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The Application of Mathematical Modeling to Predict the Financial Health of Bussinesses

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***Abstract*—The prediction of the future state of the enterprises has an irreplaceable role in corporate management; this is often the difference between the existence of a business and its future termination. Therefore, it is appropriate for businesses to know sufficiently far ahead of time about possible business failures. For more than eighty years of research several dozen methods to predict the financial health of businesses have been developed. The first part of this paper focuses on their comprehensive classification. Apart from statistical methods aim of the paper is to define, describe and analyze the use of mathematical methods to predict financial health; to explore their advantages and disadvantages.**

Keywords—Financial health; Prediction models; Linear programming; Data envelompment analysis; Linear goal programming

I. INTRODUCTION

The first works on the issue of financial health predictions began to emerge in the 1930s during the Great Depression. According to Altman [1], Smith and Winakor [2] were the first authors to seriously address bankruptcy prediction in their studies. Merwin [3] added to their knowledge. Both studies have pointed out that defaulting enterprises show significantly different values of selected financial ratios compared to successful enterprises. This basic principle was a huge breakthrough and offered considerable prospects for further research. Studies focusing on the subject matter include works of P.J. Fitzpatrick [4] that dealt with significant differences between successful and unsuccessful enterprises.

This work has become an inspiration for many applied studies that began to appear in the mid-60s of the 20th century. The most significant are the studies of W. H. Beaver [5], E. I. Altman [6], J. A. Ohlson [7], M. Tamari [8], J. G. Fulmer [9], M. E. Zmijewski [10]; T. Argenti [11] or Ch. Zavgren [12].

The studies mentioned above are traditional prediction models based on statistical methods. In addition to these, there are methods using mathematical modelling and optimization of objective function [13, 14]. These methods provide similar results as traditional models but remove some of their shortcomings. The aim of the paper is to define, describe and analyze the use of mathematical methods to predict financial health; to explore their advantages and disadvantages.

II. BUSINESS FINANCIAL HEALTH PREDICTION

A. *Methods of Business Financial Health Prediction*

As it has been mentioned, the first attempts to construct models of business financial health prediction can be dated back to the first third of the twentieth century and this process continues to this day. With the development of computing, but of course also with the increasing level of knowledge in recent years, this process has considerably speeded up [15- 17]. The goal of this subchapter is not to provide a comprehensive and definitive description of these methods or their classification, it is rather a simple description of their basic features, advantages and disadvantages.

It is provided an overview of the classification of methods used to predict the financial health of companies from selected authors. At first glance it is obvious that different methods overlap each other. The reason is prosaic, since the authors in their list reflect not only the division of the models themselves but also their historical development.

McKee [18] classifies methods or models into twelve groups. On the other hand, Moghaddam [19] has divided the methods into three main groups, which he further structured internally to eleven subgroups. In their study Balcaen and Ooghe [20] maps the development of predictive models over 35 years and classifies the methods as follows:

1. Statistical models: univariate analysis, risk index models, multiple discriminant analysis, conditional probability analysis.
2. Artificial intelligence models: survival analysis, decision trees, neural network
3. Alternative models: fuzzy rules-based classification model, multi-logit model, CUSUM model, dynamic event history analysis (DEHA), catastrophe theory or chaos theory model, multidimensional scaling (MDS), linear goal programming (LGP), multi-criteria decision aid approach (MCDA), rough set analysis, expert systems, self-organizing maps (SOM)

Based on multiple research and comparative studies, the author decided to classify prediction models as five mutually consistent groups, which are further broken down as is shown in Fig. 1.

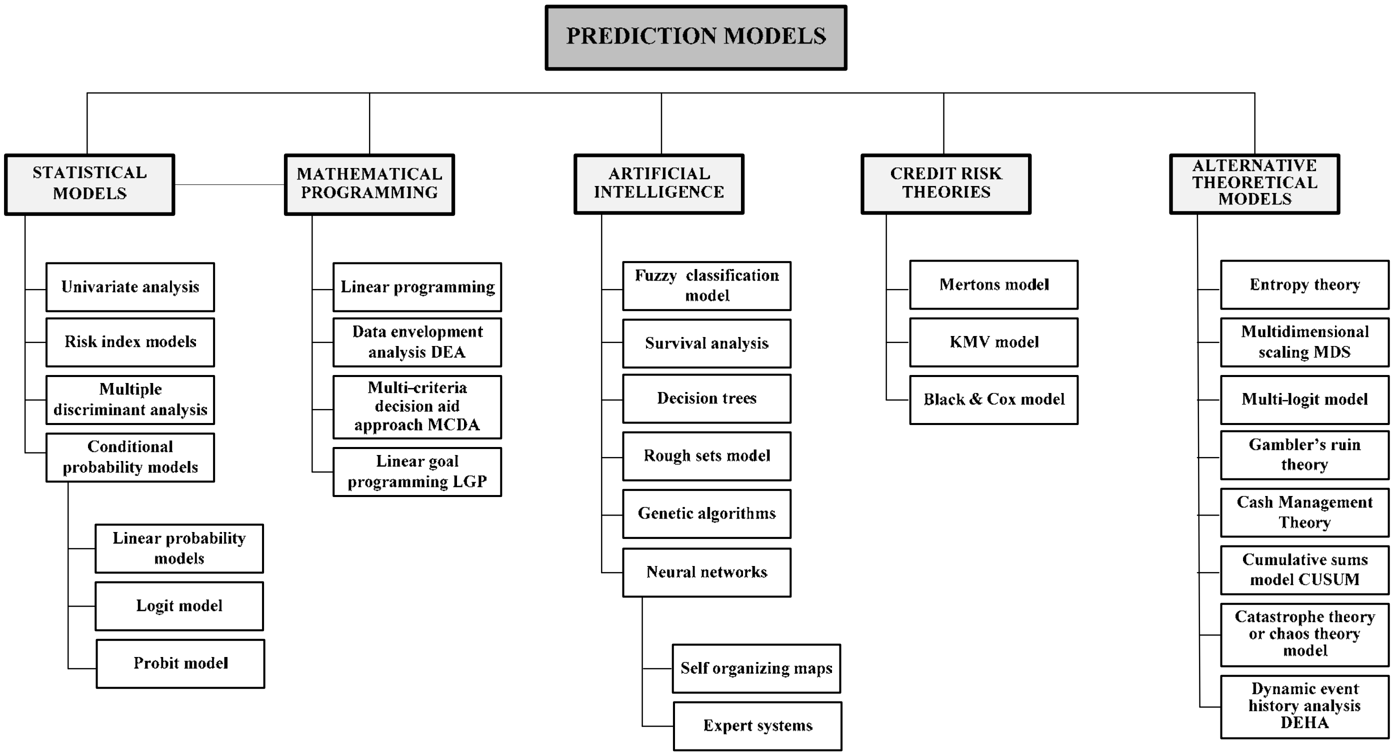

Fig. 1 Classification of prediction models.

As can be seen from Fig. 1, there are at least five major predication models and more than twenty subgroups. Statistical models are among the best known and most widely used prediction models, but in addition to these, there are other methods that provide similar and / or better results (prediction of future financial distress). One such method is mathematical modeling.

B. *Mathematical Modeling to Predict the Financial Health*

Mathematical programming methods (abbreviated MPM) allow you to predict the future financial situation of an enterprise based on the optimization of the criteria-based function. For example, if the ratio of correctly classified entities is set as a criteria-based function, it could be able to transfer the issue to an optimizing task of mathematical programming. Hand [21] offers the use of linear programming to maximise the score corresponding to the entities that have been correctly classified. Comparative studies dealing with MPM and logistic regression usually show that the results of model quality are very similar. Logistic regression, however, has a better position considering comfort and distribution.

Linear programming

The task of linear programming is of a special type that searches for the bounds to the function extreme. This is a situation in which the function the extreme of which we are looking for is linear and, at the same time, the set, on which the extreme is being looked for is defined by a system of linear inequalities.

One of the most commonly used algorithms for solving the tasks of linear programming is the simplex method. The principle of the simplex method is to find a certain admissible solution and subsequently go through the set of all the admissible solutions. As the set of all the admissible solutions is given by the system of linear inequalities, it is an intersection of many affine final half-spaces, i.e. polyhedrons. The algorithm searches for the optimum solution by gradual movement through the peaks of the given polyhedron, i.e. the admissible solution maximizing (or minimizing) the purpose function [22]. The procedure for the tasks of maximizing character is as follows: First, an arbitrary peak of the polyhedron representing the pre-set admissible solution is found. Second, it is necessary to find out how would the value of the purpose function change if we moved from the peak to any of the edges coming out of it. If there is no positive increment of the purpose function value of any of the edges, we are already in the optimum solution. Otherwise we will select one of the edges in the direction of which the objective function is growing. If these are half lines, the task does not have any solution and it is not bound. If this is a line segment, we will move along it to another peak of the polyhedron. There the value of the purpose function is higher and we can continue repeating the previous procedure until we find the optimum solution or until we find out that the task is not bound [23]. The biggest disadvantage of the simplex method is that its time demands that are exponentially dependent on the number of inputs or on the size of an input. It is, however, polynomial time complexity that is considered to be the border of practical solvability. Thus, for more extensive tasks, the algorithm does not have to find the solution in the real time at all.

One of the first ones to apply linear programming for the predictions of financial soundness were Mangasarian [24], Freed and Glover [25], Hand [21] and Nath et al. [26]. An example of the use of linear programming for predicting the financial soundness of enterprises is the model created by Thomas [27] who divided enterprises into two groups: financially sound and failing. He solved the task or the purpose function as the minimum of the sum of absolute deviations. Another possibility is to understand linear programming as the minimization of the maximum error.

The disadvantage of applying linear programming for the prediction of enterprise financial soundness is the need to introduce a large number of new variables. To be more precise, new variables for each observation in a data file have to be created [28, 29]. In the field of prediction, data is usually extensive. This requires the creation of ten to hundred thousand new variables. The advantage of applying linear programming for the purpose of prediction of enterprise financial soundness is the possibility of simple introduction of additional conditions. By adding other inequalities, it is possible to specify particular requirements for the final prediction function. It can, for example, be required that the effect of a certain set of variables is to be bigger than the effect of other variables, etc.

Data Envelopment Analysis (DEA)

Recently new procedures started to be applied to measure the relative effectiveness of enterprises. The set of such procedures, methods and algorithms is generally called Data Envelopment Analysis (DEA). These are the methods representing a special area of linear programming application. The DEA is a linear programming method initially developed for reviewing management efficiency and non-profit institution planning (e.g. schools, hospitals, etc.). Later on it started to be also used in other fields. Using DEA models, we can compare different enterprises on the basis of their work efficiency. We can, however, also compare the efficiency of work of organisational units within a single enterprise (e.g. the efficiency of work of individual bank branches and sub-branches). These methods are based on solving a system of linear programming tasks that determine the ratio efficiency of a set of enterprises. The basic ideas come from Malmquist [30]. These were later reformulated by A. Charnes, et. al. [31] and R. D. Banker, et al. [32]. Zhu [33] may be currently considered to be the most influential author publishing in the given field.

The objective of the given methods is to eliminate or even exclude the subjectivity by measuring outputs in relation to the inserted inputs. The process of selecting inputs and outputs identified for benchmarking purposes makes the process of analysis objective and limits subjectivity. Using a linear mathematical model the weights expressing the efficiency of a relevant enterprise are assigned to the inputs and outputs of individual enterprises. The models valuing relevant enterprises are of the same design. Yet with a different efficiency they will have different weight values. Based on the given weights the enterprises will be compared and ranged. Since the weights are index numbers it does not matter in what units they are expressed.

Calculation-wise all the models can be oriented either on inputs (input-oriented) or on outputs (output-oriented). As for the input oriented models we evaluate the efficiency of enterprises on the basis of input variables (total assets, operating costs, headcount, etc.). Those enterprises whose optimum value of the purpose function equals one, work efficiently within the group of enterprises under observation and those enterprises that have the optimum value of the purpose function lower than one, work inefficiently [34, 35]. The value then shows the need of proportional reduction (i.e. improvement) of inputs so that the given non-effectively working enterprise becomes efficient, i.e. using DEA models one can not only define the percentage of efficiency of enterprises but, most importantly, one can acquire the information on how such enterprises should "improve" their activities in order to become more efficient. On the other hand, as for the output-oriented models, we evaluate the efficiency of enterprises based on output quantities (revenues, profit, production volume, etc.). Those enterprises whose the optimum value of the purpose function equals one, work efficiently within the group of enterprises under observation and those that have the optimum value of the purpose function higher than one work inefficiently. In the case of the input-oriented models, an increase in some or all output quantities shall be regarded as an "improvement" of activities of inefficiently working enterprises.

The first ones to propose the use of DEA analysis for the financial analysis of an enterprise or for the prediction and evaluation of enterprise financial soundness were Cielen et al. [36], Premachandra et al. [37] or Sueyoshi and Goto [38].

The hotspot of DEA analysis application to predict the financial soundness of enterprises is the selection of the input and output variables. Should there be absolute or relative indicators of financial analysis or their combination? The authors state that there should not be more than ten indicators and that, prior to performing the analysis, correlation analysis should be conducted in order to eliminate any useless redundancy of variables. The example we state is the model of Premachandra et al [37] who used nine variables.

Input Variables

- Total debt / Total assets
- Short-term account payables / Total assets

Output Variables

- Cash flow / Total assets
- EAT / Total assets
- Working capital / Total assets
- Short-term assets / Total assets
- EBIT / Total assets
- EBIT / Interest cost
- Market value of equity / Book value of equity

When compared to the statistical approaches, DEA has the following unique properties which make it a successful tool for corporate failure prediction. DEA does not require the meeting of particular statistical preconditions of the variables used in the model. DEA may process various inputs and outputs in a single mathematical model. Moreover, it quantifies the efficiency of each of the enterprises separately and, in the form of a score, determines its ratio of efficiency or inefficiency when compared with other enterprises [39, 40]. On the basis of Malmquist's index, the ratio of work efficiency of individual enterprises can even be monitored time-wise, i.e. DEAs make it possible to review the dynamic change of enterprise success or failure in a period of time. DEA does not need any substantial sizes of samples to evaluate insolvency which are usually required by statistical and econometric approaches. Moreover, DEA answers the question about the steps to be taken by enterprises in order to become effective, i.e. how and by how much they should reduce the input parameters or increase the output parameters to minimise the risk of failure in the future. A disadvantage of the analysis, on the other hand, may be a small set of enterprises required for analysis purposes, on the basis of which it is difficult to generalise results. Another disadvantages may be calculation demands. And, finally, another disadvantage is the ambiguity related to the selection of the input and output variables of DEA models used to predict the financial soundness of enterprises.

Linear Goal Programming (LPG)

Gupta et al. [41] have applied the LGP method to the issue of bankruptcy classification. LGP is one of several methods derived from mathematical programming. The LGP model formulates the differences within a group and between the groups of failing and non-failing enterprises and, based on such differences, determines a score for each enterprise and boundary points for the discrimination of the both groups. It creates the hyper-plane by having the observations that are lying within the boundaries are at the same time the furthest from the boundaries. This leads to more detailed differentiation between the groups. The boundary point is determined by: (i) Maximisation of the weighted sum of distances between observations and the modified boundary and (ii) Minimisation of the weighted sum of boundary breaches.

The LGP method has several advantages. The first one is that it does not require any limiting statistical preconditions like MDA. Moreover, it is a flexible and easy-to-understand tool.

Multi-Criteria Decision Aid Approach (MCDA)

Zopoudinis [42], Zopoudinis and Dimitras [43] applied the MCDA methods to predict enterprise failure. MCDA approach allows for the evaluation of level of enterprise risk based on financial indicators and qualitative information related to an enterprise. Zopoudinis [42], for instance, analysed several strategic criteria such as management quality, level of research and development, and market trend, and classified enterprises based on their level of risk.

MGHDIS method (Multi-Group Hierarchical Discrimination) method is another extension of MCDA method [43] that may be used as a classification model of enterprise failure. This method derives a set of two additive utility functions, one for the failing and one for the sound enterprises, and such functions are used to place enterprises in either the group of failing or non-failing enterprises. In the process of classification it is assumed that the preferences of a decision-maker related to the placing of an enterprise in the group of failing or non-failing enterprises, are monotonous functions of attributes. There are two different classification procedures. According to the first procedure, the enterprise is classified as failing if the utility of such enterprise corresponding to the function of utility of the failing enterprise group is higher than the utility corresponding to the function of non-failing enterprise utility. The second procedure of classification includes the determination of a rule of optimum classification. It represents the specification of the optimum boundary value for the difference between the effectiveness corresponding to the effectiveness function of non-failing enterprises and the effectiveness corresponding to the effectiveness function of failing enterprises (difference in effectiveness) which results in the best possible classification result of MGHDIS model. According to the optimum classification rule, an enterprise is classified as failing when the difference of effectiveness is lower than the optimum boundary value. The most critical advantage of the MCDA and MGHDIS methods is the fact that they make it possible to evaluate the risk of enterprise failure using quantitative as well as qualitative information.

III. CONCLUSION

Business failures are part of the life cycle of each economy. The way in which an enterprise is confronted with deterioration in financial health depends on the excellence of corporate governance. The most favorable option is to prevent the financial crisis. Financial health predictions models have more than 80 years of development; from simple statistical methods to artificial intelligence imitating human thinking.

The presented paper was focused on the application of mathematical modeling to the prediction of financial health. The four main categories have been categorized and described: linear programming, data envelopment analysis, linear goal programming and multi-criteria decision aid approach. These methods have several undisputed advantages over statistical methods; there is no need for such a large sample of enterprises, a higher specification of the predictive function, or the inclusion of qualitative variables. All these benefits clarify the predictions of the future financial health of the company and allow for appropriate countermeasures. Whatever the method has its drawbacks, such as large calculation demands.

In summary, the methods of mathematical modeling can be a substitute for conventional prediction models; on the other hand, by using both of these methods, more accurate forecasts of the future financial position of the enterprise can be achieved.

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