The Analysis of Vibration for High-Speed Train-Ballastless Track-Bridge base on a hybrid FE-SEA method

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Abstract. In this study, the methods for combining statistical energy analysis (SEA) and the finite element method (FEM) for the vibration analysis of structures are studied. Using the two methods simultaneously isn't entirely extend a primarily low frequency method, the finite element method, and high frequency method, SEA, to the mid frequency region are addressed. This approach is intended to extend the frequency range for a FEM based vibration analysis . A new finite element elementl for elevated slab ballastless track is proposed in which the new model can be used for modeling the track structural constituents of elevated slab ballastless track. Using finite element method and Hamilton theory, the coupled equation of vehicle-track-bridge can be established. In calculating example, both the rail displacement induced by single four-layer beam model. Specifically, it showed that the method yields very good result and high performance in the numerical example of previous research.

Introduction

A hybrid method combining FE and SEA was recently presented for predicting the steady-state response of vibro-acoustic systems. The new method is presented for the analysis of complex dynamic systems which is based on partitioning the system degrees of freedom into a ''global'' set and a ''local'' set. The global equations of motion are formulated and solved by using the finite element method (FEM).The local equations of motion are formulated and solved by using statistical energy analysis (SEA) [2-4]. The power input from the global degrees of freedom. This paper deduces the theory for the beam element [1], and Train-Ballastless Track-Bridge System provides an application, and it showed that the method yields very good results.

Hybrid FE-SEA Method

Finite Element Method. Each beam is modeled by using a single hierarchical finite element. With this approach the vertical/corner displacement of the generic beam element is written in the form

$$\begin{aligned} U(x,t) &= q_1(t)\left(1-\frac{3x^2}{L^2}+\frac{2x^3}{L^3}\right)+q_2(t)x\left(1-\frac{2x}{L}+\frac{x^2}{L^2}\right)+q_3(t)\left(\frac{3x^2}{L^2}-\frac{2x^3}{L^3}\right)+q_4(t)\left(-\frac{x^2}{L}+\frac{x^3}{L^2}\right) \\ &+\sum_{n=1}^{j} q_{n+4}(t)\sin\left(\frac{np\,x}{L}\right)+\sum_{n=j+1}^{\infty} q_{n+4}(t)\sin\left(\frac{np\,x}{L}\right) \end{aligned} \tag{1}$$

Where $q_1(t)$. $q_2(t)$. $q_3(t)$. $q_4(t)$ represent the nodal displacements of the beam element. And $q_{n+4}(t)$ represents the amplitude of the nth ''internal'' or hierarchical admissible function. . The equations of motion which govern the amplitudes $q_n(t)$ follow from the application of Lagrange's equation.

$$\left(-w^2M + iwC + K\right)q = Dq = F \tag{2}$$

which F is the generalized force, M , C , K represent the quality,damping and stiffness of the structure.The displacement functions $f_n(x)$ are partitioned into two sets which are nominally referred to as "global" and "local" displacement functions . So Eq. 1 can be modified to the form

$$U(x,t) = \sum_{n=1}^{N_g} q_n^g(t) f_n^g(x) + \sum_{n=1}^{N_l} q_n^l(t) f_n^l(x) \tag{3}$$

Need for a detailed deterministic model.The somewhat vague definition of the global and local admissible functions can perhaps be clarified by considering about the wavelength. Having partitioned Eq.1 in the form of Eq.3,it follows that the system equations of motion, Eq.2, can be partitioned as follows

$$\begin{pmatrix} D_{gg} & D_{gl} \\ D_{gl}^T & D_{ll} \end{pmatrix} \begin{pmatrix} q^g \\ q^l \end{pmatrix} = \begin{pmatrix} F^g \\ F^l \end{pmatrix} \tag{4}$$

Eq.(4) can be modified as follows

$$\left(D_{gg} - D_{gl} D_{ll}^{-1} D_{gl}^T\right) q^g = F^g - D_{gl} D_{ll}^{-1} F^l \tag{5}$$

$$D_{ll}\ q^l = F^l - D_{gl}^T q^g \tag{6}$$

Eq.5, Eq.6 are respectively used to describe "global" and "local" models. The global equations of motions—Eq.5 are solved by using FEM; While the local equations of motions—Eq.6 are solved by using SEA .

Solution of the Global Equation. Acoording to R.S.Langley's article [2], where $D_{gl} D_{ll}^{-1} D_{gl}^T$, $D_{gl} D_{ll}^{-1} F^l$ are deduced. The forms are as follows

$$\left(D_{gl} D_{ll}^{-1} D_{gl}^T\right)_{mn} = \sum_{j=1}^{n_l} \sum_{k=1}^{n_l} \left(D_{gl}\right)_{mj} \left(D_{ll}^{-1}\right)_{jk} \left(D_{gl}\right)_{nk} \tag{7}$$

$$\left(D_{gl}\right)_{mj(k,r)} \left(D_{gl}\right)_{nk(k,r)} = w^4 \int_{v_r} \int_{v_r} r\left(x\right) r\left(x'\right) f_m^{gT}\left(x\right) \times f_{j(k,r)}^l\left(x\right) f_{j\ (k,r)}^{lT}\left(x\right) f_n^g\left(x'\right) dx dx' \tag{8}$$

The contribution to Eq.8 arising from inertia dominated local modes can be written in the form. A significant negative effective mass results from Eq.9 of the subsystem local motions.

$$\left(D_{gl} D_{ll}^{-1} D_{gl}^T\right)_{mn} = -w^2 \sum_{r=1}^{n_s} \sum_{k=1}^{n_l} j_{rmn}^2\left(k\right)\ ,\quad w_k^r = w \tag{9}$$

$$\left(D_{gl} D_{ll}^{-1} D_{gl}^T\right)_{mn} = -i \frac{pw^3}{2} \sum_{r=1}^{n_s} j_{rmn}^2 n_r\ ,\quad w_k^r \approx w \tag{10}$$

Where j_{rmn}^2 represents the average value of $j_{rmn}^2\left(k\right)$.This result implies that the local modes act to damp the global modes.

$$\left(D_{gl}D_{ll}^{-1}F^{l}\right)=\sum_{r=1}^{N_s}\left(-w^2\right)\int_{v_r} r\left(x\right)f_m^{gT}\left(x\right)\sum_{r=1}^{N_s}q_{j(k,r)}^{lB}f_{j(k,r)}^{l}\left(x\right)dx \qquad (11)$$

Solution of the Local Equation. This allows for a significant reduction in the number of degrees of freedom in the model. SEA is based on an equilibrium power balance at subsystem level. The power-balance equations for the subsystem can be written as

$$wh_rE_r+w\sum_{s=1}^{N_s}h_{rs}n_r\left(\frac{E_r}{n_r}-\frac{E_s}{n_s}\right)=P_r, \qquad r=1,2,...,N_s \qquad (12)$$

Where h_r, n_r, E_r, and P_r are respectively the loss factor, modal density, energy, and external power input to subsystem *r*, and h_{rs} is termed the coupling loss factor between subsystem *r* and subsystem *s*. Then the power input to the complete set of modes in subsystem *r* (P_r) has the form [2].

$$P_r=\frac{w^4pn_r}{4}\sum_{m=1}^{N_g}\sum_{n=1}^{N_g}\int_{v_r}\int_{v_r} r\left(x\right)r\left(x'\right)f_m^{gT}\left(x\right)\times f_{j(k,r)}^{l}\left(x\right)f_{j\ (k,r)}^{lT}\left(x\right)f_n^{g}\left(x'\right)q_mq_n^{*}dxdx \qquad (13)$$

The power input which appears in Eq. 12 can be estimated from Eq. 14 once the global mode responses q_m have been calculated by using the method described abolve.

The CRTS track-bridge Element Model

The vehicle-ballastless track-bridge system is separated into vehicle- track subsystem and track-bridge subsystem. Each vehicle is taken as four independent driven wheel element and track - bridge subsystem is taken as a single "four layer beam" element. The rail have freedom of N_r (q_{ri}). The moving wheel element is shown in Fig.1. q_{ci} is vertical displacement of car body i; q_{ti}, q_{wi} is vertical displacement of bogie i and number i wheel. Subscripts r,s,f,b represent respectively rail, track slab, concrete supporting layer, bridge. The Track-bridge is shown in Fig.2. The number of DOF of the element is $N=N_r+N_s+N_f+N_b$.

$$\Phi(x)=\left\{j_{r1}(x)\ \ j_{r2}(x)\ \ ...\ \ j_{rN_r}(x)\ \ j_{s1}(x)\ \ j_{s2}(x)\ \ ...\ \ j_{sN_s}(x)\ \ j_{f1}(x)\ \ j_{f2}(x)\ \ ...\ \ j_{fN_f}(x)\ \ j_{b1}(x)\ \ j_{b2}(x)\ \ ...\ \ j_{bN_b}(x)\right\}_{(1\times N)}$$

Where $\Phi(x)$ represents the admissible function, $\underline{a}^e$ is generalized coordinate which constitutes the time-dependent amplitude of the admissible function. And q_{rn} $(n=1...N_r)$ epresents respectively nodal displacement of rail. $j_{rn}(x)$ represents the admissible function of the rail . The subscript r in the expression of the rail admissible function is replaced with s, f, b, Respectively, represents the admissible function of track slab, concrete supporting layer, bridge. Where $j_{r1}(x)=1-\frac{3}{l^2}x^2+\frac{2}{l^2}x^3$ $j_{r2}(x)=x-\frac{2}{l}x^2+\frac{1}{l^2}x^3$, $j_{r3}(x)=\frac{3}{l^2}x^2-\frac{2}{l^2}x^3$, $j_{r4}(x)=-\frac{1}{l}x^2+\frac{1}{l}x^3$, $j_{rn}(x)=\sin\frac{(n-4)p\,x}{L}$, $(n=5,6...\mathrm{N}_r)$.

Stiffness matrix, mass matrix and damping matrix of these two kinds of elements and the vertical vibration equation set of the vehicle-ballastless track-bridge time-dependent system were established by means of finite element method and Lagrange equation [5].The dynamical equations can be solved by Newmark-β method. The vehicle parameters are taken from CRH3 and track structure parameters are from two-line slab track of single cell box girder.

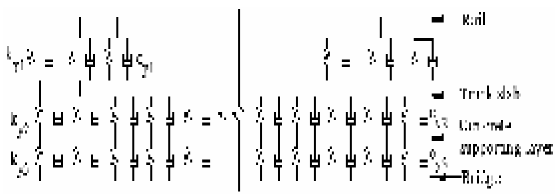

Fig.1 The moving wheel element

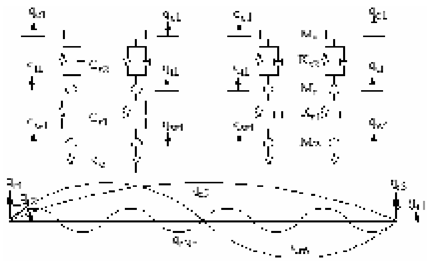

Fig.2 Track-bridge element

The train concludes one moving wheel element. Its speed is 300km/h. The viaduct is one-span and each span is 32-meter long. The number of DOF of the rail is 35(<=3000Hz) that is belong to the ‘‘global’’ degrees of freedom. The frequency range considered is 5000Hz, so that the response of rail must be augmented with a “local’’ response for 3000-5000Hz. The number of DOF of the track slab and concrete supporting layer is 35 respectively. And bridge is 10. The surface roughness of Sato [7] that can be input finite analysis model. The range for Wavelength is 0.005-0.25mm. It is helpful to consider separately the contribution arising from the local modes. A significant negative effective mass results from Eq.9 of the range of 0.005-0.01($w_k^r =\ w$). The local modes act to damp the global modes from Eq.10 in the range of 0.01-0.1($w_k^r \approx w$). The contribution arising from local modes is not considered in the range of 0.1-0.25 ($w_k^r\ f\ \ w$), as will normally be negligible. The vibration curves are shown in Fig.3-8.

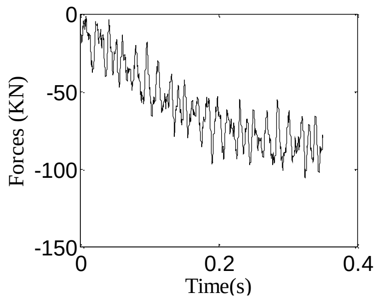

Fig.3 wheel-rail interaction forces

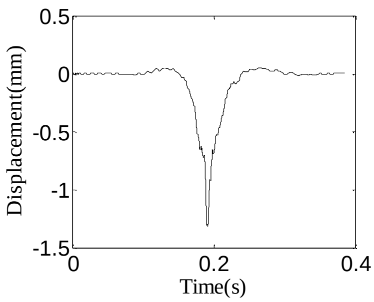

Fig.4 rail displacement

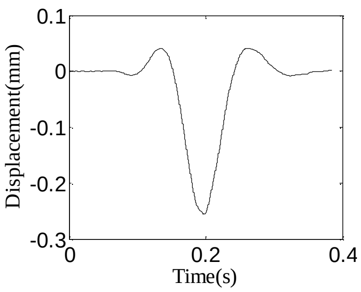

Fig.5 rail displacement

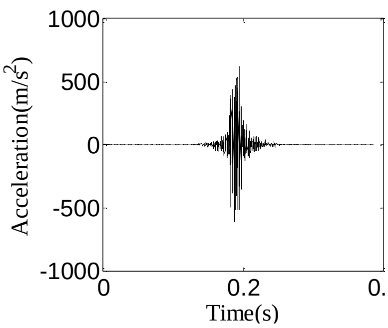

Fig.6 rail acceleration

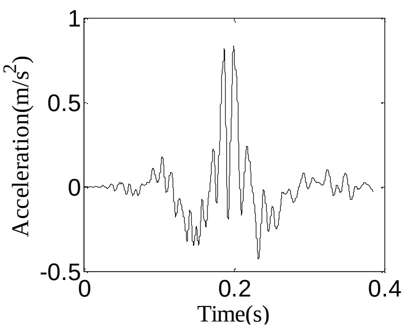

Fig.7 bridge acceleration

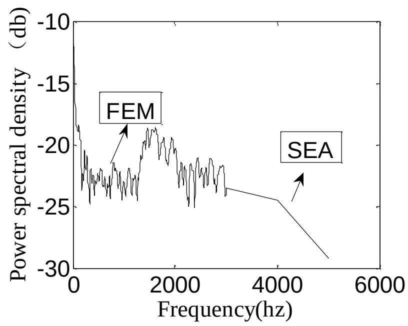

Fig.8 PSD curve for rail Speed

Conclusion

(1)When the train passing by, the vibration acceleration, displacement of rail and bridge have evident peak. So it is easy to identify the composite type of vehicle and via the acceleration time-history distribution, the time of arrival, passing and departure for train is easily defined. And according to the length of train and the passing time, the speed of metro vehicle can be deduced.

(2) Because of the vibration reduction effect from fastenings, from the vibration time-history pictures of track and box girder, the time of wheels' arrival, passing and departure can not be recognized. But the vibration time-history pictures still present evident wavelet because of the impact load.Power spectral density curve for rail speed is max at about 1400hz that is the first frequency of rail.

(3)In contrast, a conventional hierarchical finite element model of the system , the hybrid prediction where the beam elements are less degrees of freedom. The model has advantages of convenience for programming and high efficiency for computing.

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