Mathematical Modelling of Fluid Motion in Borewell Filtering Devices

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Abstract— The use of equations known in filtering theory to determine parameters of fluid motion through filters of various designs used in the oil industry to clean up the products from mechanical impurities is described. Analytical studies and calculations have been carried out to determine the speed of fluid motion in filters of frame-bar, ringed, perforating types, as well as through the packing of gravel filters. The selection of new boundary conditions simplifies the solution of the resulting differential equations; and the increase in the calculated parameters by 5-10% (Hou method) eliminates the probability of error. Thus, the obtained values of the filtering parameters allow to correctly choose the size and geometry of the filter elements.

Keywords— filtration flows, differential equations, boundary conditions, average weighted potential, geometry of filter element, mechanical impurities

I. INTRODUCTION

In the practice of borewell operation, various types of filters are used: frame-rod, annular, perforated, gravel and others.

To solve practical problems of calculating filtration flows in a borewell, a single method for all types of filter designs is used. It is called the average weighted potential method (AWP) [1], known in the technical literature as the Hou method [2]. Using this method, the value of debit underestimated by 5-10% is got. Therefore, it is enough to increase the result obtained by 7% to get as close as possible to the exact borewell flow rate.

II. METHODS AND MATERIALS

First the operation of the frame-rod filter is described. It consists of alternating vertical cracks and impenetrable walls (Fig. 1.). Obviously, by virtue of its symmetry, the surfaces of AD and BC will be the surfaces of the liquid current. The circular surface CD is an equipotential surface on which the potential of the filtration velocity will be \( \phi = kP/\mu \), where \( k \) is the formation permeability; \( P \) is the reduced pressure; \( \mu \) is the dynamic viscosity of the fluid.

It is known that the potential of plane-parallel linear filtration in an isotropic medium with permeability \( k \) satisfies the Laplace equation in polar coordinates \( r, \theta \) [3, 4]:

\[
\frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \frac{1}{r} \frac{\partial \phi}{\partial \theta} \right) = 0
\]  

(1)
The boundary conditions for the Laplace equation for a frame-rod filter are as follows:

$$\frac{\partial \phi}{\partial \vartheta}\bigg|_{\vartheta=0} = 0, \quad \frac{\partial \phi}{\partial \vartheta}\bigg|_{\vartheta=\theta_0} = 0, \quad \text{где } \theta_0 = \alpha + \beta$$

$$\frac{\partial \phi}{\partial r}\bigg|_{r=r_c} = 0, \quad 0 \leq \theta \leq \alpha$$

$$\phi\bigg|_{r=r_c} = \varphi_c, \quad \text{где } \varphi_c = -\frac{kP_c}{\mu} = \text{const}$$

The exact solution to this problem will be given by the conformal mapping method. However, we are interested in the Hou method, which is the same for all filter designs. Therefore, instead of the exact boundary condition (5), the approximate boundary condition will be considered (6)

$$\frac{\partial \phi}{\partial r}\bigg|_{r=r_c} = -V_0 = \text{const}$$

where $V_0$ – a certain, yet unknown, constant (the minus sign in (6) is because the fluid flow is directed to the center of the well). This constant will be selected so that the average value of the potential at the boundary BE satisfies the condition:

$$<\phi>\bigg|_{r=r_c} = \frac{1}{\beta} \int_0^{\beta} \varphi(r_c, \vartheta)d\vartheta = \varphi_c$$

Condition (5) was fulfilled for the arithmetic mean value of the potential $\varphi$.

The Laplace equation (1), satisfying the boundary conditions (2), (3), (4), (6), is a differential equation with separable variables and has the form:

$$\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial \vartheta^2} = 0$$

where $\lambda_n = \frac{n\pi}{\theta_0}$, $r = \frac{R}{r_c}$ и $r_0 = \frac{R}{r_c}$ – dimensionless value.

The unknown quantity $V_0$ is found by calculating the potential value averaged on the BE arc. For this, the obtained value of potential (8) into formula (7) was substituted, from which we find that:

$$V_0 = \frac{\beta \theta_0}{r_c} \frac{\varphi_c - \varphi_0}{\beta^2 \ln \tau_0 - 2 \sum_{n=1}^{\infty} \frac{\lambda_n}{\tau_0 - \lambda_n} \sin^2(\lambda_n \alpha)}$$

The flow rate of the borewell will find using the found value $V_0$ according to the formula:

$$Q = N2V_0SH = 2\pi \frac{kH}{\mu} \frac{P_{II} - P_c}{\ln \frac{R + 1}{2}}$$

where $N$ – количество щелей на 1 погонный метр, $S$ – площадь щели, $H$ – высота фильтра.
The considered method suggests increasing the value of the Q output by 7%.

An annular filter consists of alternating horizontal cracks and impermeable rings (Fig. 2.). By virtue of the symmetry, the surfaces AD and BC can be considered as current surfaces. The circular cylindrical surface CD is an equipotential surface, on it the potential of the filtration rate is equal to some given constant. Like the first part of the paper, the problem is reduced to solving the Laplace equation (11) with respect to the potential φ(r, z) in cylindrical coordinates [5]:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{\partial^2 \phi}{\partial z^2} = 0 \]

(11)

The boundary conditions in the region of ABCD (Fig. 2.) for equation (11) will be as follows:

\[ \frac{\partial \phi}{\partial z} |_{Z=0} = 0, \quad \frac{\partial \phi}{\partial z} |_{Z=Z_0} = 0 \]

(12)

\[ \phi |_{r=R} = \phi_{II} \text{, } \text{rôle } \phi_{II} = - \frac{kP_I}{\mu} = \text{const} , \]

(13)

\[ \frac{\partial \phi}{\partial r} |_{r=r_c} = 0 \text{, } \int_{l_u}^{z_0} \int_{Z=Z} \phi_{e} \text{, } \text{rôle } \phi_{e} = - \frac{kP_c}{\mu} = \text{const} \]

(15)

Fig. 2. Frame-core filter scheme used in water production borewells: \( r_c \) – borewell radius; \( l_u \) – half the height of the slit; \( l_c \) – half the height of the impermeable wall; \( R \) – the radius of the power circuit; \( z_0 = l_u + l_c \).

For an approximate solution of this problem, the AWP method is applied. For this, instead of the boundary condition (15), the boundary condition is taken (16):

\[ \frac{\partial \phi}{\partial r} |_{r=r_c} = - V_0 = \text{const}, \]

(16)

where \( V_0 \) – some yet unknown constant (the minus sign in (16) means that the movement is directed to the center of the borewell).

The selection method is found so that the average value of the potential at the boundary AE satisfies the condition

\[ \langle \phi \rangle |_{r=r_c, 0 \leq Z \leq l_u} = \frac{1}{l_u} \int_{l_u}^{Z=0} \phi(r_c, Z) dZ = \phi_{c} \]

(17)

The Laplace equation (11) satisfying the boundary conditions (12), (13), (14), (16) is solved by the method of separation of variables and we get:

\[ \phi(r, z) = \sum_{n=1}^{\infty} \frac{a_n(r)}{\lambda_n} \sin \left( \frac{\lambda_n l_u}{\lambda_n} \right) \cos \left( \frac{\lambda_n z}{\lambda_n} \right) \]

(18)

where

\[ a_n(r) = \frac{\left( J_0(\lambda_n r) \right)}{K_0(\lambda_n R)} \]

(19)
where \( I_\nu, I_1 \) – modified Bessel functions; \( K_\nu, K_1 \) – Macdonald functions [6].

Substituting the value of potential (18) into formula (17), we get:

\[
V_0 = \frac{I_{u} \theta}{r_c I_{u} \theta \ln \frac{R}{r} - \sum_{n=1}^{\infty} \frac{a_n(r_c) \sin^2(\lambda_n R)}{a_n^2}}
\]

Using the obtained value of the filtration rate \( V_0 \), the flow rate is determined:

\[
Q = NV_0 2\pi c 2I_{u} = NV_0 S = 2\pi \frac{kH}{\mu} \frac{\theta_0}{P \pi} - P_c \frac{1}{r_c} \frac{1}{2} \lambda
\]

where \( N \) – number of slots, \( S \) – slot area, \( H \) – height of the filter,

\[
\lambda = -\frac{4}{2} \sum_{n=1}^{\infty} \frac{a_n(r_c) \sin^2(\lambda_n R)}{a_n^2}
\]

Computational experiments showed that the flow rate increases with increasing filter duty cycle and approaches the asymptotic value at 20-30% duty cycle (Fig. 3). Therefore, there is no practical need for ringed filters with a higher duty cycle. In practice, indeed, they use filters of the ringed construction with a duty cycle from 20% to 30% [7].

For a perforation-type filter, the flow rate equation is obtained by analogous calculations considering its features (Fig. 3).

The holes in the cylindrical body of this type of filters can have different geometric shapes and sizes. The problem of calculating the flow rate for the row arrangement of perforation holes is described. These holes have two perpendicular axes of symmetry, one of which is parallel to the axis of the borewell; and they include a rectangle, circle, ellipse, and others (Fig. 3).

Fig. 3. The scheme of a filter perforated construction fragment with a row arrangement of perforations. On the left, the filter segment of an elementary flow area, BB, C1C – D area of the filter surface, O1h – the axis of symmetry of the wellbore, h – the height of the segment, \( \theta_0 \) – the angle of the segment solution, \( \sigma \) – the fourth part of the perforation hole.

Due to this symmetry, the surfaces ABCD, AB1B1D1, AA1B1B and DD1C1C are current surfaces. The circular cylindrical surface AA1D1D is an equipotential surface on which the potential of the filtration rate \( \phi \) is equal to a given constant. As in the two previous cases, the problem reduces to solving the Laplace equation in cylindrical coordinates \( r, \theta, z \):

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = 0
\]

Initial conditions:

\[
\frac{\partial \phi}{\partial z} = 0, \quad \frac{\partial \phi}{\partial z} = 0, \quad \frac{\partial \phi}{\partial \theta} = 0, \quad \frac{\partial \phi}{\partial \theta} = 0
\]

\[
\phi(R, \theta, z) = \phi_{II}, \quad r \theta \phi_{II} = \frac{kP_{II}}{\mu} = \text{const},
\]

\[
\begin{cases}
\phi(r_c, \theta, z) = \phi_c, \quad \text{within} \sigma \\
\frac{\partial \phi}{\partial r} = 0, \quad \text{outside} \sigma
\end{cases}
\]

where \( \phi_c = \frac{kP_c}{\mu} = \text{const} \).

The final result for determining the flow rate (for the case of a rectangular perforation hole) will be:
\[ Q = N V_0 4 S \sigma r_c = N V_0 4 S = 2 \pi \frac{k H}{\mu} \frac{P_{II} - P_c}{R} \frac{1}{r_c^2} \lambda, \quad (27) \]

\[ \lambda = \frac{8}{r_c S \sigma} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{W_{m n}}{r_c^2} \left( \frac{r_c}{r_c} \right)^2 \quad (28) \]

where \( N \) – total number of perforations, \( S = \sigma r_c \) – the area of the perforation, \( H \) – the height of the filter.

\[ \text{Fig. 4. Comparison of filters of various designs.} \]

Filter of perforation design, quality of perforation holes around 32 holes, solution angle of holes – 2.87 degrees, hole height – 115 mm, number of holes per 1 meter vertically from 1 to 8; 2. Ringed filter, slot height – 1.5 mm, number of slots per meter height from 1 to 666; 3. Frame-rod filter design, the angle of the slit solution –25 degrees, the number of slits is from 1 to 14.

The calculations were carried out according to the formulas for determining the flow rates for frame-bar, ringed and perforation types of filters (borewell radius 100 mm, power supply radius 200 m) [10].

IV. CONCLUSION

Filtering devices do not fully solve the problem of protecting the unit of electric submersible centrifugal pump, tubing from the harmful effects of mechanical impurities, mainly represented by quartz sand, loose particles of proppant, killing fluid, etc. When using such devices in order to prevent the formation of traffic jams at the bottom of a well, various containers and fishing chambers are often used [11].

However, the percentage of success of such measures depends on the concentration and fractional composition of suspended particles [12]. Therefore, the greatest effect on the protection of pumping and other equipment from mechanical impurities is achieved with a comprehensive solution to the problem.

It will be important is the work to reduce the water-cut of production borewells, to control the selection, etc.

References


