Analyzing of dynamic characteristics for discrete S-PCNN

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Abstract: Pulse Coupled Neutral Network (PCNN) is a new artificial neural network with biological mechanism, and has been widely applied in many areas such as image processing and so on, but its dynamic characteristics have not been analyzed effectively. In this paper, the dynamic characteristics about pulse period, refractory period and capture period in a simplified PCNN are discussed by mathematical analysis. Simulation results verify the correctness of the analysis conclusions.

Introduction

In 1990, Eckhorn, et al [1] studied the phenomenon that the visual cortex neurons in cats' brain can burst synchronous pulses, and brought forward a linking model for this phenomenon, and then Johnson, et al [2] presented Pulse Coupled Neutral Network (PCNN) model by simplifying Eckhorn model. PCNN has been widely applied in the areas such as image segmentation [3], image fusion [4], object detection [5], optimization calculation [6] and so on.

In a PCNN, the neurons exchange messages by pulse-streams. The independent neuron fires periodical pulses, but when receiving coupled pulses from its neighbor neurons, the neuron will fire a pulse in advance. This is to say the neuron will be captured by neighbor neurons. So the neurons with similar external stimulus will burst synchronous pulses through the mutual capture effect. The effectiveness for PCNN in practical applications is closely related to model parameters, but the dynamic characteristics for PCNN have not been studied effectively. Experimental method is the most common way to set the model parameters. In literature [7], the pulse period was analyzed in continuous PCNN, not any literatures about capture characteristics by mathematical analysis can be searched. In this paper, the mathematical analysis about the dynamic characteristics in terms of pulse period, refractory period and capture period for a simplified discrete PCNN (S-PCNN) model is discussed, and the simulation results verify the correctness of the analysis conclusions.

Simplified-PCNN Model

Due to the high degree of nonlinear and complicated interaction of PCNN, it is very difficult for a qualitative analysis. Currently a popular simplified PCNN model (S-PCNN) is described with the following equations (1)-(5):

$$F_j(n) = S_j \tag{1}$$

$$L_j(n) = V_j^L \sum_k W_k^j Y_k(n-1) \tag{2}$$

$$U_j(n) = F_j(n)\left[1 + \beta_j L_j(n)\right] \tag{3}$$

$$\theta_j(n) = \theta_j(n-1)\,\mathrm{e}^{-\alpha_j^T} + Y_j(n-1)V_j^T \tag{4}$$

$$Y_j(n) = \begin{cases} 1, U_j(n) > \theta_j(n) \\ 0, \ \ otherwise \end{cases} \tag{5}$$

The S-PCNN consists of three parts: the receptive field, the modulation field and the pulse generator. In the receptive field, the neuron receives input pulses from neighbor neurons, the pulses are transmitted through the feedback input channel *F* and link input channel *L*. *W* is the linking weight matrix of synapses between neurons, S_j is the external stimulation signal, V^L is magnitude parameter of the channel *L*. In the modulation field, $U_j(n)$ is the internal activity by modulation between *L* channel and *F* channel, β is the link constant. In the pulse generator, when $U_j(n)$ is greater than the dynamic threshold $\theta_j(n)$, the neuron will fire a pulse, and then the threshold will increase to a high level, and then attenuates exponentially also, α^T and V^T are its attenuation parameter and magnitude parameter respectively.

Analysis of dynamic characteristics for S-PCNN

Pulse period: suppose the neuron fires a pulse at n_1 time, and again fires a pulse at n_2 time without coupled pulses inputs, or again fires a pulse at n_{12} time with coupled pulses inputs, then the interval $(n_1, \ n_2]$ is called non-excitation pulse period, and $(n_1, \ n_{12}]$ is called excitation pulse period, obviously, $n_{12} \le n_2$.

Refractory period and capture period: within the interval $(n_1, \ n_2]$, if the neuron receives the external pulses inputs at $n' \in [n_1', \ n_2']$, and then its internal activity *U* will increase, then the neuron will fire a pulse in advance, the interval $[n_1', \ n_2']$ can be called capture period. Due to the threshold of the neuron attenuates exponentially, $n_1' > n_1$, $n_2' = n_2$. The interval $[n_1+1, \ n_1'-1]$ is called refractory period, it means that the neuron won't fire even if it receives the external pulses at this interval.

2.1 Pulse period or pulse phase

Suppose at $n = n_1$, neuron fire a pulse at the first time. Then $Y(n_1) = 1$, and the neuron satisfies $U(n_1) > \theta(n_1)$ through the iteration, that is $U(n_1) \approx \theta(n_1)$. And suppose at $n = n_2$, the neuron fires a pulse for the second time. Because of $U(n_2) \approx \theta(n_2)$, the following equations can be derived:

$$S \approx (\theta(n_1) + V^T e^{\alpha^T}) e^{-(n_2-n_1)\alpha^T} \qquad (6)$$

$$\theta(n_2) = \theta(n_1) e^{-(n_2-n_1)\alpha^T} + V^T e^{-(n_2-n_1-1)\alpha^T} = (\theta(n_1) + V^T e^{\alpha^T}) e^{-(n_2-n_1)\alpha^T} \qquad (7)$$

$$n_2 = n_1 + \left\lceil \frac{1}{\alpha^T} \ln \frac{\theta(n_1) + V^T e^{\alpha^T}}{U(n_2)} \right\rceil \qquad (8)$$

The neuron fires a pulse for the m_{th} times at n_m, and $T(n_m)$ is the instantaneous period:

$$n_m = n_{m-1} + \left\lceil \frac{1}{\alpha^T} \ln \frac{\theta(n_{m-1}) + V^T e^{\alpha^T}}{U(n_m)} \right\rceil \qquad (9)$$

$$T(n_m) = n_m - n_{m-1} = \left\lceil \frac{1}{\alpha^T} \ln \frac{\theta(n_{m-1}) + V^T e^{\alpha^T}}{U(n_m)} \right\rceil \qquad (10)$$

When the neuron receives coupled pulses, $\theta(n_{m-1}) \approx U(n_{m-1}) = F(n_{m-1}) = S$, and $U(n_m) = F(n_m) = S$, and then the non-excitation pulse period can be derived:

$$T(n_m)=\begin{cases}\left\lceil \dfrac{1}{\alpha^T}\ln\dfrac{\theta(0)}{S}\right\rceil, m=1\\ \left\lceil \dfrac{1}{\alpha^T}\ln\left(1+\dfrac{V^T e^{\alpha^T}}{S}\right)\right\rceil, m\geq 2\end{cases} \tag{11}$$

When not receiving coupled pulses, the neuron will start to fire pulses with a stable period from the second time. The n_m at which the neuron fires a pulse for the m_{th} times can be derived:

$$n_m=\begin{cases}\left\lceil \dfrac{1}{\alpha^T}\ln\dfrac{\theta(0)}{S}\right\rceil,\ m=1\\ \left\lceil \dfrac{1}{\alpha^T}\ln\dfrac{\theta(0)}{S}\right\rceil+(\text{m-1})\left\lceil \dfrac{1}{\alpha^T}\ln\left(1+\dfrac{V^T e^{\alpha^T}}{S}\right)\right\rceil, m\geq 2\end{cases} \tag{12}$$

2.2 Refractory period and capture period of neuron

Suppose the neuron fires a pulse at n_m, if there are no external pulses inputs, the neuron will fire a pulse again at n_{m+1}. Suppose coupled pulses are input to the neuron, it will fire a pulse in advance when $t\in(n_m,n_{m+1})$.

$$t=n_m+1+\left\lceil \frac{1}{\alpha^T}\ln\frac{\theta(n_m+1)}{F(t)(1+\beta L(t))}\right\rceil=n_m+1+\left\lceil \frac{1}{\alpha^T}\ln\frac{\theta(n_m+1)}{S(1+\beta L(t))}\right\rceil \tag{13}$$

The time span of refractory period T_D and the time span of capture period T_C are:

$$T_D=(t-1)-n_m=\left\lceil \frac{1}{\alpha^T}\ln\frac{\theta(n_m+1)}{S(1+\beta V^L\sum Y_{ij}(t)W_{ij})}\right\rceil \tag{14}$$

$$T_C=(n_{m+1}-n_m)-T_D=1+\left\lceil \frac{1}{\alpha^T}\ln\frac{\theta(n_m+1)}{S}\right\rceil-\left\lceil \frac{1}{\alpha^T}\ln\frac{\theta(n_m+1)}{S(1+\beta V^L\sum Y_{ij}(t)W_{ij})}\right\rceil \tag{15}$$

The top integral function is generally ignored in equation (15):

$$T_C=1+\left\lceil \frac{1}{\alpha^T}\ln\left[1+\beta V^L\sum_j Y_{ij}(t)W_{ij}\right]\right\rceil \tag{16}$$

Simulation results and analysis

3.1 Validation test of pulse period analysis

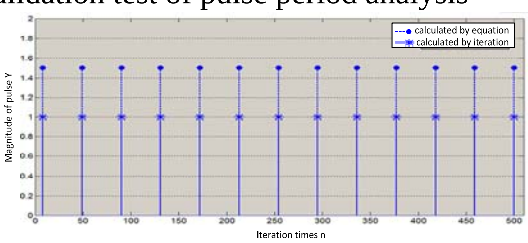


Fig.1 temporal phases of neuron firing pulses

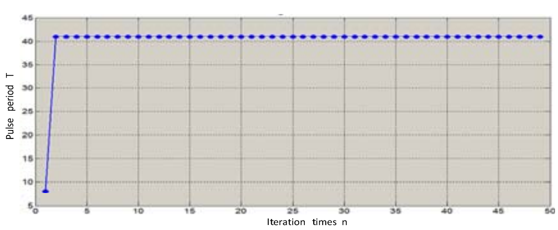


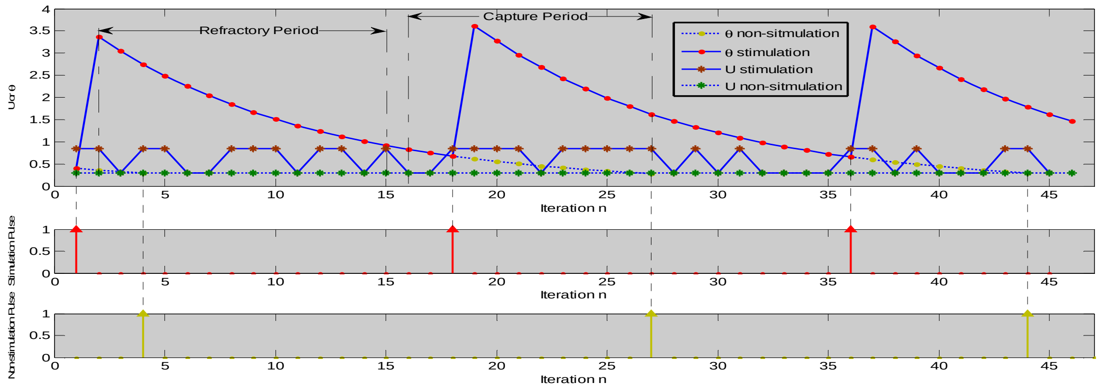
Fig.2 the changes of neuron pulse period

Set $\alpha^T = 0.1, V^T = 10, I = 0.2, \theta(0) = 0.4$, temporal phases are shown in Fig.1. There are two kinds of pulse temporal phase. One is calculated by iterations, the other is obtained by the equation (11). In order to distinguish between the two pulses, the magnitudes are set to 1 and 1.5. It can be observed the calculation results are consistent with the actual results. After firing for the second time, the neuron fires pulse with a stable period. The pulse period, $T_1 = 8$, $T_2 = 41$ can be obtained.

3.2 Validation test of capture characteristics analysis

Set $W = [1\ \ 0.5\ \ 1;1\ \ 0\ \ 1;1\ \ 0.5\ \ 1]$, $\alpha^T = 0.1, V^T = 3, V^L = 1, I = 0.3, \theta(0) = 0.4$ $\beta = 0.3$. Suppose that if the neuron receives external pulses as inputs, then it can receives pulses from its 8 neighbor neurons at the same time (i.e. $\sum Y_{ij}(t)W_{ij} = 6$). As shown in Fig.3, after the neuron firing a pulse, the threshold will increase rapidly then attenuate exponentially. If the neuron doesn't receive external pulses, its internal activity U will be a constant. Otherwise, it will increase to a high level. An attenuation interval of threshold θ is consisted of capture and refractory period. In the capture period, threshold θ has decayed below U. And if the neuron receives pulses as inputs, it will fire a pulse in advance. In the refractory period, threshold θ is still above U, so it won't fire any pulse in advance even if it receives pulses as inputs. The time span of capture period is unrelated to the external stimulation signal.

Fig.3 the changes of firing time in the cases of non-excitation and



Conclusions

This paper analyzed the dynamic characteristics for S-PCNN. The equations of the pulse period and pulse phase were deduced. It's showed that neuron will start to fire pulses with a stable period from the second time. The equations of the time span of refractory period and capture period were deduced. When the accumulation value of coupled pulses is fixed, the time span of capture period is also fixed. The experimental results showed that the calculation results by the equations are consistent with the iterations.

Acknowledgments

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