( q_{i}(\bar{r}) \) — air humidity at point \( \bar{r} \); \( q_{i}(\bar{r}) \) — saturated steam humidity at the same point; \( a \) - numerical coefficient; \( \psi_i(\bar{r}, m) \) — assigned distribution of drops at the same point; \( q_{i}, (\bar{r}, m) \) — water content for assigned distribution of drops. Formation of crystals in the sublimation nuclei \( I_1(\bar{r}, m, t) \) can be described by the same formula.

Condensation and sublimation are described by formulas

\[
\left( \frac{\partial f_i}{\partial t} \right)_{cr} = -\frac{\partial}{\partial m} f_i(\bar{r}, m, t) \frac{dm}{dt},
\]

(27)

\[
\left( \frac{\partial f_i}{\partial t} \right)_{cr} = -\frac{\partial}{\partial m} f_i(\bar{r}, m, t) \frac{dm}{dt}.
\]

(28)

The speed rate for changing masses of cloud particles can be determined using the speed rate for changing their radii:

\[
\frac{dm}{dt} = 4 \pi \varrho r^2 \frac{dr}{dt}.
\]

(29)

Formulas for describing formation (evaporation) can be written as

\[
\frac{dr}{dt} = D_{\varrho} m \frac{1}{\varrho} \frac{\mu}{r} M \cdot P - r, \frac{1}{P} \mu \frac{e}{e_n} (f - 1),
\]

(30)

\[
\frac{dr}{dt} = D_{\varrho} m \frac{1}{\varrho} \frac{\mu}{r} M \cdot P - r, \frac{1}{P} \mu \frac{e}{e_n} (f - 1),
\]

(31)

where \( D \) - coefficient of molecular steam diffusion; \( r_e \) and \( r_i \) - sizes (radii) of drops and ice particles respectively; \( P \) - air pressure; \( \varrho, \mu, \mu, \mu, M \) - molecular masses of vapor and air; \( e, e_n, e_n \) - saturated steam pressure at a cloud temperature above water and ice; \( f \) - relative air humidity.

Function \( L_{\Omega}(\bar{r}, m, t) \) describes formation of artificial crystals under the influence of a crystallized reagent. Its type depends on the shape and properties of the source of artificial particles and properties of particles themselves while modelling
natural evolution of the cloud \( I_{ab}(r, m, t) = 0 \). A specific type of function \( I_{ab}(r, m, t) \) depending on the mode of reagent application is given.

The model takes into account electrification of cloud particles based on thunderstorm development patterns and charge separation factors due to waterdrop congelation, grains and hailstones growth, and interaction of hailstones with ice crystals and supercooled drops.

When solving model equations, splitting and running methods were used.

The presented numerical model makes it possible to calculate the evolution, time of thermodynamic parameters and spectra of droplets and crystals in the cloud.

The main module involves initialization of the model, and calculation of cloud characteristics in a cycle over time.

B. Calculation formulas for extinction coefficients

To calculate aerosol attenuation, software was developed. Its input parameters are radiation wavelength, particle size, particle concentration, complex refractive index.

Propagation and attenuation of optical radiation in the atmosphere are described using optical characteristics: attenuation, absorption and scattering coefficients, optical thickness of the medium, radiation indicatrix, geometrical parameters of backscattering, polarization characteristics, etc. \([5-8]\).

When electromagnetic radiation propagates in the layer of aerosol particles with distribution function \( f(r) \) and length \( \Delta H \), its output power changes according to the Bouguer law:

\[
P = P_0 \exp(-\alpha_e \Delta H)
\]

where \( P_0 \) - radiation power; \( \alpha_e \) - volume radiation attenuation coefficient, \( \text{m}^{-1} \). The extinction coefficient is a sum of scattering and absorption coefficients:

\[
\alpha_e = \alpha_s + \alpha_a,
\]

According to the theory of aerosol electromagnetic radiation scattering, attenuation, scattering and absorption coefficients are efficiency factors of aerosol attenuation, scattering and absorption:

\[
\sigma_e = \pi \cdot r^2 \cdot K_e(r, \lambda, m_p),
\]

\( r \) - particle radius, \( \lambda \) - wavelength, \( m_p = n + i\chi \) - integral particle refraction index (\( n \)-refraction index, \( \chi \)-absorption index).

Attenuation \( K_e(\lambda, m_p, r) \), scattering \( K_s(\lambda, m_p, r) \) and absorption \( K_a(\lambda, m_p, r) \) coefficients are calculated according to the scattering theory \([12]\).

III. RESULTS

A. Simulation results for particle spectra

Using the model, thermodynamic, microstructural and electrical parameters of convective clouds for various atmospheric states were calculated.

Dimensions of the spatial domain were 40 km in horizontal coordinates and 16 km in vertical ones. The grid step in the X, Y coordinates was 500 m, in the Z coordinates – 250 m. The X axis is directed to the east, Y - to the north, Z - vertically. The cloud was initiated by thermal impulses near the earth surface with overheating = 1 °C.

The model of a convective cloud with detailed microphysical parameters allows for studying formation of microstructural characteristics of clouds, formation of precipitation particles, accumulation of electrical charges and electrical coagulation of cloud particles. The results obtained reflect nonlinear effects of cloud physics which cannot be studied and evaluated using simpler models, for example, parameterized microphysics.

Fig. 1 shows an example of 3-D cloud parameters simulation. Water iso-surface of 0.5 g/m³ (1) and ice content of 0.1 g/m³ (2) are shown in the vertical section. Several contour lines in the vertical plane intersecting the cloud reflect ascending flows. There is an upward flow in the center of the cloud at the velocity of 30 m/s.

Fig. 2 shows distribution of drops and ice particles by their sizes at the 40th minute of cloud formation at \( z = 8 \text{ km} \). There are small drops and ice crystals of different sizes. The second maximum lies within \( r \geq 100 \mu\text{m} \).

B. Microwave radiation extinction calculation results for ice particles

To estimate microwave radiation extinction in the cloud environment, scattering and absorption factors for ice spheres with radii varying from 0.1 \( \mu\text{m} \) to 100 \( \mu\text{m} \) in the wavelength range from 0.4 \( \mu\text{m} \) to 10 cm were calculated.

Table 1 and Fig. 3 show some calculation results for crystals at \( r = 0.5 \mu\text{m} \).
Properties of IR radiation attenuation by small and large ice particles depending on the wavelength can be different (Fig. 3). For large ice crystals, absorption slightly depends on the wavelength.

Taking into account the spectra of liquid and solid particles in the cloud which were obtained using the cloud model, radiative attenuation in the “anvil” of a powerful convective cloud consisting of small ice crystals was calculated.

In the cloud environment consisting of ice crystals, microwave radiation power can be reduced several times. In some cases, radiation attenuation is 2–3 times more due to scattering than due to absorption.

IV. CONCLUSION

Thermodynamic and microstructural characteristics of convective clouds were calculated using a three-dimensional nonstationary numerical model. The numerical model helps calculate characteristics of the spectra of liquid and solid particles in different parts of the developing cloud.

Microstructural parameters of the mature cloud were studied. Drop and ice particle spectra were determined.

Size distribution functions for drops and crystals were calculated based on absorption and scattering of electromagnetic radiation in the cloudy environment and actual spectra.

Microwave radiation extinction in the cloudy environment was studied at different wavelengths and particle sizes.

References


