Abstract—Since the 2000s, we can talk about rapid development of high technologies. Technically complex devices equipped with a variety of systems and sensors became an essential part of our daily life and their absence is difficult to imagine.

Technological processes, which until recently were carried out only with the help of manual labor, are increasingly replaced by automated and robotic systems that allow one to increase the efficiency of a particular production.

It is impossible to imagine modern engineering, medicine or industry without such systems. The agro-industrial complex of the Russian Federation is not an exception.

Robotic systems are becoming more and more popular in the field of dairy production. They perform both milking and preparation of the animal's udder for milking. Such operations are quite difficult to perform for robotic devices, because they have to work with a living creature, which behavior is not predictable, and does not fit into the programmed algorithm. However, every year the interest in such installations is growing.

One of the most important aspects in solving this problem is equipping robotic installations with systems of their working bodies positioning – manipulators with high accuracy and efficiency.

Keywords—dairy cattle breeding; robot; optical positioning; manipulator
udder washing and massage, wiping the udder dry and milking-off the first trickles of milk.

The first two operations are not difficult to mechanize and automate, and as for the last two ones it would be better for the operator of a milking parlor to control the quality of the whole technological operation of pre-milking udder preparation. The most important operations here are udder washing and massage. It is their correct performing that ensures milking reflex, intensity and completeness of milking. To perform this operation perfectly and with minimal labor costs, it is necessary to use pre-milking udder preparation installation, which work is easiest to coordinate with the milking unit of "Carousel" type.

The most promising ones are robotic installations with a positioning system of the manipulator. The udder preparation installation is a set of the machine and the manipulator with a working body directly engaged in washing and massage of udder. Partial robotization of pre-milking, the udder preparation process has a number of advantages in comparison with manual operations [12, 13].

An important part of any robotic system is a positioning system controlling its mechanical elements. There are many principles and approaches to its implementation. It is especially difficult to create it in systems used for working with animals. Due to the increasingly common use of milking robots, this task has become particularly relevant. Effective operation of the positioning system will ensure optimal work of the manipulator serving an animal.

II. STATEMENT OF THE RESEARCH PROBLEM

When comparing the advantages and disadvantages of box milking robots and milking parlors, the most promising thing, in our opinion, is the combination of traditional milking systems with robotic elements performing individual technological operations. This combination allows one not to stop milking parlour operation, even in case of robots’ breakage. In addition, it enables one to refine in more detail the design of a mechanical part and positioning systems, which will permit complete removal of milking parlor operators in future.

It should be noted that there are practically no such developments in Russia, and the first step in the transition to robotic milking on large dairy farms and complexes can be the installation of pre-milking preparation of udder [12, 13].

Thus, the first robotic milkers, which appeared in 2007 in Russia, were designed to service a small number of animals, three installations were intended for 187 animals [14]. Currently, the company "DeLaval" developed a project of a robotic farm for 5000 heads but growth of the serviced livestock caused an increase in the number of robots involved in milking process [14].

Milking is not the only example of using robots in agriculture and animal husbandry in particular. The company "Pellonpaja" presented a series of bunker robots for distribution of Pellon feed. Such systems are also proposed by the companies "Mullerup A/S", "Delaval", "Trioliet". The company "Lely", developed a robot performing the shift of feed [15].

In crop production, robotic systems have also found their application. Robot BoniRob (Bosch) independently works in the fields and cleans weeds. Robot Prospero (USA) was created for automatic planting. Machines have been created for harvesting berries and plants [16].

However, despite the variety of robotic devices created, it should be noted that all of them are highly specialized. Almost all known robots are designed to perform one specific task [16]. A major difficulty in achieving the universality of robots is designing their "senses", that is, systems carrying out purposes determination along with their classification, navigation, and positioning.

In the development of robotic technology, different types of sensors are used. They are the basis of robot’s "senses", and they all use physical principles underlying their work.

Thus, designing of robotic milking systems required development of a system that accurately determines the coordinates of udder teats. Most manufacturers of milking robots use a laser as a basis for the positioning system of manipulators.

For aiming a working body of udder preparation installation, we assume using an optical scheme. Preliminary studies show the possibility of its application. [13, 14, 17]

In addition, this method will create a dynamic system that will reduce injury of animals and improve aiming accuracy in mechanical elements positioning of robotic installation. However, at the first stage, it is necessary to determine theoretically the coordinates of cow’s udder in the coordinate system related to the robotic installation of pre-milking udder preparation in order to develop the algorithm and reduce the aiming error of the manipulator optical positioning system.

III. AIM OF THE RESEARCH

Theoretical determination of animal’s udder coordinates under the optical method of manipulator positioning in the coordinate system is related to the robotic installation of pre-milking udder preparation.

IV. RESEARCH RESULTS

The optical positioning system uses an image obtained from two cameras, so it is necessary to solve the task, which is to get equations relating the coordinates of an aiming object with the displacement of the image relative to the center of the image obtained from cameras.

We consider the Cartesian coordinate system related to the elements of the unit by pointing the x axis along one of the walls and the object located in the camera field (fig. 1). In this case, the coordinates $x_{object}$ and $y_{object}$ are to be determined.
Fig. 1. Coordinate system related to the installation of pre-milking udder preparation.

The planes in which cameras are located form an oblique-angled coordinate system with a vertex at point $O'$. In this case, the position of the object is defined by two radius vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. $\mathbf{r}_1$ is the radius-vector of the beginning of the oblique-angled coordinate system $x'O'y'$ and $\mathbf{r}_2$ is the radius-vector of the aiming object relative to the beginning of the oblique-angled coordinate system (fig. 2).

Let us define coordinates of both vectors. To do this, we will mark the plane of the unit, indicating coordinates of main elements.

According to the notations selected, cameras have coordinates $K_1(x_0; d + d_0)$ and $K_2(x_0; -d_0)$. The coordinates of vector $\mathbf{r}_1$ will coincide with the coordinates of point $O'$. The coordinates of this point can be defined as the point of intersection of two lines lying in the intersecting planes of the survey. To do this, it is necessary to make equations of these straight lines. We apply the equation of a straight line passing through one point with a known angular coefficient:

$$y - y_0 = k \cdot (x - x_0).$$

Angular coefficient $k$ determines the angle of a straight line relative to the positive direction of the $x$ axis:

$$k = \tan \varphi.$$

For the straight lines selected, we obtain two angular coefficients:

$$k_1 = \tan \alpha;$$

$$k_2 = -\tan \beta.$$

And, accordingly, two equations:

$$\begin{cases}
y - (d + d_0) = \tan \alpha (x - x_0), \\
y - (-d_0) = -\tan \beta (x - x_0).
\end{cases}$$

Solving the system related to unknown coordinates $x$ and $y$, we obtain coordinate values of point $O'$ and, accordingly, coordinate values of radius vector $\mathbf{r}_1$:

$$\begin{cases}
x_1 = \frac{x_0 (\tan \alpha + \tan \beta) - d - 2d_0}{\tan \alpha + \tan \beta}, \\
y_1 = \frac{\tan \beta (d + d_0) - d_0 \tan \alpha}{\tan \alpha + \tan \beta}.
\end{cases}$$

Fig. 2. Position of radius-vectors $\mathbf{r}_1$ and $\mathbf{r}_2$.

Fig. 3. a – position of the survey object in the oblique-angle coordinate system; b – survey object image displacement relative to the center of the image.
Let us consider radius vector $\vec{r}_2$ in the coordinate system $x'O'y'$. This coordinate system is oblique and the coordinates of vector $\vec{r}_2$ in this coordinate system will be determined by the points $x'_2$ and $y'_2$ (fig. 3a). By defining coordinates $x'_2$ and $y'_2$, we can define the coordinates of vector $\vec{r}_2$ in the Cartesian coordinate system $xOy$.

To determine the coordinates of vector $\vec{r}_2$ in the oblique-angle coordinate system, we must associate them with the displacement of the survey object relative to the optical axis of the camera. These displacements are related to the displacement of the object image and are determined by optical characteristics of the camera:

$$\frac{H}{h} = \frac{b}{a},$$

where $h$ is an image displacement (fig. 3b), $b$ is the distance from the object to the camera, $a$ is the distance from the optical center of lens to the camera matrix.

The camera lens is an optical system with a certain focal length $F$, for which the formula of a thin lens is performed:

$$\frac{1}{F} = \frac{1}{a} + \frac{1}{b}.$$  

Combining equations (2) and (3), we obtain:

$$\frac{1}{F} = \frac{1}{b}\left(\frac{H}{h} + 1\right).$$

The relation of coordinates $x'_1$ and $y'_1$ is determined on the basis of geometric considerations (fig. 4):

For the convenience of calculations lengths, $O'K_1$ and $O'K_2$ are defined as: $O'K_1 = d_1$ and $O'K_2 = d_2$. These lengths can be calculated by knowing the coordinates of extreme points. Cameras have a rigidly fixed position in the Cartesian coordinate system at points with coordinates $K_1(x_0; d + d_0)$ and $K_2(x_0; -d_0)$, and the coordinates of the point $O'$ are determined by formulas (1):

$$O'K_1 = d_1 = \sqrt{(x_0 - x_1)^2 + (d + d_0 - y_1)^2},$$

$$O'K_2 = d_2 = \sqrt{(x_0 - x_1)^2 + (-d_0 - y_1)^2}.$$

Then the coordinates of points $x'_2$ and $y'_2$ in the coordinate system will have values:

$$\begin{align*}
x'_2 &= d_2 - \frac{b_2}{\tan \phi}, \quad H_2 = d_2 - \frac{b_2}{\tan \phi} - x'_2, \\
y'_2 &= d_1 + \frac{b_1}{\tan \phi}, \quad H_1 = y'_2 - d_1 + \frac{b_1}{\tan \phi}.
\end{align*}$$

Considering this and the expression for a thin lens (4), we obtain:

$$\begin{align*}
\frac{1}{F} &= \frac{1}{b_1}\left(\frac{y'_2 - d_1 + \frac{b_1}{\tan \phi}}{h_1} + 1\right) \Rightarrow \\
&\Rightarrow b_1 = \frac{1}{F}\left(\frac{y'_2 - d_1 + \frac{b_1}{\tan \phi}}{h_1} + 1\right) \\
\frac{1}{F} &= \frac{1}{b_2}\left(\frac{d_2 - \frac{b_2}{\tan \phi} - x'_2}{h_2} + 1\right)
\end{align*}$$

According to figure 4, the relation of lengths $b_1$ and $b_2$ with coordinates $x'_1$ and $y'_1$ will be expressed as follows:

Fig. 4. Relation of the survey object coordinates to its displacement relative to the camera axis
\[ \begin{align*}
\sin \varphi &= \frac{b_1}{x_2}, \\
\sin \varphi &= \frac{b_2}{y_2}
\end{align*} \Rightarrow \begin{align*}
x_2' &= \frac{b_1}{\sin \varphi}, \\
y_2' &= \frac{b_2}{\sin \varphi}
\end{align*} \tag{7}

Combining equations for a thin lens (5) and (6) with the system of equations (7), we obtain:

\[ \begin{align*}
\frac{1}{F} \left( \frac{b_1}{\sin \varphi} - d_1 + \frac{b_1'}{tg \varphi} \right) &= b_1, \\
\frac{1}{F} \left( \frac{d_2 - b_2}{\tan \varphi} - \frac{b_2'}{\sin \varphi} \right) &= b_2.
\end{align*} \tag{8}

The resulting system is reduced to a linear form and is solved in relation to unknown \( b_1 \) and \( b_2 \):

\[ \begin{align*}
b_1 \left( Fh_1 - \frac{1}{tg \varphi} \right) - \frac{b_2}{\sin \varphi} &= h_1 - d_1, \\
b_1 \left( Fh_2 + \frac{1}{tg \varphi} \right) + \frac{b_1'}{\sin \varphi} &= d_2 + h_2.
\end{align*} \tag{9}

For ease of calculations, we denote: \( M_1 = Fh_1 - \frac{1}{tg \varphi} \), \( M_2 = Fh_2 + \frac{1}{tg \varphi} \), \( N = \frac{Fh_1}{\sin \varphi} \); then the system (8) takes the following form:

\[ \begin{align*}
b_1 M_1 - b_2 N &= h_1 - d_1, \\
b_2 N + b_1 M_2 &= h_2 + d_2.
\end{align*} \tag{10}

Values \( b_1 \) and \( b_2 \) are the solution of the system (9):

\[ \begin{align*}
b_1 &= \frac{M_2 (h_1 - d_1) + N (h_2 + d_2)}{M_2 M_1 + N^2}, \\
b_2 &= \frac{M_1 (h_1 + d_1) + N (h_2 - d_2)}{M_2 M_1 + N^2}.
\end{align*} \tag{11}

Resulting coordinates allow us to find the module of vector \( \vec{r}_2 \). We can find the module value with the help of the cosine theorem (fig. 4):

\[ |\vec{r}_2| = \sqrt{x_2'^2 + y_2'^2 + 2x_2' y_2' \cos \varphi}. \tag{12}\]

Fig. 5. Angle between vector \( \vec{r}_2 \) in the oblique-angle coordinate system and the positive direction of \( x \) axis in the coordinate system related to the installation.

We can find the coordinates of vector \( \vec{r}_2 \) in the Cartesian system related to the unit, as the projection of this vector in the horizontal and vertical directions. These projections depend on the angle \( \Theta \), which value is equal to (fig. 5): \( \Theta = \beta - \varphi' \).

Based on geometric relations, the value of angle \( \varphi' \) is found from a right triangle formed by the \( x' \) axis, segment \( b_2 \) and vector \( \vec{r}_2 \) (fig. 5):

\[ \sin \varphi' = \frac{b_2}{|\vec{r}_2|} \Rightarrow \varphi' = \arcsin \left( \frac{b_2}{|\vec{r}_2|} \right) \]

Thus:

\[ \Theta = \beta - \arcsin \left( \frac{b_2}{|\vec{r}_2|} \right). \tag{13}\]

The ratio (12) takes into account the absolute value of the angle. According to figure 5, we can see that it runs in the negative direction. To take it into consideration in the projections, we add a minus sign, i.e.:
\[
\Theta = - \left( \beta - \arcsin \left( \frac{b_x}{|F_2|} \right) \right) 
\] (14)

Then projections of vector \( \vec{r}_2 \) on the axis of the Cartesian coordinate system will have the form:

\[
\begin{align*}
    x_2 &= |F_2| \cos \Theta, \\
    y_2 &= |F_2| \sin \Theta.
\end{align*}
\] (15)

Thus, we can find the coordinates of the object under consideration in the coordinate system related to the unit as the sum of two vectors \( \vec{r}_1 \) and \( \vec{r}_2 \):

\[
\begin{align*}
    x_{\text{object}} &= x_0 (tg \alpha + tg \beta) - d - 2d_s |F_2| \cos \Theta, \\
    y_{\text{object}} &= \frac{tg \beta (d + d_s) - d_s tg \alpha}{tg \alpha + tg \beta} + |F_2| \sin \Theta.
\end{align*}
\] (16)

V. CONCLUSIONS AND SUGGESTIONS

The resulting expressions (16) are suitable for a single object which is in the view of two cameras. In case of fourudder teats, these formulas can be applied for each teat separately. Along with it, an images’ shift relative to the center of camera images will be determined for each teat separately. It should also be noted that different teats, depending on the size of an udder and the size of an animal, can be shifted differently relative to the camera axis. Therefore, this factor must be taken into account by correcting the equations obtained for different variants of a teat location relative to the camera axis.

References