The Boundary Element Method of Testing Wood Moisture Content Problem in The Heterogeneous and Asymmetric Case

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***Abstract*—In this paper, we discussed this engineering problem that used planar capacitance sensor measuring lumber moisture content. And a new mathematical model was introduced which could set up the relationship between the distribution of dielectric constant and planar capacitance sensor in the heterogeneous asymmetric cases. At the same time, the numerical example was given and we matched the function of capacitance value and dielectric permittivity of medium. Accordingly, through measuring capacitance value of capacitance sensor, we could get the distribution situation of dielectric permittivity, thus obtaining the moisture content distribution of media such as wood. It could not only greatly cut down measuring work, but also avoided the waste of media resources, such as wood.**

Keywords: Capacitance sensor; Capacitance value; Moisture content; Dielectric constant; Mathematical model; Boundary element method

I. INTRODUCTION

Wood moisture content detection has been a hot research topic for many years. Either too high or too low moisture content can cause quality problems of wood products, such as cracks, being out of shape. Capacitance sensor [1] is a kind of electronic device which can put determined physical quantity and its change law into capacitance value and capacitance value change law. Its outstanding advantage is to make non-contact measurement to objects to be tested. With the physical characteristics that the moisture content of the medium such as wood, directly influencing the dielectric constant of the medium, we can obtain the dielectric constant of the media through measuring the capacitance value. Furthermore, we get the moisture content of medium [2].

First of all, from the physical methods of the wood moisture content detection, in developed countries, the wood moisture content detection has developed to the degree of continuous, non-contact, high accuracy detection [3-5] by using electromagnetic method. It needs build huge database to store data. If one of the conditions changes, remeasurement is required.

Secondly, from the progress of mathematics models, a new mathematics model which has a wider application range has been established with no special requirements for the wood shape, the distribution of moisture content and the position of plate electrode and wood. It was a three-dimensional Laplace equation with join conditions and boundary integral connections, thereby satisfying detection requirements in the heterogeneous and asymmetric case. With the new method, moisture content of measured media can be obtained indirectly by detecting capacitance value [6], among which, most can be achieved by numerical calculation. At last only a corresponding experimental operation between dielectric constant and moisture content is needed.

In the end, from the calculation methods, in the past, this problem was calculated by the finite element method or the finite difference method, etc, and the artificial boundaries were all added. The purpose was to ensure the number of unknown domain limited. However, the boundary element method made the differential equations into boundary integral equations. Its advantage was the dimension of solving problem was reduced one dimension. In this way, the input data quantity and an unknown quantity of algebraic equations were greatly reduced; at the same time, the discrete only exists in the boundary and an unknown quantity only appears in the boundary. It is not only suitable for our problems, but error produced only on boundary. So the accuracy would be high; and the artificial boundary was not needed. The calculation error was greatly reduced.

II. SOME BASIC NOTATIONS

a) The occupied spacial region of the measured wood is B , and it is disjoint with electrode plate B_1 and B_2 .

b) $\varepsilon = \varepsilon(x, y, z)$ indicates dielectric constant of wood at point (x, y, z) and its size is related to its spatial position.

c) $\tilde{\varepsilon}$ indicates dielectric constant in the air.

d) $V = V(x, y, z)$ indicates electric potential at point (x, y, z) in electric field, and electric potentials at B_1 and B_2 are denoted by V_1 and V_2 respectively.

e) $E = E(x, y, z)$ indicates electric field intensity at point (x, y, z) .

f) C indicates capacitance value of detecting capacitor.

g) q_1 and q_2 indicate respectively electric quantities of B_1 and B_2 .They are the function of $\varepsilon(x, y, z)$, and $C = \dfrac{q_1}{V_1 - V_2}$ [7].

h) $\rho(x,y,z)$ indicates density of free charge in insulator.

i) $\vec{D}$ indicates electric displacement vector, and $\vec{D}=\varepsilon\vec{E}$ [7].

j) The source point and type point of the boundary are denoted by r' and r.

III. THE ESTABLISHMENT OF MATHEMATICAL MODEL

A. On $R^3-B_1-B_2-B$, observe and study $V=V(x,y,z)$.

Due to the fact that this is no free charge in $R^3-B_1-B_2-B$, in this region V satisfies

$$\nabla^2 V=0 \cdot$$

Obviously, this equation has no requirement for wood shape.

B. At infinity, it has

$$\lim_{\sqrt{x^2+y^2+z^2}\to\infty} V(x,y,z)=0$$

C. To the detection of heterogeneous and asymmetric case, observe and study the interior of B, by Gaussian theorem

$$\nabla\cdot\vec{D}=\rho$$

Due to the fact that this is no free charge within B, this region has

$$\nabla\cdot\vec{D}=\nabla\cdot[\varepsilon\nabla V]=0$$

After finishing, $V=V(x,y,z)$ satisfies

$$\nabla\varepsilon(x,y,z)\cdot\nabla V+\varepsilon(x,y,z)\cdot\nabla^2 V=0$$

Apparently, this equation has no requirement for wood shape.

D. On the internal and external boundary of B, it has

$$V\big|_{\partial B_{\text{in}}}=V\big|_{\partial B_{\text{out}}},\ \varepsilon\frac{\partial V}{\partial n}\bigg|_{\partial B_{\text{in}}}=\tilde{\varepsilon}\frac{\partial V}{\partial n}\bigg|_{\partial B_{\text{out}}}$$

Among them, $V\big|_{\partial B_{\text{in}}}$ and $V\big|_{\partial B_{\text{out}}}$ indicate respectively limit values when V trends to ∂B on the internal and external boundary of ∂B. $\frac{\partial V}{\partial n}\Big|_{\partial B_{\text{in}}}$ and $\frac{\partial V}{\partial n}\Big|_{\partial B_{out}}$ indicate respectively limit values when directional derivative of V external normal line trends to ∂B on the internal and external boundary of ∂B.

E. On B_1 and B_2, $V(x,y,z)$ is observed and studied.

Suppose that electrode plate B_1 and B_2 are conductor plates, when they reach electrostatic equilibrium, the above electric potentials must be equivalent, then

$$V|_{B_1}=V_1,\ V|_{B_2}=V_2$$

In the cases of heterogeneous wood moisture content and asymmetric wood disposition, values of V_1 and V_2 cannot be given randomly. Otherwise, quantity of electric charge stored in V_1 and V_2 will be unequal during the charging process. If basic principle of capacitor's capacitance is violated, capacitance C cannot be calculated accordingly.

F. On B_1 and B_2, carried charge is observed and studied.

Due to the fact that electrode plate is very thin, we may suppose that electric charge are distributed on the upper part of electrode plate. Take any point at B_1 as a thin layer of cylindrical surface, as illustrated in Fig.1. The lower surface of the cylinder is on ∂B_1, the upper surface is denoted by ΔS ,and the flank of the cylinder is denoted by Δh . Applying Gaussian theorem on the surface of the cylinder, we have

$$q_1=\oiint_{B_1}\tilde{\varepsilon}E_1\cdot ds=\tilde{\varepsilon}\oiint_{B_1}|gradV|ds$$

Thus, a definite solution of a partial differential equation satisfied by electric potential $V(x,y,z)$ can be derived, as follows

$$\begin{cases}\nabla^2 V=0,\quad (x,y,z)\in(R^3-B_1-B_2-B)\\ \nabla\varepsilon\cdot\nabla V+\varepsilon\cdot\nabla^2 V=0,(x,y,z)\in B\\ V|_{B_1}=V_1, V|_{B_2}=V_2\\ \tilde{\varepsilon}\oiint_{B_1}|gradV|ds=-\tilde{\varepsilon}\oiint_{B_2}|gradV|ds\\ \varepsilon\frac{\partial V}{\partial n}|_{\partial B_{in}}=\tilde{\varepsilon}\frac{\partial V}{\partial n}|_{\partial B_{out}}, V|_{\partial B_{in}}=V|_{\partial B_{out}}\\ \lim\limits_{x^2+y^2+z^2\to\infty}V(x,y,z)=0\end{cases}$$

Take $G=\frac{1}{4\pi R}$ [8]to meet the basic differential equation $\nabla^2 V=0$ .That is $\nabla^2 G=0$ .And $R=|r'-r|$ represents the distance between r' and r. In regional boundary integral, use Green formula to establish the boundary integral equation, as follows

$$C(r)V(r)=\iint_{\partial B in}(G\frac{\partial V(r')}{\partial n}-V(r')\frac{\partial G}{\partial n})ds$$

where

$$C(r)=\begin{cases}\frac{1}{2}, r\in surface\ of\ \partial B_{in}\\ \frac{1}{4}, r\in arris\ of\ \partial B_{in}\\ \frac{1}{8}, r\in angle\ of\ \partial B_{in}\end{cases}$$

$$C(r)V(r)=\iint_{\partial B\,out}(G\frac{\partial V(r')}{\partial n}-V(r')\frac{\partial G}{\partial n})ds+\iint_{\partial B_1}G\frac{\partial V(r')}{\partial n}ds$$
$$-\iint_{\partial B_1}\frac{\partial G}{\partial n}ds+\iint_{\partial B_2}G\frac{\partial V(r')}{\partial n}ds-V_2\iint_{\partial B_2}\frac{\partial G}{\partial n}ds$$

where

$$C(r)=\begin{cases}\frac{1}{2}, r\in surface\ of\ \partial B_{out}\\ \frac{3}{4}, r\in arris\ of\ \partial B_{out}\\ \frac{7}{8}, r\in angle\ of\ \partial B_{out}\end{cases}$$

$$[C(r)+\iint_{\partial B_1}\frac{\partial G}{\partial n}ds]=\iint_{\partial Bout}(G\frac{\partial V(r')}{\partial n}-V(r')\frac{\partial G}{\partial n})ds$$
$$+\iint_{\partial B_1}G\frac{\partial V(r')}{\partial n}ds+\iint_{\partial B_2}G\frac{\partial V(r')}{\partial n}ds-V_2\iint_{\partial B_2}\frac{\partial G}{\partial n}ds$$

$$[C(r)+\iint_{\partial B_2}\frac{\partial G}{\partial n}ds]V_2=\iint_{\partial Bout}(G\frac{\partial V(r')}{\partial n}-V(r')\frac{\partial G}{\partial n})ds$$
$$+\iint_{\partial B_1}G\frac{\partial V(r')}{\partial n}ds+\iint_{\partial B_2}G\frac{\partial V(r')}{\partial n}ds-V_1\iint_{\partial B_1}\frac{\partial G}{\partial n}ds$$

where $C(r)=1$.

IV. A NUMERICAL EXAMPLE AND BOUNDARY ELEMENT METHOD FORMAT

A. This example is considered in even and symmetrical cases. Its purpose is to use the testing results to test the reasonability of the model.

B. The occupied spatial regions of two electrode plates are respectively

$$B_1=\{(x,y,z)\big|-3\le x\le 3,-6\le y\le -1,0\le z\le 2\}$$
$$B_2=\{(x,y,z)\big|-3\le x<3,1\le y\le 6,0\le z\le 2\}$$

C. The occupied spatial region of the wood is

$$B=\{(x,y,z)\big|-3\le x\le 3,-6\le y\le 4,3\le z\le 5\}$$

, as Fig. 2 shows. The length unit of the above spatial regions is centimeter. Among them, the above rectangular region represents wood, the below two rectangular regions represent electrode plate. We take step size $\Delta x=\Delta y=\Delta z=1$, and denote $V(i,j,k)=V_{i,j,k}$ .The wood dielectric constant of every point is ε . Due to the symmetry of the structure, we know that $V(x,y,z)$ is symmetrical about the yoz plane. We assume $V_1=1$ and $V_2=a$(a is undetermined constant.). If through the numerical calculation, the obtained value of a is very close to -1 .It means the mathematical model we established is reasonable.

D. After discreting on the border, the boundary integral equation[9] is obtained. In wood internal we have

$$\left[C_l\pi-\sum_{\Gamma_l}\left(\iint_{\sigma_{\Gamma_l}}\frac{\cos\alpha}{R^2}ds\right)\right]V_l=\sum_{\substack{j=1\\j\ne l}}^{186}\left[V_j\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{\cos\alpha}{R^2}ds\right)\right]$$
$$+\sum_{j=1}^{186}\frac{\varepsilon}{\tilde{\varepsilon}}\left[\frac{\partial V_j}{\partial n}\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{1}{R}ds\right)\right]$$

where

$$C_l=\begin{cases}6, r\in surface\ of\ \partial B_{in}\\ 3, r\in arris\ of\ \partial B_{in}\\ 1.5, r\in angle\ of\ \partial B_{in}\end{cases}$$

Γ_l stands for the No. of Triangle unit , whose vertex is V_l . σ_{Γ_l} stands for the area whose Numbers is Γ_l .In wood outside we have

$$\left[C_l\pi-\sum_{\Gamma_l}\left(\iint_{\sigma_{\Gamma_l}}\frac{\cos\alpha}{R^2}ds\right)\right]V_l$$
$$=\sum_{\substack{j=1\\j\ne l}}^{186}\left[V_j\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{\cos\alpha}{R^2}ds\right)\right]+\sum_{j=1}^{186}\frac{\varepsilon}{\tilde{\varepsilon}}\left[\frac{\partial V_j}{\partial n}\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{1}{R}ds\right)\right]$$
$$+\sum_{\substack{j=1\\(\partial B_1)}}^{42}\left[\frac{\partial V_j}{\partial n}\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{1}{R}ds\right)\right]+\sum_{\substack{j=1\\(\partial B_2)}}^{42}\left[\frac{\partial V_j}{\partial n}\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{1}{R}ds\right)\right]$$
$$+3V_1\sum_{\substack{j=1\\(\partial B_1)}}^{60}\left(\iint_{\sigma_{\Gamma_j}}\frac{\cos\alpha}{R^2}ds\right)+3V_2\sum_{\substack{j=1\\(\partial B_2)}}^{60}\left(\iint_{\sigma_{\Gamma_j}}\frac{\cos\alpha}{R^2}ds\right)$$

where

$$C_l=\begin{cases}6, r\in surface\ of\ \partial B_{in}\\ 9, r\in arris\ of\ \partial B_{in}\\ 10.5, r\in angle\ of\ \partial B_{in}\end{cases}$$

On B_1 we have

$$\left[12\pi-3\sum_{\substack{j=1\\(\partial B_1)}}^{60}\left(\iint_{\sigma_{\Gamma_j}}\frac{\cos\alpha}{R^2}ds\right)\right]V_1$$
$$=\sum_{\substack{j=1\\j\ne l}}^{186}\left[V_j\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{\cos\alpha}{R^2}ds\right)\right]+\sum_{j=1}^{186}\frac{\varepsilon}{\tilde{\varepsilon}}\left[\frac{\partial V_j}{\partial n}\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{1}{R}ds\right)\right]$$
$$+\sum_{\substack{j=1\\(\partial B_1)}}^{42}\left[\frac{\partial V_j}{\partial n}\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{1}{R}ds\right)\right]+\sum_{\substack{j=1\\(\partial B_2)}}^{42}\left[\frac{\partial V_j}{\partial n}\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{1}{R}ds\right)\right]$$
$$+3V_2\sum_{\substack{j=1\\(\partial B_2)}}^{60}\left(\iint_{\sigma_{\Gamma_j}}\frac{\cos\alpha}{R^2}ds\right)$$

On B_2 we have

$$\left[12\pi-3\sum_{\substack{j=1\\(\partial B_2)}}^{60}\left(\iint_{\sigma_{\Gamma_j}}\frac{\cos\alpha}{R^2}ds\right)\right]V_2$$
$$=\sum_{\substack{j=1\\j\ne l}}^{186}\left[V_j\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{\cos\alpha}{R^2}ds\right)\right]+\sum_{j=1}^{186}\frac{\varepsilon}{\tilde{\varepsilon}}\left[\frac{\partial V_j}{\partial n}\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{1}{R}ds\right)\right]$$
$$+\sum_{\substack{j=1\\(\partial B_1)}}^{42}\left[\frac{\partial V_j}{\partial n}\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{1}{R}ds\right)\right]+\sum_{\substack{j=1\\(\partial B_2)}}^{42}\left[\frac{\partial V_j}{\partial n}\left(\sum_{\Gamma_j}\iint_{\sigma_{\Gamma_j}}\frac{1}{R}ds\right)\right]$$

$$+3V_1 \sum_{\substack{j=1 \\ (\partial B_1)}}^{60} \left(\iint_{\sigma_{\Gamma_j}} \frac{\cos\alpha}{R^2} ds \right)$$

Through programming and calculating, we have the numerical results of capacitance value which is about the dielectric constant ε. It is obvious that the value of V_2 is very close to -1 .It means the mathematical model we established is reasonable. These results are shown in table 1.

table 1 the numerical results of C about ε

ε	**C**	V_2
1	26.5312	-1
2	26.1113	-1.0104
3	27.4158	-1.006
4	26.9513	-1.0146
5	27.6897	-1.0132
6	28.1085	-1.0152
7	28.4975	-1.0148
8	28.6633	-1.0138
9	28.8105	-1.0124
10	28.8926	-1.0103
11	28.9991	-1.0082
12	29.0570	-1.0058
13	29.0906	-1.0030
14	29.1224	-1.0005
15	29.1475	-0.9972
16	29.1566	-0.9950
17	29.1614	-0.9920
18	29.1705	-0.9893
19	29.1844	-0.9863
20	29.1956	-0.9836

Function image is showed in figure 3. At the same time, fitting the function about capacitance value C and dielectric permittivity ε . It is

$$C = -0.0142\varepsilon^2 + 0.4414\varepsilon + 25.8493$$

Conclusion

Based on the non-uniform asymmetrical condition, we establish a new mathematical model. This model is more general. Compared with the method before, its outstanding is that most of the work can be completed by calculations, greatly reducing the size of the database and reducing the measurement work. Secondly, for the numerical method, we only discrete on the border and unknown quantity only exist on the border. It is obvious that unknown quantity is lower and this method is more suitable for the needs of the problem. Finally, from the calculation results, they are consistent with the actual physical results. At the same time, in the calculation process, the outstanding of the numerical method are fully reflected, such as a little an unknown quantity, a low dimension, a fast operation speed and a high precision.

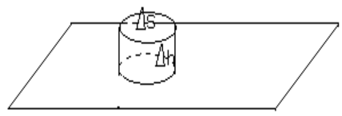

Fig.1 thin layer of the cylinder on ∂B_1

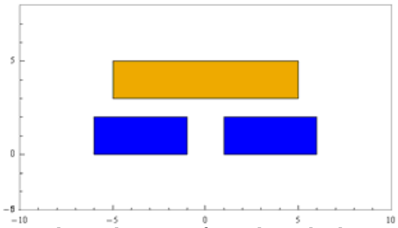

Fig.2 occupied spatial regions of two electrode plates and wood

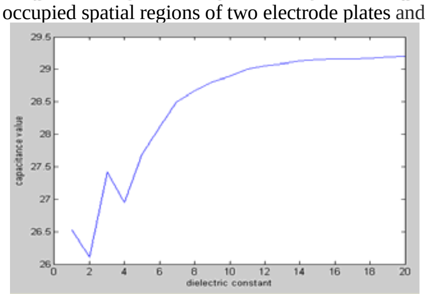

Fig.3 scatter data diagrams of C about ε

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