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Vibration Analysis of Cultivator Modification

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***Abstract*—The paper presents the vibration analysis of 'modification' of a cultivator representing a manometric tubular spring (MTS) taking into account the rigid tip. It also considers the mathematical model of damped vibrations. The MTS dynamic model represents a thin-walled curved bar making in-plane vibrations within the central axis curvature. The MTS vibration equations are obtained according to the D'Alembert principle in projections on a normal and on a tangential component. The Bubnov-Galerkin method is used to solve the obtained equations.**

***Keywords*—Manometric tubular spring, vibrations, cultivator.**

I. INTRODUCTION

Mechanical pressure gages are used to measure pressure in compliance with the standard documentation. However, their use is also reflected in the following studies devoted to the use of manometric tubular spring (MTS) – sensitive elements of manometers in various technical fields [1, 2, 3]. The use of MTS in agricultural machinery allows reducing the draught resistance of tillage machines due to vibration when interacting with soil [4], as well as increasing the quality of soil cultivation by adjusting the rack stiffness [5, 6].

II. OBJECTS AND METHODS

The theoretical study of vibration parameters of a cultivator's working body representing manometric tubular spring was carried out on the basis of higher mathematics, physics and theoretical mechanics. The results were analyzed via mathematical statistics.

III. ANALYSIS OF THE STUDY

The principal diagram of a cultivator 'modification' (Fig. 1) was designed for the theoretical study.

This 'modification' represents MTS with a rigid tip and has the following geometric characteristics: central angle – 180 degrees, curvature radius – 500 mm, major semiaxis of cross-section profile – 25 mm, minor semi-axis of cross-section profile – 12.5 mm, wall thickness – 2.5 mm.

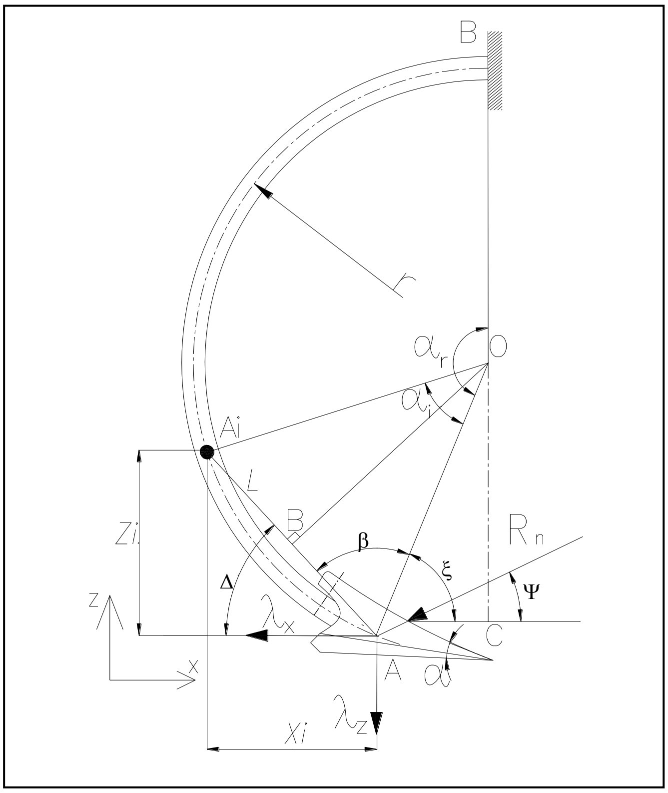

Fig. 1. Principal diagram of a cultivator 'modification'

The study of frequency characteristics of MTS with a rigid tip is covered in works [7, 8]. The designed mathematical model allows considering the influence of damping fluid viscosity and a tip mass. MTS represents a curved bar making in-plane vibrations within the central axis curvature.

The system of the MTS motion equations is obtained by projecting all forces, including inertia forces, on a normal and on a tangential component.

$$m(\varphi)\frac{\partial^2 w}{\partial t^2}+\left(1+\frac{b}{R}\right)\beta\frac{\partial w}{\partial t}-\frac{\partial Q}{R\partial\varphi}+\frac{N}{R}=0$$
$$m(\varphi)\frac{\partial^2 u}{\partial t^2}-\frac{Q}{R}-\frac{\partial N}{R\partial\varphi}=0 \qquad (1)$$

where $m(\varphi)$ – linear density (weight); β – damping liquid resistance coefficient; Q – shear force; N – axial force; M – bending moment; b – major semiaxis of MTS cross-section; R – curvature radius of the central axis; $d\varphi$ – angle of infinitely small element cut out from a curved bar; w – motion in cross direction; u – motion in axial direction.

The given system of differential equations takes into account the following boundary conditions. In the base of a manometric spring, at φ=0, the axial and cross motion, as well as the rotation angle of a tube cross-section equal zero (main boundary conditions).

On a free end (φ=γ), near the concentrated weight the MTS curvature may be neglected. Such assumption leads to the following boundary conditions on a free end (natural boundary conditions):

$$M(\gamma)=0;$$
$$N(\gamma)=-m_{нак}\frac{\partial^2 u}{\partial t^2};$$
$$Q(\gamma)=-m_{нак}\frac{\partial^2 w}{\partial t^2}$$

Initial conditions are not importance since only frequency characteristics of MTS are defined.

The problem is solved by the Bubnov-Galerkin method [9]. Motions u and w are set as follows:

$$u=\psi_1(\varphi)a_1(t)+\psi_2(\varphi)a_2(t)+\ldots+\psi_n(\varphi)a_n(t)=\sum_{i=1}^{n}\psi_n(\varphi)a_n(t)$$
$$w=\xi_1(\varphi)b_1(t)+\xi_2(\varphi)b_2(t)+\ldots+\xi_n(\varphi)b_n(t)=\sum_{i=1}^{n}\xi_n(\varphi)b_n(t) \qquad (2)$$

where $a_1,a_2...a_n,b_1,b_2...b_n$ – unknown functions of a variable t;

$\psi_1,\psi_2...\psi_n,\xi_1,\xi_2...\xi_n$ – basic functions of a variable φ.

The following power functions are used as basic functions:

$$\psi_i(\varphi)=\varphi^i;\quad i=1...n.$$
$$\xi_i(\varphi)=\varphi^{j+1};\quad j=1...n. \qquad (3)$$

Such function system is complete, functions are linearly independent and satisfy the main boundary conditions. However, the functions are not orthogonal and hence functions 5-8 may be considered in approximations u and w. Let us fulfill the orthogonality condition of the system of equations (1) with the system of basic functions (3):

$$\begin{cases}\int_0^\gamma m(\varphi)\frac{\partial^2 w}{\partial t^2}\zeta_j\,d\varphi+\int_0^\gamma\left(1+\frac{b}{R}\right)\beta\frac{\partial w}{\partial t}\zeta_j\,d\varphi-\int_0^\gamma\frac{\partial Q}{R\partial\varphi}\\ \int_0^\gamma m(\varphi)\frac{\partial^2 u}{\partial t^2}\psi_i\,d\varphi-\int_0^\gamma\frac{Q}{R}\psi_i\,d\varphi-\int_0^\gamma\frac{\partial N}{R\partial\varphi}\end{cases} \qquad (4)$$

Let us present the integral $\int_0^\gamma\frac{\partial Q}{R\partial\varphi}\zeta_j d\varphi$ as follows:

$$\int_0^\gamma\frac{\partial Q}{R\partial\varphi}\zeta_j d\varphi=\frac{1}{R}\int_0^\gamma\frac{\partial}{\partial\varphi}(Q\cdot\zeta_j)d\varphi-\frac{1}{R}\int_0^\gamma Q\frac{\partial\zeta_j}{\partial\varphi}d\varphi=\frac{1}{R}\big(Q(\gamma)\cdot\zeta_j(\gamma)\big)-\frac{1}{R}\big(Q(0)\cdot\zeta_j(0)\big)-\frac{1}{R}\int_0^\gamma Q\frac{\partial\zeta_j}{\partial\varphi}d\varphi \qquad (5)$$

Due to boundary conditions $\zeta_j(0)=0$, the integral $\int_0^\gamma\frac{\partial Q}{R\partial\varphi}\zeta_j d\varphi$ may be written as:

$$\int_0^\gamma\frac{\partial Q}{R\partial\varphi}\zeta_j d\varphi=\frac{1}{R}\left(-m_{нак}\frac{\partial^2 w}{\partial t^2}\cdot\zeta_j(\gamma)\right)-\frac{1}{R}\int_0^\gamma Q\frac{\partial\zeta_j}{\partial\varphi}d\varphi \qquad (6)$$

In a similar manner, the integral can be transformed as follows $\int_0^\gamma\frac{\partial N}{R\partial\varphi}\psi_i\,d\varphi$:

$$\int_0^\gamma\frac{\partial N}{R\partial\varphi}\psi_i\,d\varphi=\frac{1}{R}\int_0^\gamma\frac{\partial}{\partial\varphi}(N\cdot\psi_i)d\varphi-\frac{1}{R}\int_0^\gamma N\frac{\partial\psi_i}{\partial\varphi}d\varphi=\frac{1}{R}\big(N(\gamma)\cdot\psi_i(\gamma)\big)-\frac{1}{R}\big(N(0)\cdot\psi_i(0)\big)-\frac{1}{R}\int_0^\gamma N\frac{\partial\psi_i}{\partial\varphi}d\varphi \qquad (7)$$

Finally it is presented as:

$$\int_0^\gamma\frac{\partial N}{R\partial\varphi}\psi_i\,d\varphi=\frac{1}{R}\left(-m_{нак}\frac{\partial^2 u}{\partial t^2}\cdot\psi_i(\gamma)\right)-\frac{1}{R}\int_0^\gamma N\frac{\partial\psi_i}{\partial\varphi}d\varphi \qquad (8)$$

Let us transform the orthogonality condition (3) taking into account (5) and (7):

$$\int_0^\gamma m(\varphi)\frac{\partial^2 w}{\partial t^2}\zeta_j\,d\varphi+\frac{m_{\text{нак}}}{R}\frac{\partial^2 w}{\partial t^2}\cdot\zeta_j(\gamma)+\int_0^\gamma\left(1+\frac{b}{R}\right)\beta\frac{\partial w}{\partial t}\zeta_j\,d\varphi+$$
$$+\frac{1}{R}\int_0^\gamma Q\frac{\partial\zeta_j}{\partial\varphi}d\varphi+\int_0^\gamma\frac{N}{R}\zeta_j\,d\varphi=0, \quad (9)$$
$$\int_0^\gamma m(\varphi)\frac{\partial^2 u}{\partial t^2}\psi_i\,d\varphi+\frac{m_{\text{нак}}}{R}\frac{\partial^2 u}{\partial t^2}\cdot\psi_i(\gamma)-\int_0^\gamma\frac{Q}{R}\psi_i\,d\varphi+\frac{1}{R}\int_0^\gamma N\frac{\partial\psi_i}{\partial\varphi}d\varphi=0.$$

It is also possible to get the system of equations (9) in a different way. Thus, if in system (1) the bulk weight considering the tip weight is presented as $m(\varphi)$, then the expression to determine the MTS weight with a tip will be as follows:

$$m=m_{\text{тр}}+m_{\text{нак}}\eta(\varphi-\gamma), \quad (10)$$

where m – MTS weight with a tip, $m_{\text{тр}}$ – tube weight, $\eta(\varphi-\gamma)$ – Heaviside unit step function.

To determine the bulk weight (weight per unit length) there is a need to make the following transformations:

$$m_L=\frac{\partial m}{\partial S}=\frac{\partial m}{R\partial\varphi}=m(\varphi)+\frac{m_{\text{нак}}}{R}\delta(\varphi-\gamma), \quad (11)$$

where S – length of the MTS central axis, $\delta(\varphi-\gamma)$ – Dirac delta function.

By substituting the distributed weight (10) into (3), and by considering the boundary conditions on the end of MTS (for free end) – $M(\gamma)=0$; $N(\gamma)=0$; $Q(\gamma)=0$; we get the following:

$$\int_0^\gamma m(\varphi)\frac{\partial^2 w}{\partial t^2}\zeta_j\,d\varphi+\frac{m_{\text{нак}}}{R}\frac{\partial^2 w}{\partial t^2}\cdot\zeta_j(\gamma)+$$
$$+\int_0^\gamma\left(1+\frac{b}{R}\right)\beta\frac{\partial w}{\partial t}\zeta_j\,d\varphi+\frac{1}{R}\int_0^\gamma Q\frac{\partial\zeta_j}{\partial\varphi}d\varphi \quad (12)$$
$$+\int_0^\gamma\frac{N}{R}\zeta_j\,d\varphi=0,$$
$$\int_0^\gamma m(\varphi)\frac{\partial^2 u}{\partial t^2}\psi_i\,d\varphi+\frac{m_{\text{нак}}}{R}\frac{\partial^2 u}{\partial t^2}\cdot\psi_i(\gamma)-\int_0^\gamma\frac{Q}{R}\psi_i\,d\varphi+\frac{1}{R}\int_0^\gamma N\frac{\partial\psi_i}{\partial\varphi}d\varphi=0.$$

Apparently, the systems of equations (9) and (12) are similar. The expressions for cross and axial forces may be presented within motion u and w for a rod element as follows:

$$Q=\frac{\partial}{\partial\varphi}\left(\frac{EJ(\varphi)K_k(\varphi)}{(1-\mu^2)R^3}\left(\frac{\partial u}{\partial\varphi}-\frac{\partial^2 w}{\partial\varphi^2}\right)\right),$$
$$N=\frac{ES(\varphi)}{(1-\mu^2)R}\left(\frac{\partial u}{\partial\varphi}+w\right). \quad (12)$$

where E – elasticity modulus of a tube material; $J(\varphi)$ – second area moment, in any coordinate φ; $K(\varphi)$ – Karman constant in any coordinate φ; μ – Poisson's ratio; $S(\varphi)$ – cross-sectional area in any coordinate φ.

Considering the tip weight in a boundary condition on a free MTS end the orthogonality condition will be presented as follows:

$$\begin{cases}\int_0^\gamma m(\varphi)\frac{\partial^2 w}{\partial t^2}\xi_i d\varphi+\frac{m_{нак}}{R}\frac{\partial^2 w}{\partial t^2}\xi_i(\gamma)+\int_0^\gamma\left(1+\frac{b}{R}\right)\beta\frac{\partial w}{\partial t}\xi_i d\varphi+ \\ +\int_0^\gamma\frac{\partial}{\partial\varphi}\left(H_R\left(\frac{\partial u}{\partial\varphi}-\frac{\partial^2 w}{\partial\varphi^2}\right)\right)\frac{\partial\xi_i}{\partial\varphi}d\varphi+\int_0^\gamma D_R\left(\frac{\partial u}{\partial\varphi}+w\right)\xi_i d\varphi=0, \\ \int_0^\gamma m(\varphi)\frac{\partial^2 u}{\partial t^2}\psi_i d\varphi+\frac{m_{нак}}{R}\frac{\partial^2 u}{\partial t^2}\psi_i(\gamma)-\int_0^\gamma\frac{\partial}{\partial\varphi}\left(H_R\left(\frac{\partial u}{\partial\varphi}-\frac{\partial^2 w}{\partial\varphi^2}\right)\right)\psi_i d\varphi+ \\ +\int_0^\gamma D_R\left(\frac{\partial u}{\partial\varphi}+w\right)\frac{\partial\psi_i}{\partial\varphi}d\varphi=0.\end{cases} \quad (13)$$

where $H_R=\frac{EJ(\varphi)K(\varphi)}{(1-\mu^2)R^4}$; $D_R=\frac{ES(\varphi)}{(1-\mu^2)R^2}$;

Substituting the motion approximation (2) into (13) the system 2n of standard linear differential equations with the second-order constant coefficients in unknown functions $a_1(t)...a_n(t), b_1(t)...b_n(t)$, is presented as follows:

$$a_n(t)=c_n e^{rt},$$
$$b_n(t)=f_n e^{rt},$$

where c_n, f_n and r – unknown constants.

First and second derivative:

$$\dot{a}_n(t)=rc_n e^{rt},$$
$$\ddot{a}_n(t)=r^2c_n e^{rt},$$

$$\dot{b}_n(t) = r f_n e^{rt},$$
$$\ddot{b}_n(t) = r^2 f_n e^{rt}/$$

The nontrivial solution is possible when the main determinant of the system equals zero. Let us write down the determinant as follows:

$$\begin{vmatrix} F_{1,1} & \dots & F_{1,n} & C_{1,1} & \dots & C_{1,n} \\ \dots & & \dots \quad \dots & \dots & \dots & \dots \\ F_{2n,1} & \dots & F_{2n,n} & C_{2n,1} & \dots & C_{2n,n} \end{vmatrix} = 0 \qquad (14)$$

where

$$F_{1,1} = r^2 m(\varphi)\int_0^{\gamma} \xi_1\xi_1 d\varphi + r^2 \frac{m_{нак}}{R}\xi_1\xi_1 + r\left(1+\frac{b}{R}\right)\beta\int_0^{\gamma}\xi_1\xi_1 d\varphi + D_R\int_0^{\gamma}\xi_1\xi_1 d\varphi - H_R\int_0^{\gamma}\frac{\partial^3\xi_1}{\partial\varphi^3}\frac{\partial\xi_1}{\partial\varphi}d\varphi,$$

$$F_{1,n} = r^2 m(\varphi)\int_0^{\gamma} \xi_n\xi_1 d\varphi + r^2 \frac{m_{нак}}{R}\xi_n\xi_1 + r\left(1+\frac{b}{R}\right)\beta\int_0^{\gamma}\xi_n\xi_1 d\varphi + D_R\int_0^{\gamma}\xi_n\xi_1 d\varphi - H_R\int_0^{\gamma}\frac{\partial^3\xi_n}{\partial\varphi^3}\frac{\partial\xi_1}{\partial\varphi}d\varphi,$$

$$C_{1,1} = D_R\int_0^{\gamma}\frac{\partial\psi_1}{\partial\varphi}\xi_1 d\varphi + H_R\int_0^{\gamma}\frac{\partial^2\psi_1}{\partial\varphi^2}\frac{\partial\xi_1}{\partial\varphi}d\varphi,$$

$$C_{1,n} = D_R\int_0^{\gamma}\frac{\partial\psi_n}{\partial\varphi}\xi_1 d\varphi + H_R\int_0^{\gamma}\frac{\partial^2\psi_n}{\partial\varphi^2}\frac{\partial\xi_1}{\partial\varphi}d\varphi,$$

$$F_{2n,1} = D_R\int_0^{\gamma}\xi_1\frac{\partial\psi_{2n}}{\partial\varphi}d\varphi + H_R\int_0^{\gamma}\frac{\partial^3\xi_1}{\partial\varphi^3}\psi_{2n}d\varphi,$$

$$F_{2n,n} = D_R\int_0^{\gamma}\xi_n\frac{\partial\psi_{2n}}{\partial\varphi}d\varphi + H_R\int_0^{\gamma}\frac{\partial^3\xi_n}{\partial\varphi^3}\psi_{2n}d\varphi,$$

$$C_{2n,1} = r^2 m(\varphi)\int_0^{\gamma}\psi_1\psi_{2n}d\varphi + r^2\frac{m_{нак}}{R}\psi_1\psi_{2n} + D_R\int_0^{\gamma}\frac{\partial\psi_1}{\partial\varphi}\frac{\partial\psi_{2n}}{\partial\varphi}d\varphi + H_R\int_0^{\gamma}\frac{\partial^2\psi_1}{\partial\varphi^2}\psi_{2n}d\varphi,$$

$$C_{2n,n} = r^2 m(\varphi)\int_0^{\gamma}\psi_n\psi_{2n}d\varphi + r^2\frac{m_{нак}}{R}\psi_n\psi_{2n} + D_R\int_0^{\gamma}\frac{\partial\psi_n}{\partial\varphi}\frac{\partial\psi_{2n}}{\partial\varphi}d\varphi + H_R\int_0^{\gamma}\frac{\partial^2\psi_n}{\partial\varphi^2}\psi_{2n}d\varphi.$$

From this condition, let us determine *r*. The change of vibration damping parameters at various number of functions within the motion approximation was studies to assess the convergence of the solution. For satisfactory results, it was sufficient to ensure five basic functions (Fig. 2, 3).

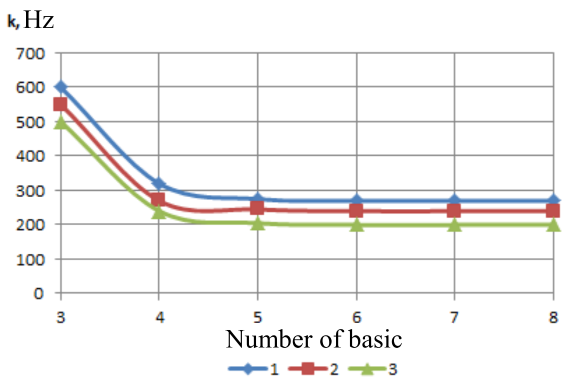

Fig. 2. Change of vibration frequencies

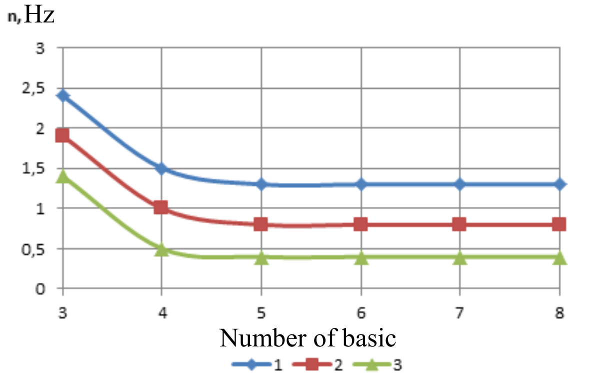

Fig. 3. Change of vibration damping coefficient

The MANOMETER software package was designed to calculate the damping parameters. Fig. 4 shows its interface.

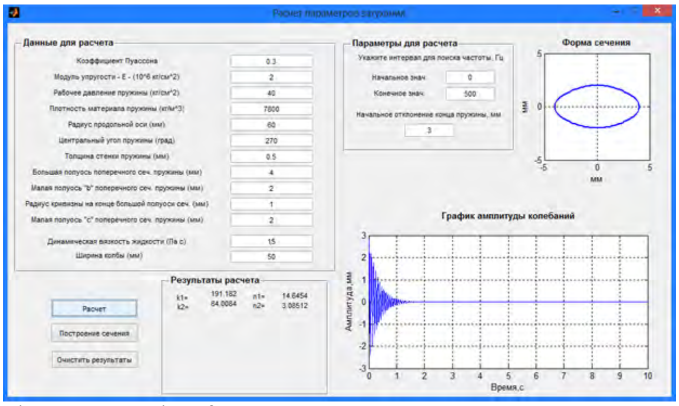

Fig. 4. Program interface

The designed mathematical model considers the damping liquid as a continuous medium with the absolute viscosity coefficient being its key feature. Let us accept the following: the cultivated soil will be considered as a continuous medium.

The work [10] describes the obtained experimental dependence (15), which makes it possible to determine the absolute viscosity coefficient depending on absolute humidity of middle loamy soils.

$$\eta = 5627{,}569 \cdot \omega^2 - 722124{,}877 \cdot \omega + 1{,}598 \cdot 10^7 \qquad (6)$$

Let us use the experimental absolute viscosity coefficient at the following humidity (Tab. 1).

TABLE I. DEPENDENCE OF VISCOSITY ON ABSOLUTE HUMIDITY OF MIDDLE LOAMY SOILS

Absolute humidity, %	Absolute viscosity coefficient, 10^6 Pa*s
10	12
15	6
20	4
25	2

The results of the study regarding the influence of a tip weight on the frequency of damping vibrations are shown in Fig. 2.

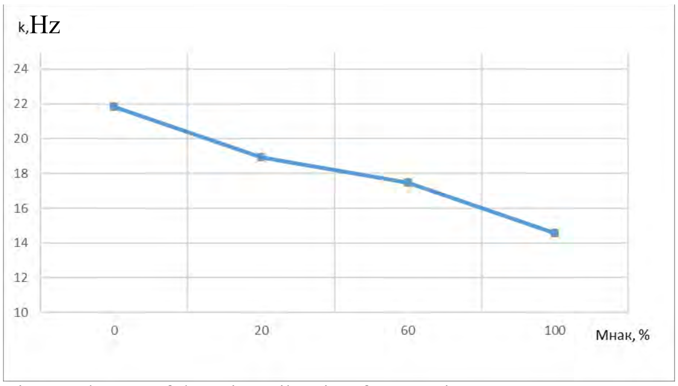

Fig. 5. Change of damping vibration frequencies.

The analysis of the given dependence shows that the increase of a tip weight leads to the decrease in the frequency of damping vibrations. The tip weight does not influence the damping vibration coefficient.

IV. CONCLUSIONS

The aperiodic motion will be observed with the stated dynamic viscosity of the loamy soil. The absolute viscosity resulting in aperiodic motion (25 Pa*s) is defined. Such absolute viscosity is reached at 100% of absolute humidity of a loamy soil, and thus makes it possible to conclude that during operation of a cultivator the aperiodic motion will only be observed.

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