One-factor and two-factor dynamic hedging of futures contracts with different maturities for emissions allowances

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Abstract: Unexpected market information have a different speed change to market price of futures contracts with different maturities, and the paper estimates one-factor and two-factor dynamics hedge ratios and hedging effectiveness evaluation. One-factor and two-factor hedge ratios of futures contracts with different maturities for emissions allowances have time-varying trends. Compared with one-factor hedging, with an increase of span period, market participations can achieve a slight effect on risk reduction of portfolio revenues of futures contracts with different maturities by using two-factor hedge ratios, and especially two-factor hedging policy exhibits better hedging effectiveness for longer-term span period of futures contracts with different maturities for emissions allowances.

Introduction

Greenhouse gas (GHS) emissions are an ever-creasing hot topic in the 21st century for the alarming phenomena of global warming and climate deterioration. Most of scientists and politicians generally accept emissions trading scheme is a cost-effective scheme. Since the launch of European Union emissions trading scheme (EU ETS) in 2005, CO2 emissions allowances have become valuable commodities which can be traded in the CO2 emissions allowances markets. In recent years emissions allowances have become the most promising and quickly growing markets in the global commodities markets.

Brennan and Crew (1997), Schwartz (1997) propose hedge ratios of long-term and short-term commodities futures contracts are equal to the sensitivity of basic factor of commodities futures prices [1, 2]. Neuberger (1999) presents optimal hedging ratios of long-maturity and short-maturity futures contracts by using cumulative effect of linear regression method and market hedgers achieve minimum risk of commodity futures portfolio [3]. Merkoulova and Derroon (2003), Lien and Wang (2004) propose hedging policy of commodities futures contracts by using term structures of convenience yields and then market participants decrease the price risk and rolling hedging risk of commodity spot contracts [4, 5]. Larcher and Leobacher (2003) find an optimal hedging policy of long-term and short-term futures contracts when the commodity market prices follow a Gaussian process, and then investors can reduce hedging risk of futures contracts with different maturities [6]. Buhler, Korn and Schoebel (2004) provide time-varying hedge ratios under the cost-of-carry theory, Brennan- Schwartz (1985) model and Schwartz (1997) model. Their empirical results show two-factor hedge ratios are higher than one-factor hedge ratios, and then investors can decrease the downside risk of assets portfolio returns between spot and futures [7]. Hilliard and Huang (2005) propose multi-periods hedging policy for the three commercial commodities (shot oil, light oil and unleaded gasoline) and then market participants can achieve better hedging effectiveness by using minimum-variance hedging policy with mean-reversion model [8]. Leobacher (2008) find a more generalized hedging method of short-term futures contracts when commodity futures prices followed Ornstein-Uhlenbeck mean reversion process and geometric Brownian motion [9]. Compared with hedging accuracy of commodities futures contracts with the cost-of-carry theory, Prokopczuk (2011) presents that two-factor hedging policy can attain higher hedging effectiveness by using two-factor model with the introduction of stochastic convenience yields [10].
Futures contracts of emissions allowances provide risk management tools for market participants, however unexpected market information has a significant asymmetric leverage effect on futures prices of emissions allowances. Favorable and unfavorable market information can induce obvious difference on futures prices with different maturity for emissions allowances. Unexpected market information will have different speed changes in futures prices with time-varying delivery dates. Accordingly it is important for market participants that how to determine the hedge ratios accurately and that how to optimize portfolio sizes of futures assets for emissions allowances.

**Data Source**

Emissions allowances markets in European Union have existed two stages: the pilot phase (2005-2007), and the Kyoto phase (2008-2012). Various exchange markets introduce different emphasis on spot and futures trading products for CO$_2$ emissions allowances. Since EU governments prohibited borrowing and banking trading policy between Pilot and Kyoto phase, spot prices of emissions allowances fell to zero from October 2006 to December 2007. Data samples are from the most liquid and promising spot and futures exchange platforms under the EU ETS. The spot trading in Bluenext exchange was introduced on June 24, 2005. Now Bluenext exchange has become the most liquid platform for CO$_2$ spot trading. The futures trading in European Climate Exchange (ECX) which is merged by ICE on August 2010, has started on April 22, 2005. Now ECX (ICE) has become the most liquid and largest platform for CO$_2$ futures and options trading in the world.

One European Union allowance (EUA) has the right to emit one tone CO$_2$ into the atmosphere under the EU ETS. The minimum trading volumes for each futures contract are equal to 1,000 tons CO$_2$ equivalent. We choose time-varying daily settlement prices for EUA futures contracts with different delivery dates from December 2010 to December 2014. Since the launch of futures contracts with different maturities from December 2013 and December 2014 were started on April 8, 2008. Considered the continuity and availability of numerical samples, we select the data samples cover the period from April 8, 2008 to December 20, 2010 in the Kyoto phrase.

**One-Factor Dynamic Hedging Effectiveness**

**one-factor dynamic hedge ratios**

Key factors of assets hedging effectiveness are that hedging assets are the tradable commodities and financial investment assets. Based on favorable and unfavorable market information induced from futures markets, a bona fide hedger can optimize and adjust assets portfolio sizes of futures contracts with different maturities for emissions allowances. If emissions allowances markets exhibit no market frictions, no market transaction cost, no market arbitrage opportunities and non-defaulted market risks, the instantaneous market interest rates have important impacts on futures prices for emissions allowances assets according to the cost-of-carry theory. In the non-restrictive assumptions, Cox, Ingersoll, and Ross (1981) test that commodities prices in forward and futures contracts are of bigger difference because commodity spot prices and random market interest rates have a non-zero variance [11]. Since the formation of emissions allowances quotas has a certain production cycle, unexpected market information increases excess market demand and then emissions allowances market induces certain a market scarcity of emissions allowances quotas. The increase of spot prices for emissions allowances is higher speed than theoretical futures prices on the basis of cost-of-carry model, and then emissions allowances market exists market arbitrage opportunities, accordingly market participants reduce earning risk through adjusting portfolio sizes of futures assets for emissions allowances. Schwartz (1990) proposes that risk premium of market prices and convenience yields have significant impacts on commodities futures prices, and convenience yields were the implied gains from holding commodities spot [12]. Inventory level is correlative with convenience yields, lower inventory level induces higher convenience yields, while higher inventory level induces lower convenience yields. Compared with other market factors, Schwartz (1997) find that commodity convenience yields exhibit larger value, the volatility of
annual convenience yields for crude oil and copper metals were 0.372 and 0.249, while the volatility of market interest rates were 0.0081 [1]. Fama and French (1987) show that convenience yields of agricultural products have a significantly seasonal property [13]. Chang and Wang et al. (2012) propose a new N-factor affine term structure model for CO2 futures prices and their empirical results indicate that CO2 futures prices and convenience yields follow a significant mean-reversion process [14].

Assumed $F_{t,T_1}, F_{t,T_2}$ denote long-maturity and short-maturity futures contracts of emissions allowances, and the delivery date $T_1 < T_2$. On the basis of cost-of-carry theory, Gibson and Schwartz (1990) [12], Bessmeltender (1995) [15], Liu and Tang (2011)[16] propose short-maturity futures contracts replacing spot contracts, and then long-maturity futures prices are equal to

$$F_{t,T_2} = e^{(r-cy)(T_2-T_1)}F_{t,T_1}$$

(1)

Where $cy$ denotes constant convenience yields of emissions allowances, $r$ denotes a free-risk of market interest rates. Assumed $h_1$ denotes one-factor hedge ratios, based on the cost-of-cost theory, accordingly dynamic hedge ratios are equal to [1] [7, 8]

$$h_1 = \frac{e^{-(r-cy)(T_2-T_1)} \frac{\partial F_{t,T_2}}{\partial S}}{\frac{\partial F_{t,T_1}}{\partial S}}$$

(2)

On the basis of estimating hedge ratios by Brennan–Schwartz (1985) model, dynamic hedge ratios of futures contracts with different maturities are equal to

$$h_1 = e^{-(r-cy)(T_2-T_1)} \frac{F_{t,T_2}}{F_{t,T_1}}$$

(3)

**Empirical evidence of dynamic hedge ratios**

$F_1$ denotes the EUA futures contracts that are closest to maturity, $F_2$ denotes the second closest to maturity, and $F_3, F_4, F_5$ are defined similarly. In the figures 1, 2 and 3, $h_{12}$ denotes one-factor hedge ratios between short-maturity futures contracts $F_1$ and long-maturity futures contracts $F_2$, $h_{13}$ denotes one-factor hedge ratios between short-maturity futures contracts $F_2$ and long-maturity futures contracts $F_3$, other hedge ratios are defined similarly. Seen from the figure 1, 2 and 3, one-factor hedge ratios of futures contracts with different maturities exhibit time-varying trends, dynamic hedge ratios have similar motion trends. Take an example for dynamic hedge ratios $h_{12}$, the maximum hedge ratio is 0.9854, the minimum hedge ratio is 0.9050, and the average hedge ratio is 0.9424 in the observation period of data samples. With an increase of maturity span for futures contracts, the average of dynamic hedge ratios exhibits a gradually reduce trend, its standard deviation shows a gradually enlarged trend and the one-factor hedge ratios have a left-skewed and heavy-tail feature.

![Fig 1 one-factor hedge ratios of futures contracts with different maturities for emissions allowances (F1 and F2,F3,F4,F5)](image-url)
Table 1 statistical analysis of one-factor hedge ratios of futures contracts with different maturities for emissions allowances

<table>
<thead>
<tr>
<th>h</th>
<th>average</th>
<th>Standard deviation</th>
<th>maximum</th>
<th>minimum</th>
<th>skewness</th>
<th>kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>h_{12}</td>
<td>0.9424</td>
<td>0.0190</td>
<td>0.9854</td>
<td>0.9050</td>
<td>-0.2223</td>
<td>1.9300</td>
</tr>
<tr>
<td>h_{13}</td>
<td>0.8771</td>
<td>0.0337</td>
<td>0.9426</td>
<td>0.8174</td>
<td>-0.1135</td>
<td>1.8085</td>
</tr>
<tr>
<td>h_{14}</td>
<td>0.7974</td>
<td>0.0428</td>
<td>0.8654</td>
<td>0.6981</td>
<td>-0.2869</td>
<td>1.7431</td>
</tr>
<tr>
<td>h_{15}</td>
<td>0.7376</td>
<td>0.0464</td>
<td>0.8112</td>
<td>0.6273</td>
<td>-0.1877</td>
<td>1.7179</td>
</tr>
<tr>
<td>h_{23}</td>
<td>0.9304</td>
<td>0.0174</td>
<td>0.9585</td>
<td>0.8972</td>
<td>-0.0344</td>
<td>1.7883</td>
</tr>
<tr>
<td>h_{24}</td>
<td>0.8456</td>
<td>0.0299</td>
<td>0.8896</td>
<td>0.7535</td>
<td>-0.5177</td>
<td>1.7383</td>
</tr>
<tr>
<td>h_{25}</td>
<td>0.8758</td>
<td>0.0357</td>
<td>0.9187</td>
<td>0.8089</td>
<td>-0.6582</td>
<td>2.0936</td>
</tr>
<tr>
<td>h_{34}</td>
<td>0.9087</td>
<td>0.0192</td>
<td>0.9378</td>
<td>0.8163</td>
<td>-1.3959</td>
<td>4.6807</td>
</tr>
<tr>
<td>h_{35}</td>
<td>0.8402</td>
<td>0.0245</td>
<td>0.8771</td>
<td>0.7335</td>
<td>-0.9472</td>
<td>3.4015</td>
</tr>
<tr>
<td>h_{45}</td>
<td>0.9245</td>
<td>0.0102</td>
<td>0.9448</td>
<td>0.8984</td>
<td>-0.1940</td>
<td>2.0938</td>
</tr>
</tbody>
</table>

hedging effectiveness estimation

Compared with the minimum variance of unhedged portfolio returns, we use percentage reduction in the variance of hedged portfolio returns in order to evaluate the hedging effectiveness (HE) [17].

\[
HE = \frac{Var(U_t) - Var(H_t)}{Var(U_t)}
\]  

(4)

Where \(Var(U_t)\) denotes the variance of unhedged portfolio returns, \(Var(H_t)\) denotes the variance of optimally hedged portfolio returns. When futures contracts of emissions allowances completely reduce hedging risks of assets portfolio returns, we can obtain HE=1 which exhibits a 100% reduction in the variance of hedging portfolio returns, whereas we can obtain HE=0 when hedging portfolio returns don’t eliminate risk. The higher HE shows better hedging effectiveness between futures contracts with different maturities.

Table 2 one-factor hedging effectiveness of futures with different maturities for carbon emissions futures

<table>
<thead>
<tr>
<th>HE</th>
<th>0.9592</th>
<th>0.9448</th>
<th>0.8381</th>
<th>0.7801</th>
<th>0.9817</th>
</tr>
</thead>
</table>

Fig. 2 one-factor hedge ratios of futures contracts with different maturities for emissions allowances (F_2 and F_3,F_4,F_5)

Fig. 3 one-factor hedge ratios of futures contracts with different maturities for emissions allowances (F_3 and F_4,F_5, F_4 and F_5)
seen from table 2, compared with unhedged portfolio returns between futures contracts with different maturities, market participants optimize and adjust portfolio sizes of futures assets, and then the hedging risks of futures contracts with different maturities have a significant decrease by using equation (4). Take an example for short-maturity futures contracts $F_1$ with long-maturity futures contracts $F_2,F_3,F_4,F_5$, the revenues variance of hedging portfolio reduce 0.9592, 0.9448, 0.8381, 0.7801. With an increase of span periods between futures contracts, the percentage reduction of revenue risks of futures assets for emissions allowances exhibits a significant decline, accordingly the hedgers can achieve better hedging effectiveness by using one-factor hedge ratios.

### Two-Factor Hedging Effectiveness

#### Two-factor hedge ratios

Gibson and Schwartz (1990) present commodities spot prices and convenience yields are important variables for commodity futures prices [12]. Convenience yields are unobservable state variables which spot holders achieve implied investment earnings in order to respond unexpected market demand. Two-factor futures pricing model of emissions allowances is equal to [12]

$$
F(S_t, \delta_t, T) = S_t \exp(-\delta_t \frac{1-e^{-\alpha(T-t)}}{k} + A(T-t))
$$

Where $k$ denotes mean-reversion speeds of convenience yields, $\delta$ denotes instantaneous convenience yields. Based on an change of convenience yields and time-to-maturity, two-factor Hedging policy of futures contracts with different maturities is to flexibly adjust assets portfolio between short-maturity and long-maturity futures contracts. Assumed portfolio sizes between short-maturity and long-maturity futures assets for emissions allowances are $h_{12}, h_{22}$, two-factor hedge ratios are equal to [1] [7] [10]

$$
\begin{align*}
\frac{\partial F_{t,T}}{\partial S_t} &= e^{-\gamma(T-t)} \frac{\partial F_{t,T}}{\partial S_t} \\
\frac{\partial F_{t,T}}{\partial \delta_t} &= e^{-\gamma(T-t)} \frac{\partial F_{t,T}}{\partial \delta_t}
\end{align*}
$$

Equation (5) substitute into equation (6) and (7), we can give the following equation [1] [10]

$$
\begin{align*}
\frac{\partial F_{t,T}}{\partial S_t} &= (1 - \frac{1 - e^{-k(T-t)}}{1 - e^{-k(T-t)}}) \frac{e^{-\gamma(T-t)}F_{t,T}}{F_{t,T}} \\
\frac{\partial F_{t,T}}{\partial \delta_t} &= (1 - \frac{1 - e^{-k(T-t)}}{1 - e^{-k(T-t)}}) \frac{e^{-\gamma(T-t)}F_{t,T}}{F_{t,T}} \\
h_{12}^2 + h_{22}^2 &= h_t^2
\end{align*}
$$

Seen from equation (8) to (10), Two-factor hedge ratios are affected by short-maturity and long-maturity futures prices, mean-reverting speeds of convenience yields, time to maturity and span period between short-maturity and long-maturity futures contracts.

**Empirical evidence of two-factor hedge ratios**

In the figures 4, 5, 6, and table 3, $h_{12}$ denotes two-factor hedge ratios between short-maturity futures contracts $F_1$ and long-maturity futures contracts $F_2$, $h_{13}$ denotes two-factor hedge ratios between short-maturity futures contracts $F_1$ and long-maturity futures contracts $F_3$, the other two-factor hedge ratios are defined similarly. Based on equation (8) - (10) estimation, two-factor hedge ratios of futures contracts with different maturities for emissions allowances exhibit time-varying trends, two-factor hedge ratios show higher volatility with the nearer span periods.
among futures contracts. Seen from table 3, with an increase of span period among futures contracts, the average of two-factor hedge ratios reduces gradually, its standard deviation shows a shrunken trends and two-factor hedge ratios have a left-skewed and heavy-tail feature. Take an example for two-factor hedge ratios $h_{12}, h_{13}, h_{14}, h_{15}$, one-factor hedge ratios are less than 1, with an increase of span period among futures contracts, one-factor hedge ratios adjust dynamically from 0.6273 to 0.9854. Under the similar span period among futures contracts, two-factor hedge ratios slightly higher than one-factor hedge ratios, and two-factor hedge ratios show dynamic adjustment range from 0.7261 to 1.1179.

### Table 3 statistical analysis of two-factor hedge ratios of futures contracts with different maturities for emissions allowances

<table>
<thead>
<tr>
<th>$hr$</th>
<th>Average</th>
<th>Standard Deviation</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$hr_{12}$</td>
<td>1.0108</td>
<td>0.1222</td>
<td>1.0749</td>
<td>0.7582</td>
<td>-1.1238</td>
<td>2.4885</td>
</tr>
<tr>
<td>$hr_{13}$</td>
<td>0.9725</td>
<td>0.1151</td>
<td>1.1179</td>
<td>0.7357</td>
<td>-1.1111</td>
<td>2.45550</td>
</tr>
<tr>
<td>$hr_{14}$</td>
<td>0.9134</td>
<td>0.1035</td>
<td>1.0057</td>
<td>0.7166</td>
<td>-1.0141</td>
<td>2.2336</td>
</tr>
<tr>
<td>$hr_{15}$</td>
<td>0.8745</td>
<td>0.0923</td>
<td>0.9521</td>
<td>0.6986</td>
<td>-0.9875</td>
<td>2.2081</td>
</tr>
<tr>
<td>$hr_{23}$</td>
<td>0.9572</td>
<td>0.1124</td>
<td>1.1066</td>
<td>0.7261</td>
<td>-1.0938</td>
<td>2.4305</td>
</tr>
<tr>
<td>$hr_{24}$</td>
<td>0.9048</td>
<td>0.1020</td>
<td>0.9962</td>
<td>0.7115</td>
<td>-0.9980</td>
<td>2.2040</td>
</tr>
</tbody>
</table>

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Table 4 shows market participants adjust assets portfolio sizes of futures contracts with different maturities for emissions allowances by using two-factor hedge ratios, compared with unhedged portfolio risk of futures contracts, and then the revenue variance of assets portfolio of futures contracts with different maturities exhibits better risks reduction. Take an example for two-factor hedge ratios \(hr_{12} - hr_{15}\), with the span period of futures contracts raised, two-factor hedging effectiveness decline 0.9611, 0.9568, 0.8879 and 0.8567. Accordingly market participants can achieve significant reduction ranges in assets portfolio risks of futures contracts for emissions allowances by using two-factor hedge ratios. Compared with one-factor hedging policy, market participants can attain a slight decline in assets portfolio risks of futures contracts, and the reduction range of assets portfolio risks of futures contracts gradually increase with the span period of futures contracts raised. The above empirical results exhibit two-factor hedging effectiveness is better than one-factor HE, especially two-factor hedging policy have more significant hedging effectiveness for longer span period of futures contracts with different delivery dates for emissions allowances.

<table>
<thead>
<tr>
<th>Futures</th>
<th>(F_i &amp; F_j)</th>
<th>(F_i &amp; F_j)</th>
<th>(F_i &amp; F_j)</th>
<th>(F_i &amp; F_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HE0</td>
<td>0.9611</td>
<td>0.9568</td>
<td>0.8879</td>
<td>0.8567</td>
</tr>
<tr>
<td>HE1</td>
<td>0.0534</td>
<td>0.2169</td>
<td>0.3075</td>
<td>0.3484</td>
</tr>
<tr>
<td>Futures</td>
<td>(F_i &amp; F_j)</td>
<td>(F_i &amp; F_j)</td>
<td>(F_i &amp; F_j)</td>
<td>(F_i &amp; F_j)</td>
</tr>
<tr>
<td>HE0</td>
<td>0.9146</td>
<td>0.8844</td>
<td>0.9159</td>
<td>0.8930</td>
</tr>
<tr>
<td>HE1</td>
<td>0.2194</td>
<td>0.3012</td>
<td>0.0275</td>
<td>0.1390</td>
</tr>
</tbody>
</table>

### Conclusion

Non-expected market information has a significant speed difference of futures prices with different maturities for emissions allowances. We propose one-factor and two-factor hedge ratios and hedging effectiveness estimation.

One-factor and two-factor hedge ratios of futures contracts with different maturities for emissions allowances exhibit time-varying trends. With an increase of span period of futures contracts, the average of one-factor and two-factor hedge ratios of futures contracts show decreased trends, and they have a left-skewed and heavy-tail feature. Compared with the risk reduction of unhedged portfolio of futures assets, market participants can achieve significant risk reduction in assets portfolio of futures contracts with different maturities by using one-factor and two-factor hedging policy. Compared with one-factor hedging policy, market participants can attain higher risk reduction in futures assets portfolio with the span period of futures contracts raised. Accordingly Two-factor hedging effectiveness is better than one-factor hedging policy, and especially two-factor hedging policy exhibits better hedging effectiveness for longer span period of futures contracts with different maturities for emissions allowances.

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