Journal of Nonlinear Mathematical Physics Volume 12, Supplement 1 (2005), 124–137 **Birthday Issue**

Goat cheese for breakfast in Istanbul or Why are certain nonlinear PDEs both widely applicable and integrable? Reminiscences of Francesco Calogero

Robin BULLOUGH

Department of Mathematics, University of Manchester
P.O. Box 88, Sackville Street, Manchester, M60 1QD, UK
E-mail: Robin.Bullough@manchester.ac.uk

This article is part of the special issue published in honour of Francesco Calogero on the occasion of his 70th birthday

Abstract

It is shown how in the early days of soliton theory 1976-the early 1980's Francesco Calogero maintained a considerable influence on the field and on the work of the author Robin Bullough in particular. A vehicle to this end was the essentially annual sequence of international conferences Francesco organised with his usual skill and with minimum fuss. Some of these conferences, the NEEDs meetings, and their mathematical contents, are here recalled – each offering its own especial pleasure to the writer.

1 Introduction

Francesco Calogero, but which *persona* should one choose? Professor of Theoretical Physics at the University of Rome "La Sapienza", expert on arms control disarmament and conflict resolution since 1963, some time Secretary General of Pugwash and consequently "Nobel Laureate" who received the Nobel Prize for Peace on behalf of that distinguished body in December 1995, subsequent Chairman of the Pugwash Council, regretful renegade as an amateur painter although his many pictures still (I believe) decorate his home[1], on his own admission a mediocre chess player (although he beat my younger son Patrick who 'wasn't bad'), multilingual – fluent in Italian, English, French (at least) and even able to chat in Russian, finding time to edit the quarterly Pugwash Newsletter, publisher of three hundred articles on Science and Society, organising (for Pugwash) 70 International meetings, – where could I even begin?

[1]Francesco gave us as a family the opportunity to stay in his empty house in Rome. Apart from the many pictures I recall that it rained continuously throughout this summer visit – no reflection on our putative hosts however!

2 Actual Beginnings

For my present purposes I can only begin where I first met Francesco, namely at the 'Conference on the Theory and Applications of Solitons' held in Tucson, Arizona, January 5-10 1976, and the persona is as Theoretical Physicist, the persona I actually know about best. Francesco is quoted in several places in Alan Newell's 'The inverse scattering transform, nonlinear waves, singular perturbations and synchronised solitons' published in the Proceedings [1] (most notably about Francesco's extension of the independent variable time t in an integrable p.d.e. by further times $\mathbf{y} = (y_1, \ldots, y_n)$. Newell [2] gives the associated integrable nonlinear evolution equation as

$$F(L^A)\begin{bmatrix} r_t \\ -q_t \end{bmatrix} + \mathbf{G}(L^A).\nabla_y \begin{bmatrix} r \\ -q \end{bmatrix} + 2\Omega(L^A)\begin{bmatrix} r \\ q \end{bmatrix} = 0 \qquad (2.1)$$

and L^A is operator equation (6.54) in Newell [2]). Remarkably Francesco is quoted also in the article by Bill Sutherland 'History of the *quantum* soliton' [3] in the same Proceedings at the reference 15.. The word *quantum* is here italicised (by me) since with Miki Wadati [4] I have recently been obliged to *define* a quantum soliton as such. But in his article [3] Sutherland is concerned only to embed Bethe's original ansatz into the general framework of solitons. At the reference 15. to a then recent preprint of F. Calogero, C. Marchioro and O. Ragnisco (published I believe as [5]) Sutherland points out that reference 15. recognizes that the momenta of the particles are the same for any arrangement of the particles so that the asymptotic form of the wave function must be of the form

$$\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n) \sim \sum_P A(P) \prod_{j=1}^{N} e^{ikP_j x_j}; \qquad (2.2)$$

the symbol P represents one of the $N!$ permutations of the k's given by $(P_1, P_2, \ldots, P_N)$. This wave function can be seen to be exactly Bethe's ansatz truly the beginning of the quantum solitons theory which has now become the Quantum Inverse Method [6].

3 Contribution to our book on solitons

From such illustrious beginnings it is remarkable to me that no paper appeared in these Proceedings from Francesco himself. Neverthless they apparently determined me to get *his* contribution to our book 'Solitons' which volume edited by myself and Philip Caudrey is already quoted by Alan Newell as 'to appear'. This article by Francesco did indeed slowly appear under the joint names of Francesco Calogero and his *alter ego* Antonio (Tony) Degasperis [7] (but read the Preface to our book [8] for the tortuously slow appearance of this particular reference, the reference [8]!). This article [7] was entitled 'Nonlinear Evolution Equations Solvable by the Inverse Spectral Transform Associated with the Matrix Schrödinger Equation' and it was exactly what this title forebodes – using generalised Wronskian techniques and one of the first applications of the matrix form of the Schrödinger equation in this connection (Wadati [9] and references also used the matrix form of the Schrödinger operator). This analysis, done first of all with full generality and extended by the many times $\mathbf{y} = (y_1, \ldots, y_n)$ augmenting time t as already described, combined to achieve *fundamentally new results*, namely systems with soliton solutions which did not move with constant speeds. Very recently [10] the two authors have extended a wholly comparable analysis to the AKNS systems which originally depended on the 2×2 matrix scattering problem of Zakharov and Shabat [11] and they have again found solitons with variable speeds in this context. In their original work [7] the two authors were already able to find their equations for the 'boomeron' soon to be followed by that for the 'zoomeron'. By replacing the dependent $N \times N$ matrix variable $Q(x,t)$ which appears as the potential in the Schrödinger problem $\psi_{xx} = [Q - k^2]\psi$ by the 2×2 matrix equation

$$U(x,t) + \sigma_n V_n(x,t) = \int_x^\infty dx' Q(x',t) \tag{3.1}$$

in which the σ_n are the three Pauli matrices and summation over n is understood the 'boomeron' equation emerges as

$$\begin{aligned} U_t(x,t) &= \mathbf{b}.\mathbf{V}_x(x,t) \\ \mathbf{V}_{xt}(x,t) &= U_{xx}(x,t)\mathbf{b} + \mathbf{a} \times \mathbf{V}_x(x,t) \\ &\quad -2\mathbf{V}_x(x,t) \times [\mathbf{V}(x,t) \times \mathbf{b}] \end{aligned} \tag{3.2}$$

in which $\mathbf{a}, \mathbf{b}$ and $\mathbf{V}(x,t)$ are three-vectors. Asymptotic conditions on the solutions are evidently

$$U(+\infty,t) = U_x(\pm\infty,t) = 0, \qquad \mathbf{V}(+\infty,t) = \mathbf{V}_x(\pm\infty,t) = \mathbf{0}. \tag{3.3}$$

Under the condition $\mathbf{a}.\mathbf{b} = 0$ this behaviour of non-constant speeds is illustrated by the Figs. (a-e) provided by J.C. Eilbeck in the book 'Solitons' [8] pp. 316-317. The one soliton solution exhibited is obtained from the explicit form

$$U(x,t) = -p[1 - \tanh[p(x - \xi(t))]], \qquad \mathbf{V}(x,t) = \hat{\mathbf{n}}(t)U(x,t) \tag{3.4}$$

and the "position" $\xi(t)$ and "polarisation" $\hat{\mathbf{n}}(t)$ (a unit vector) evolve in time according to

$$\begin{aligned} \hat{\mathbf{n}}_t(t) &= \mathbf{a} \times \hat{\mathbf{n}}(t) + 2p\hat{\mathbf{n}}(t) \times [\hat{\mathbf{n}}(t) \times \mathbf{b}] \\ \xi_t(t) &= -\mathbf{b}.\hat{\mathbf{n}}(t), \end{aligned} \tag{3.5}$$

equations which can be explicitly solved.

The 'zoomeron' is not mentioned in the article [7] but was to be treated (for example) in the two volumes, I & II of the massive compendium of work planned by Calogero and Degasperis 'Spectral transform and solitons: tools to solve and investigate nonlinear evolution equations' [12] where in Vol. I it is already given as

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\left(\frac{Z_{xt}}{Z}\right) + 2(Z^2)_{xt} = 0, \tag{3.6}$$

in a single scalar variable $Z(x,t)$. Here the solitons have an amplitude which changes with time t as well as a speed with this property. With 23 Appendices to the Vol. I this pair of volumes I & II would have covered almost everything known at 1982 on 'Spectral transforms and solitons'. Following in with the famous volume Courant-Hilbert Volume 2, the volume 2 by Calogero and Degasperis is still to appear and we look forward very much to its so doing. It is worth pointing out too that the usages 'Spectral Transform' and 'Nonlinear Evolution Equations' are, as far as I know them, themselves due to Francesco who persuasively explained both to me at the Corsica meeting (see below).

4 The Istanbul meeting

My next intersection with Francesco following the Tucson meeting (1976) must be at the NATO ASI “Nonlinear Equations in Physics and Mathematics” in Istanbul August 1-13 1977 which ASI Francesco helped to organise. With colleagues Philip Caudrey who spoke on ‘Hirota’s method of solving soliton-type equations’, Roger Dodd who spoke on ‘Prolongation structure techniques for the new hierarchy of Korteweg-de Vries equations’ and Tony Mason who spoke on ‘Perturbation theory for the double sine-Gordon equation’, work I have recently reproduced in my own paper [13], I gave a series of lectures on ‘Solitons in physics’ at Istanbul [14]. At this time I was much concerned with the value $\gamma_0 \sim 10^5$ I had elucidated for the coupling constant γ_0 for the quantised sine-Gordon equation applied to spin waves in the A-phase of ^3He below 2.6mK and subsequently I have concluded that this estimate had to be quite wrong! Anyway with John Hook [15] this *was* the value we had reached and I tried to live with this value in my paper. Philip Caudrey described a second Lax type hierarchy he had found with Roger Dodd and John Gibbon associated with the KdV equation and Roger Dodd applied the new prolongation techniques of Wahlquist and Estabrook [16, 17, 18] to the first member of the new hierarchy reported by Philip Caudrey namely to

$$(q_{4x} + \alpha q q_{2x} + (30 - \alpha) q_x^2 + 2\alpha q^3)_x + q_t = 0. \tag{4.1}$$

I have already mentioned that I have recently reproduced Tony Mason’s work [19] in my recent work on ‘Quantum information’ [13].

The Istanbul meeting was organised by Francesco and the late A.O.(Asim) Barut who also edited the Proceedings and it was held in the Robert College there that Barut had used for a couple of ASIs previously. I remember very well the iron bedsteads and the steel lockers we had available to use in the College and I remember also the problem I had with a local fisherman. He offered me Red Mullet at a wholly exceptional price amongst a spread of untagged and unpriced fish and when I vehemently protested against the price as it had emerged a crowd assembled: the problem was solved only when a voice from that crowd in perfect English suggested we simply split off the actual tax on my item but how we did that I don’t recall!

For *me* at least perhaps the most spectacular memory of Istanbul was the goat cheese offered for breakfast in Robert College itself – a title for these reminiscences! Asim had a total belief in the intervention of divine providence which allowed him to override trivial organisational matters such as this and anyway why should we *not* be offered goat cheese for breakfast in Istanbul? However less trivial was the production of the Conference Proceedings which Asim subsequently asked each of us to make a financial contribution to. In Manchester we were certainly in no position to do anything like this since we were working, as usual, on essentially zero money although the UK’s Science and Engineering Research Council (the SERC) were being very helpful with their post-doctoral support. Fortunately divine providence did indeed take a hand and the Proceedings edited by A.O. Barut duly appeared with our scientific contributions as described above.

I should point out that at this Istanbul ASI Francesco’s considerable scientific contribution was to give three lectures under the title ‘Integrable many-body problems’ something I had not at all been aware of. Francesco was not quite the first to put these in the context of the ‘Lax trick’ as he calls it (actually Moser [20] did this first). Hamilton’s equations for the one-dimensional model were expressed in terms of two matrices L and M of rank n and L is Hermitean $L = L^\dagger$, and M is anti-Hermitean $M = -M^\dagger$ exactly as in Lax [21]. Thus these Hamilton's equation become

$$\dot{L} = [M, L] \tag{4.2}$$

which is of course ultimately the famous zero curvature condition $U_t - V_x + [U, V] = 0$ (c.f. e.g. [15, 22]) and from equation (4.2) Francesco proved, as we would now expect, that the n eigenvalues of L are constants of the motion. He then identified a general class of Hamiltonians conforming to this prescription. These included an interaction between each *pair* of the particles of the many-body problem

$$V(x) = B\mathcal{P}(ax|\omega, \omega') + \text{constant} \tag{4.3}$$

and $\mathcal{P}$ is the Weirstrass function. This I thought very pretty at the time but I was also suspicious because I did not know of any real physical system where such interactions occurred in Nature! Even so Francesco's results influenced the actual way I wrote the Chapter 1 for our book 'Solitons' (see particularly [8] pp.49-53) including my reference to Airault *et al.* [23] who found direct connections between the rational solutions of the integrable KdV and Boussinesq p.d.e.s and solvable many-body problems. It is remarkable that Francesco had first of all solved the *quantum mechanical* problem with Hamiltonian

$$H = \frac{1}{2}\sum_j y_j^2 + g\sum_j \sum_{j<k}(x_j - x_k)^{-2}, \quad g > 0 \tag{4.4}$$

(y_j = momentum, $\dfrac{\partial H}{\partial y_j} = \dot{x_j}$, $\quad \dot{y_j} = -\dfrac{\partial H}{\partial x_j}$) in 1971 [24] and note again how he was referenced by Bill Sutherland.

Actually prior to the Istanbul meeting Francesco had organised the International Symposium on "Nonlinear Evolution Equations Solvable by the Inverse Spectral Transform" which took place at the prestigious Accademia dei Lincei in Rome, June 15-18 1977. My paper given with Philip Caudrey was 'The multiple sine-Gordon equations in nonlinear optics and in liquid ^3He' [25]. It embarrasses me to read this paper all over again now for the Theorem 3 is apparently quite wrong! This false theorem was 'A necessary and sufficient condition for the existence of an infinity of polynomial conserved densities for the equation $u_{xt} = F(u)$ is that $F(u) = Ae^{\alpha u} + Be^{-\alpha u}$ where α is a non-zero complex constant (and A, B are such that $F(u)$ is real). The theorem is wrong because it omits the integrable equation $u_{xt} = \exp(u) - \exp(-2u)$ which in the form

$$(\log h)_{xy} = h - h^{-2} \tag{4.5}$$

is the compatibility condition for the Gauss equations

$$\begin{aligned} r_{xx} &= (h_x r_x + \lambda r_y)/h \\ r_{xy} &= hr \\ r_{yy} &= (h_y r_y + \lambda^{-1} r_x)/h \end{aligned} \tag{4.6}$$

found by the Romanian mathematician G.Tzitzéica [26, 27] in 1910 in differential geometry: Mikhailov [28] first gave the infinite set of polynomial conserved densities for it.[2] Despite this embarrassing error much of the paper has proved correct although I have already mentioned the very strong misgivings for the coupling constant $\gamma_0 \approx 10^5$ we offered for the coupling constant for the B-phase of liquid ${}^3He$ at temperatures of order 2.6mK already in that paper [25].

There was a next brief interaction with Francesco during early 1978 namely at the 'Symposium on Nonlinear (Soliton) Structure and Dynamics in Condensed Matter' in Oxford, UK, June 27-29, 1978. The Organising Committee was S. Aubry, A.R. Bishop, R. Blinc, A. Bruce, R. Bullough, R.J. Elliott, J. Krumhansl, A Luther, T. Schneider, R. Stinchcombe, H. Thomas and S. Trullinger and I gave 'Solitons in Mathematics: Brief History' with Roger Dodd as well as 'Perturbation Theory of the Double sine-Gordon Equation' with P.W. Kitchenside, A.L. Mason and P.J. Caudrey. With Tony Degasperis Francesco gave the paper 'Novel Class of Nonlinear Evolution Equations Solvable by the Spectral Transform Technique. Including the So-Called Cylindrical KdV Equation'. The paper is relatively short (three pages only) and starts with the spectral problem

$$-\psi_{xx}(x,z) + [x + u(x)]\psi_{xz} = z\psi(x,z). \tag{4.7}$$

A Jost Function $f(z)$ is defined under boundary conditions for (4.7) involving the standard Airy functions $Ai(y)$ and $Bi(y)$ and the solution of the inverse problem is stated. The Remark which follows is 'There is a biunivocous correspondence between $u(x)$ and $f(z)$; and a constructive procedure is now available involving only linear calculus to construct f from u (direct problem); see eqs. (1-5) and u from f (inverse problem); see eqs. (7-10). The function $f(z,t)$ is then solved for

$$u_t = \alpha_0(t)u_x + \alpha_1(t)[u_{xxx} - 6u_x u - 4xu_x - 2u] \tag{4.8}$$

and several evolution equations related to (4.8) by change of variable are exhibited. A special case of one of these is the cylindrical KdV given as

$$q_t + q_{yyy} - 6q_y q + (2t)^{-1}q = 0. \tag{4.9}$$

The final § 3. gives very interesting conservation laws for (4.8). (At my next meeting with Francesco I *still* need to ascertain the meaning of "biunivocous"!).

In November 1978 and until October 1979 I took up a Visiting Professorship at NORDITA, Copenhagen, Denmark. With Alan Luther, NORDITA, and see Chapter 12 of [8], we planned a solitons and integrable systems programme and the first name we each had in mind for visitors to NORDITA under that programme was that of Francesco Calogero. Ever reliable, he did arrive – in the depths of the Danish winter earflaps outstanding and (as I remember him) almost wholly frozen. I believe he spoke well on that occasion but I can no longer remember the subject matter now.

Otherwise, after Istanbul (1977) and Oxford (1978) my next strong interactions with Francesco were at Lecce, Italy and then contiguously in time, at the Cargèse Summer School on Corsica, June 24-July 7, 1979. Somehow both Francesco and I *had* managed to

[2]Further bibliography for the Tzitzéica equation is contained in the author's paper for the ISLAND 2 Meeting to appear in the Glasgow Mathematical Journal during 2005.

take part in the first of the Lecce (Italy) meetings June 20-23, 1979 beforehand. We were both on the Scientific Committee for this meeting which in practice served to “launch” Marco Boiti and Flora Pempinelli into the solitons field. In that meeting I gave the paper ‘Geometry of the AKNS-ZS Inverse Scattering Scheme’ with Ryugo (Takashi) Sasaki who I had met in Copenhagen. It was from this work with Takashi that I was finally able to write the published form of the Chapter 1 for our book ‘Solitons’ [8] edited by myself and Philip Caudrey!

At this Lecce meeting in 1979 Francesco gave the first paper – entitled ‘Nonlinear Evolution Equations Solvable by the Spectral Transform: Some Recent Results’. Francesco was actually on leave at Queen Mary College, London up to June 1979, and Tony Degasperis spent the year in Manchester supported by a Science and Engineering Research Council (SERC) Visiting Fellowship – a happy arrangemant for all parties and for Francesco Calogero’s family in particular (I believe I was actually *commuting* weekly to Copenhagen during 1978 to 1979 so strong interaction with Tony at least (and with Philip Caudrey) was nevertheless wholly practicable!).

In his paper [29] which was a ‘solitons’ paper rather than a ‘many-body problem’ paper Francesco again treated both an NEE with x-dependent coefficients and the “cylindrical KdV” equation in the form (see equation (4.9))

$$u_t + u_{xxx} - 6u_x u + (2t)^{-1}u = 0. \qquad (4.10)$$

For the latter he explained as at the Oxford meeting beforehand why, in order to solve the Cauchy problem for (4.10) in the class of functions $u(x,t)$ which vanish asymptotically, it was necessary to introduce a novel spectral transform which he exhibited and discussed. (The background to this article [29] is, apart from the Oxford paper, set in the eight references [2, ..., 9] in the paper [29] with one exception work all done with Tony Degasperis).

The x-dependent coefficients problem he discusses in [29] interested me a lot because independently I had got on to a closely related problem with M. Lakshmanan [30]. Of course it is the behaviour of the conservation laws which is of greatest interest (in our paper [30] we made a small mistake which however is corrected in later papers).

The actual passage from Lecce to the meeting at Cargèse, Corsica on 24 June was exhausting to me and I got to Cargèse very early on the morning of 25 June. Therefore it was salutary to hear from Francesco that my first lecture, done with zero preparation and almost straight off the plane, was received by one of the Cargèse participants as ‘the best lecture he had ever heard’! The later lectures [31] proved rather hard going for the bulk of the participants however – as the organisers were to tell me! Despite the actual title of this NATO ASI which was “Bifurcation Phenomena in Mathematical Physics and Related Topics” these organisers (C. Bardos, Université de Paris-Nord and D. Bessis, Saclay) had asked me to lecture on ‘Inverse scattering and its applications’ – which I did appealing at that time to everything that I then knew about the action-angle variables of the infinite dimensional integrable models.

With further new results since Istanbul Francesco lectured on ‘Solvable many-body problems and related mathematical findings (and conjectures)’. Among other things he proved that the Hamiltonian, equation (4.4), taken in classical form, has a unique equilibrium configuration (up to permutations) the set of numbers x_j which are the n zeros of the Hermite polynomial of order n

$$H_n(x_j) = 0, \qquad j = 1, \ldots, n. \tag{4.11}$$

He also proves that the general solution of the classical equations of motion from equation (4.4) is completely periodic with period 2π and highlights a number of curious properties of this and related dynamical problems. A theme is the relationship between the motion of poles and zeros of special solutions of nonlinear and linear partial differential equations, and related "solvable" many-body problems as in his [32]. Finally he mentions that his results for the Hermite polynomials generalise to all of the classical polynomials (Laguerre, Jacobi) and to the Bessel functions.

For me my interaction with Francesco at Cargèse is summarised in the final § 6 of the published form of my lectures [31]. I believe that the whole of the § 6 was generated from discussion with Francesco (others perhaps but primarily Francesco). The reference to Moser's solving the many-body problem with x^{-2} interactions (equation (3.4)) is directly related to his, Francesco's, actual lectures; the reply to a criticism by Francesco on p. 340 of the published form is explicit evidence of our discussions. What the § 6 does not say is that I believe I was finally converted to 'spectral transforms' (STs) and 'inverse spectral transforms' (ISTs) by Francesco at this meeting. 'Nonlinear evolution equations' (NEEs) are for me his also as I noted. In this context I now also recall the joke pointed out by Francesco at the Istanbul meeting. Our luggage (and my rucksack in the lecture theatre) generally bore the tag IST (for Istanbul!).

The next interaction was in 1980 at Chania, Crete: with his friend Andonis Verganelakis, Nuclear Research Centre "Democritos", Physics Department, University of Athens and A. Papaderos of the Orthodox Academy of Crete (Francesco worked on nuclear many-body problems during 1967 to 1975) Francesco had organised the meeting 'Workshop on Nonlinear Evolution Equations and Dynamical Systems' held at the Orthodox Academy. Still very much concerned with physical applications of solitons, I gave a paper 'Some problems applicable and inapplicable' mainly about spin chains and especially with $CsNiF_3$ (which has a sine-Gordon limit) but also (at the inapplicable level) about the vanishing curvature conditions of our generalised AKNS systems [33]. Francesco's scientific contribution was 'Isospectral matrices' referring to *three* of his papers currently in the press 'Matrix differential operators and polynomials' (J.Math.Phys.), 'Isospectral matrices and polynomials' (Nuovo Cimento B) and 'Finite deformations of certain isospectral matrices' (Lett. Nuovo Cimento).

The importance of this Chania meeting really is that it is evidently the very first of the NEEDs meetings which Francesco has organised on a semi-regular basis ever since (see below). My personal documentation (my CV) shows that I went also to the second meeting in Chania, Crete in 1983 and gave a paper and this would be a NEEDs meeting NEEDs2. The NEEDs1 meeting is fraught in my recollection by the fact that Olympic Airlines (flying Athens – Crete) went on strike and I had to change carriers: this led me to purchase a new air-ticket which 'because he said he would' (as Francesco actually reminded me) Francesco refunded not without difficulty. Fortunately my own travel insurance came up with the cost of that ticket and I was able to pay the money back to Francesco soon afterwards. I cannot remember whether it was from the first or the second Chania meeting and the long trip down the Samaria gorge in each case that I retained this picture of Pierre

Sabatier, Montpellier, who had traversed the whole 18km of the gorge, white in the face and apparently very ill indeed. Fortunately he soon recovered.

At this stage the period that had involved at least an annual interaction with Francesco drew to a close: my theoretical interests, always based in physical applications anyway, moved to a slightly more different direction from his I suppose (notably to the statistical mechanics of solitons). Also by 1989 anyway Francesco became Secretary General to the Pugwash Conference on Science and World Affairs establishing a Pugwash Office located in the Accademia dei Lincei. The Fifth Workshop in 'Nonlinear Evolution Equations and Dynamical Systems' (NEEDs5) took place in Kolymbari, near Chania, Crete and I had missed NEEDs3 and NEEDs4 at Gallipolli (Italy 1985) and Balaruc near Montpellier (France 1987). I was in touch with Francesco to the extent that my very able assistant Zhuhan Jiang gave a paper (on the many-dimensions problem) at NEEDs5. Francesco's scientific contribution was his short paper 'C-Integrable Generalisation of a System of Nonlinear PDE's Describing Nonresonant N-Wave Interactions' [44]. By this time he had already written his significant paper 'Why are certain nonlinear PDEs both widely applicable and integrable?' [34] which heads this appreciation and this was a theme he continued with up to the KdV meeting in Amsterdam in 1995 where we both contrived again to be present together.

In some ways it has been this paper [34] by Francesco which has gained greatest resonance from me. For in the early discoveries of solitons in Manchester first reported in, for example, [35, 36] we were once again concerned with physical phenomena namely with the physical phenomenon of 'self-induced transparency'. I picked up the Maxwell-Bloch envelope system of equations for SIT from E.L. Hahn at Flagstaff, Arizona in 1969 [15] and my research background was in theoretical nonlinear optics and the emergent field of quantum optics. By 1973 my very able research group in UMIST, Manchester had solved both the sine-Gordon and the SIT equations for their multi-soliton solutions and gone on to both introduce, and to solve the initial value problem for, the system we called the 'Reduced Maxwell-Bloch (RMB) Equations' [37]. Since then I have realised just how remarkable these several solutions really were for from the RMB equations stems the whole *sequence* of integrable families of equations RMB, SIT (also called Maxwell-Bloch (MB)), sine-Gordon and nonlinear Schrödinger equation, integrability being handed down the sequence by the slowly varying envelope and phase approximation (SVEPA) or the non-relativistic limit in particular.

Francesco's paper [34] provides a heuristic but still very illuminating explanation of this 'miracle' of a both widely applicable and integrable equation such as the $1+1$ dimensional sine-Gordon equation (Francesco himself offers as example the nonlinear Schrödinger equation: my choice of sine-Gordon is exactly because of the last paragraph!). I think it was at the Korteweg-deVries equation KdV '95 meeting in Amsterdam that I first actually heard Francesco make his key point that it was heuristically sufficient for the integrability of any 'universal' equation, deriving by a rigorous if asymptotic analysis from a large class of equations, that one *at least* of the large class was integrable. This was the viewpoint he worked from with considerable illumination for me in his papers [34, 38, 39, 40] focussing in particular on nonlinear PDEs whose linear part is dispersive and whose nonlinear part is, in some very weak sense, analytic.

My most recent interactions with Francesco have included NEEDs12 (NEEDs in Leeds) 21-28 June 1998, Leeds, England and the Cambridge (UK) meeting forming a part of the integrable systems programme at the Isaac Newton Institute organised by Vladimir Zakharov together with Sasha Mikhailov and Paolo Santini. In the NEEDs12 meeting I spoke under the title 'Quantum integrabilty and quantum chaos in the micromaser' presenting joint work by myself, N.M. Bogoliubov (St. Petersburg) and R.R. Puri (Bombay) [41]. Apparently Francesco gave no paper at this NEEDs12 meeting but I am sure I am right in believing that he appeared there! However everything is absolutely clear about the Cambridge meeting: this is the NEEDs15 meeting and Francesco spoke on the 'Solvable Three-Body Problems and Painlevé Conjectures' [42] and I spoke on 'Quantum integrable and non-integrable nonlinear Schrödinger models for realisable Bose-Einstein Condensation in $d+1$ dimensions ($d = 1, 2, 3$)' [43]. The two titles rather illustrate the separation which had developed between the research interests of myself and Francesco – particularly as the *classical* Nonlinear Schrödinger model had proved to be so fundamental to his, Francesco's, analysis of widely applicable and integrable systems.

Sadly, as he approaches 70 years of age next year, 2005, and I approach 74 this year, 2004, it is not clear where the next interaction between us will actually take place. It will certainly be fruitful for all of my interactions with Francesco have been fruitful. Indeed I have tried to show in this appreciation how, especially in our early working years 1976-1980, Francesco Calogero had a significant influence on much of the work I actually produced. I have also made clear I believe the debt the whole community of nonlinear systems people owes to Francesco Calogero. It seems certain that in its hands the NEEDs meetings must continue in different parts of Europe well into the future.

I finish with an aphorism well illustrating Francesco's attitude to mathematics. More latterly David Crighton, who, until his untimely death in 1999, 'ran' the UK's funding agency for mathematical research in the UK, the so-called SERC (now EPSRC), had co-opted Francesco to his Mathematics Committee. Philip Caudrey and myself had been applying to this SERC Mathematics Committee more-or-less successfully for work on solitons and integrable systems since our first discoveries in this field in 1973. A typical application would be for one research assistant for two years *and the funds to go with it* and a fine example of the result of this strategy was Zhuhan Jiang mentioned reference NEEDs5 above. Francesco's viewpoint was however quite different namely that if a research problem is *really* worth doing it is one which can only be done by oneself – a piece of irrefutable logic which we have to accept even if expediency dictates quite otherwise!

Both Philip Caudrey and I dedicate this article to Francesco. We can scarcely wish him a happy retirement for we still hope for other NEEDs meetings and more illuminating papers on subjects as diverse as integrable multi-dimensional systems, on integrable many-body problems and indeed on the origin of Nature's quantum theories in particular. We hope indeed (and for the background see [52, 53]) that his present conjecture that quantization is vested in the gravitational field proves to be entirely right!

Finally the actual writer of this article, Robin Bullough, apologises for the occasional lapses of memory which Francesco will find when he reads it. A particular worry as I write is when did Francesco *actually* come to NORDITA, Copenhagen as described in the text. Is this during the hectic years of 1979-80 as stated or was it actually later? I hope Francesco can tell me directly in due course.

References

[1] Newell A C, The inverse scattering transform, nonlinear waves, singular perturbations and synchronised solitons, in Proceedings of the Conference on the Theory and Applications of Solitons, Tucson January 5-10 1976 Editors: Flaschka H and McLaughlin D W, *Rocky Mountain Journal of Mathematics* **8**, Nos. 1 and 2, (Winter and Spring 1978), 25–52.

[2] Newell A C, The inverse scattering transform in Solitons, Editors: Bullough R K and Caudrey P J, *Topics in Current Physics* **17**, Springer-Verlag, Heidelberg, 1980, 177–242.

[3] Sutherland Bill, A brief history of the quantum soliton with new results on the quantisation of the Toda lattice, in Proceedings of the Conference on the Theory and Applications of Solitons, Tucson January 5-10 1976 Editors: Flaschka H and McLaughlin D W, *Rocky Mountain Journal of Mathematics* **8**, Nos. 1 and 2, (Winter and Spring 1978), 413–428.

[4] Bullough R K and Wadati Miki, Optical solitons and quantum solitons, *J. Opt. B*: *Quantum semi-class. Opt.*, **6**, 2004, S205–S216.

[5] Calogero F *et al.*, *Lett. Nuovo Cimento*, **13**, 1975, 383

[6] Korepin V E, Bogoliubov N M and Izergin A G, Quantum Inverse Scattering Method and Correlation Functions, *Cambridge Monographs on Mathematical Physics*, Cambridge University Press, Cambridge, UK. First paperback edition, 1997.

[7] Calogero F and Degasperis A, Nonlinear Evolution Equations Solvable by the Inverse Spectral Transform Associated with the Matrix Schrödinger Equation in Solitons, Editors: Bullough R K and Caudrey P J, *Topics in Current Physics*, **17**, Springer-Verlag, Heidelberg, 1980, Chapter 9, 301–338.

[8] Solitons, Editors: Bullough R K and Caudrey P J, *Topics in Current Physics*, **17**, Springer-Verlag, Heidelberg, 1980.

[9] Wadati M, Generalised Matrix Form of the Inverse Scattering Method, in *Solitons* [8], 288–299.

[10] Calogero F and Degasperis A, New Integrable Equations of Nonlinear Schrödinger Type, *Studies in Applied Mathematics*, **113**, (1), 2004, 91–137.

[11] Zakharov V E and Shabat A B, Exact theory of two-dimensional self-focussing and one-dimensional self-modulation of waves in nonlinear media, *Zh. Eksp. Teor. Fiz*, **61**, 1971, 118–125 [English translation: *Soviet Physics, JETP*, **34**, 1972, 62–69].

[12] Calogero Francesco and Degasperis Antonio, Spectral transform and solitons: tools to solve and investigate nonlinear evolution equations. Vols I and II. North-Holland Publ. Co., Amsterdam, 1982.

[13] Bullough R K and Gibbs H M, Information storage and retrieval by stopping pulses of light, *Journal of Modern Optics* **51**, No.2, 2004, 255–284.

[14] Bullough R K, Solitons in Physics, in *Nonlinear Equations in Physics and Mathematics* NATO Advanced Study Institutes Series. Series C: Mathematical and Physical Sciences. Editor: Barut A O, (D. Reidel Publishing Company, Dordrecht, Holland), 99–133. Also Caudrey P J, 177-192; Dodd R K, 193–204; Mason A L, 205–218.

[15] Bullough R K, The optical solitons of QE1 are the BEC of QE14: has the quantum soliton arrived? Special Foundation Lecture: a personal view. National Quantum Electronics and Photonics Conference, Owens Park, University of Manchester, 8 September, 1999. *Journal of Modern Optics* **47**, No. 11, 2000, 2029–2065, see page 2052. (Erratum: *Journal of Modern Optics* **48**, No. 4, 2001, 747–748).

[16] Wahlquist H D and Estabrook F B, Prolongation Structures of Nonlinear Evolution Equations, *J. Math. Phys.*, **16**, 1975, 1–7.

[17] Estabrook F B and Wahlquist H D, Prolongation Structures of Nonlinear Evolution Equations II, *J. Math. Phys.*, **17**, 1976, 1293–1297.

[18] Estabrook F B and Wahlquist H D, Prolongation Structure, Connection Theory and Bäcklund Transformations in Nonlinear evolution equations solvable by the spectral transform, *Pitman Research Notes in Mathematics.* Editor: Calogero F, Pitman Publishing Ltd., London, 1978, 64–83.

[19] Mason A L, Perturbation theory for the double sine-Gordon equation, in *Nonlinear Equations in Physics and Mathematics*, Editor: Barut A O, Reference [14], 205–218.

[20] Moser J, Three integrable Hamiltonian systems connected with isospectral deformations, *Adv. in Math.*, **16**, 1975, 197.

[21] Lax P D, Integrals of nonlinear equations of evolution and solitary waves, *Comm. Pure Appl. Math.*, **21**, 1968, 467–490.

[22] Bullough R K, Optical solitons: twenty-seven years of the last millennium and three more years of the new? in *Mathematics and the 21st Century.* Editor: Ashour A A and Obada A-S F, World Scientific Publishing Co., 2001, 69–121.

[23] Airault H, McKean H P and Moser J, Rational and elliptic solutions of the Korteweg-deVries equation and a related many-body problem, *Comm. Pure Appl. Math.*, **30**, 1977, 95.

[24] Calogero F, Solution of the one-dimensional N-body problem with quadratic and/or inversely quadratic pair potentials, *J. Math. Phys.*, **12**, 1971, 419.

[25] Bullough R K and Caudrey P J, The multiple sine-Gordon equations in nonlinear optics and in liquid ^3He, in *Nonlinear evolution equations solvable by the spectral transform*, Pitman Research Notes in Mathematics **26**. Editor: Calogero F, Pitman Publishing Ltd., London, 1978, 180–224.

[26] Tzitzéica G, *Comptes Rendus Acad. Sci.*, **150**, 1910, 955–956. (Title believed to be: Géometrie infinitésimale – sur un nouvelle classe de surfaces).

[27] Tzitzéica G, *Comptes Rendus Acad. Sci.*, **150**, 1910, 1227–1229. (Title believed to be: Géometrie infinitésimale – sur un nouvelle classe de surfaces).

[28] Mikhailov A V, The reduction problem and the inverse scattering method, *Physica*, **3D**, 1981, 1 & 2, 73–117.

[29] Calogero F, Nonlinear Evolution Equations Solvable by the Inverse Spectral Transform: some recent results, in *Nonlinear Evolution Equations and Dynamical Systems.* Editors: Boiti M, Pempinelli F and Soliani G, Springer Lecture Notes in Physics, **120**, Springer-Verlag, Heidelberg, New York, 1980, 1–14. References [2, ..., 9] of [29] mentioned in the text are given in this list of references below as [44], ..., [51].

[30] Lakshmanan M and Bullough R K, Geometry of generalised nonlinear Schrödinger and Heisenberg ferromagnet spin equations with linearly x-dependent coefficients, *Phys. Lett.*, **80A**, 1980, 287–292.

[31] Bullough R K, Solitons: Inverse Scattering Theory and Its Applications, in *Bifurcation Phenomena in Mathematical Physics and Related Topics*, Proceedings of the NATO Advanced Study Institute at Cargèse, Corsica. Editors: Bardos C and Bessis D. Advanced Studies Institutes Series, Series C: Mathematical and Physical Sciences (D. Reidel Publishing Company, Dordrecht, Holland), 295–347.

[32] Calogero F, Motion of poles and zeros of special solutions of nonlinear and linear partial differential equations and related "solvable" many-body problems, *Nuovo Cimento*, **43B**, 1978, 177–241.

[33] Sasaki R and Bullough R K, Geometry of the AKNS-ZS Inverse Scattering Scheme, in *Nonlinear Evolution Equations and Dynamical Systems*, Lecture Notes in Physics, **120**. Editors: Boiti M, Pempinelli F and Soliani G, Springer-Verlag, Heidelberg, 1980, 314–337.

[34] Calogero F, Why are certain nonlinear PDEs both widely applicable and integrable?, in *What Is Integrability*, Editor: Zakharov V E, Springer Series in Nonlinear Dynamics, Springer-Verlag, Heidelberg, 1991, 1–62.

[35] Caudrey P J, Gibbon J D, Eilbeck J C and Bullough R K, Exact Multisoliton Solutions of the Self-Induced Transparency and Sine-Gordon Equations, *Phys. Rev. Lett.* **30**, 1973, 237–238.

[36] Eilbeck J C, Gibbon J D, Caudrey P J and Bullough R K, Solitons in non-linear optics I. A more accurate description of the 2π-pulse in self-induced transparency, *J. Phys. A* **6**, 1973, 1337–1347.

[37] Gibbon J D, Caudrey P J, Bullough R K and Eilbeck J C, An N-Soliton Solution of a Nonlinear Optics Equation Derived by a General Inverse Method, *Lett. al Nuovo Cimento* **8**, 1973, 775–779.

[38] Calogero F, Universal integrable nonlinear PDEs in *Applications of Analytical and Geometrical Methods to Nonlinear Differential Equations*, Editor: Clarkson Peter A, Advanced Studies Institutes Series, Series C: Mathematical and Physical Sciences (D. Reidel Publishing Company, Dordrecht, Holland), 109–114.

[39] Calogero Francesco, Integrable Nonlinear Evolution Equations and Dynamical Systems in Multidimensions, *Acta Applicandae Mathematicae*, **39**, 1995, 229 – 244. (Paper to the Proceedings of the KdV '95 meeting).

[40] Also see *e.g.* Calogero F and Eckhaus W, Nonlinear Evolution Equations, Rescalings, Model PDEs and Their Integrability I and II, *Inverse Problems*, **3**, 1987, 229 – 262 and *ibid.* **4**, 1988, 11 – 33.

[41] Bullough R K, Bogolyubov N M and Puri R R, Quantum Integrability and Quantum Chaos in the Micromaser, *Theoretical and Mathematical Physics* **122** No. 2, 2000, 151–169 (Translated from *Teoreticheskaya i Matematicheskaya Fizika*, **122** No. 2, 2000, 181–203.

[42] Calogero F, Solvable 3-Body Problems and Painlevé Conjectures, *Theoretical and Mathematical Physics* **133** (No. 2), 2002, 1445–1454 (Translated from *Teoreticheskaya i Matematicheskaya Fizika*, **133** (No. 2), 2002, 149–159.

[43] Bullough R K, Bogolyubov N M, Kapitonov V S, Malyshev C, Timonen J, Rybin A V, Varzugin G G and Lindberg M, Quantum Integrable and Nonintegrable Nonlinear Schrödinger Models for Realizable Bose-Einstein Condensation in $d+1$ Dimensions ($d = 1, 2, 3$), *Theoretical and Mathematical Physics* **134** (1), 2003, 47–61 (Translated from *Teoreticheskaya i Matematicheskaya Fizika*, **134** (1), 2003, 55-73.

[44] Calogero F, C-Integrable Generalization of a System of Nonlinear PDE's Describing Nonresonant N-Wave Interactions in *Nonlinear Evolution Equations and Dynamical Systems.* Editors: Carillo S and Ragnisco O, Springer Research Reports in Physics, Springer-Verlag, Heidelberg, 1990, 102 – 104.

[45] see Ref. ([12]

[46] Calogero F and Degasperis A, Extension of the spectral transform method for solving nonlinear evolution equations, *Lett. Nuovo Cimento*, **22**, 1978, 131–137.

[47] Calogero F and Degasperis A, Exact solution via the spectral transform of a nonlinear evolution equation with linearly x-dependent coefficients, *Lett. Nuovo Cimento*, **22**, 1978, 138–141.

[48] Calogero F and Degasperis A, Extension of the spectral transform method for solving nonlinear evolution equations II, *Lett. Nuovo Cimento*, **22**, 1978, 263–269.

[49] Calogero F and Degasperis A, Exact solution via the spectral transform of a generalisation with linearly x-dependent coefficients of the modified Korteweg-de Vries equation, *Lett. Nuovo Cimento*, **22**, 1978, 270–273.

[50] Calogero F and Degasperis A, Exact solution via the spectral transform of a generalisation with linearly x-dependent coefficients of the nonlinear Schrödinger equation, *Lett. Nuovo Cimento*, **22**, 1978, 420–424.

[51] Calogero F and Degasperis A, Conservation laws for classes of nonlinear evolution equations solvable by the spectral transform, *Comm. Math. Phys.*, **63**, 1978, 155–176.

[52] Calogero F, Cosmic origin of quantization, *Phys. Lett. A*, **228**, 1997, 335–346.

[53] Calogero F, Cosmic origin of quantization, *Int. J. Modern Phys.*, **18**, 2004, 519–525.