A Method for Generating the Timetable of Double-track Railway Line

ZHANG Yan
Transportation and Economics Institute
China Academy of Railway Sciences
Beijing, China
zhangyanyj@rails.cn

CUI Yanping, YANG Wentao
Transportation and Economics Institute
China Academy of Railway Sciences
Beijing, China
cuiyanp, yangwt@rails.cn

***Abstract*—Passenger and freight train scheduling problem on double-track railway line is considered by using Ant Colony Optimization (ACO) algorithm. The aim is to reasonably arrange the dispatch sequence of the trains to minimize the total run time. The constrains in train scheduling problem are considered and the model is established. Due to the complexity of train scheduling problem, this problem is solved by ACO and implemented by programming. A case study is presented to illustrate the solution. The results illustrate that the proposed method is effective to solve the scheduling problem on double-track railway line.**

Keywords-timetable; double-track railway; ACO

I. INTRODUCTION

Solving train scheduling problem by intelligent method is the key technology in the realization of railway planning intellectualization and also the most important links in improving the efficiency of transportation. This problem is NP-hard problem and solving it with exact algorithm is very difficult and time consuming.

Some algorithms of the metaheuristics method include genetic algorithm [1], neural networks [2], tabu search [3], simulation annealing [4] and ACO [5]. Although the solution quality of these algorithms is high, there is still little research on passenger and freight train scheduling problem on double-track railway line.

Noon et al. [6] showed that the train scheduling problem can be easily transformed to a Travel Salesman Problem (TSP) and the trains represent the cities. Dorigo and Gambardella [7] showed that the ACS algorithm has been more successful than the other metaheuristics in solving the TSP. K. Ghoseiri [5] has successfully developed the ACS algorithm applied in the single-track railway line. But until now, nobody consider the passenger and freight train scheduling problem in double-track railway line which has the different solution method with the problem in single-track line. In this research, it is decided to solve the passenger and freight train scheduling problem on double-track railway line using ACS algorithm.

II. DESCRIPTION OF THE PROBLEM

Train scheduling is a combinatorial optimization problem. In this problem the aim is to determine the arrival and departure times from stations on which the train passes. In this model it is supposed that scheduling problem happens on the double-track railway line, and trains are just dispatched from the first and the last station. After preparation, the trains in the beginning or end station should be dispatched immediately. Meanwhile, the speed and trip times in each track section for each train are assumed to be fixed. The dispatched trains from left to right and also dispatched trains from right to left form two independent sub-networks of the TSP, so there is no interaction in sections between trains from different direction.

In this model, we consider the potential collisions that may happen in the section and station. Meanwhile, the passenger trains should be firstly arranged, and then freight trains are considered. The important trains should be firstly arranged, such as high-speed railway trains, and then the less important trains are considered, such as slow trains. These can be expressed in the importance weight of each train.

III. THE ESTABLISHMENT OF THE MODEL

A. The Process of the Movement of the Train

Set the number of sections is m, the number of stations is m+1, start time of scheduling is td, the length of scheduling time is th, so the scheduling time range is [td, td+ th]. In this time range, totally there are n trains need to be managed, they are labeled as 1, 2… n, and their importance weight are labeled as $\omega_1, \omega_2, \cdots, \ \omega_n$. The route of train i is appointed as Ri= (ri1, ri2…rim). (If the route of train i does not includes rk, then the rk=∞, this is applied to the condition that there is branches in the track.) So the route of total trains can be labeled as in (1).

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} \qquad (1)$$

Accordingly, the travel time matrix in the section can be labeled as in (2).

$$T = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1m} \\ t_{21} & t_{22} & \cdots & t_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ t_{n1} & t_{n2} & \cdots & t_{nm} \end{bmatrix} \qquad (2)$$

Wherein, Ti means the time that train i pass its route. If the route of train i does not includes rik, then the tik=0.

TABLE I. NOTATIONS IN THE MODEL

Notations	*Description*
R	the group of trains that should be dispatched from right station to left
L	the group of trains that should be dispatched from left station to right
T	the group of total trains. ($i, j \in R$ or L or T and $T = R \cup L$)
k	track section *"k"* is a section of track that connects two stations *"k"* and *"k+1"*
S	set of section($k \in S$)
t_{ik}	travel time: train *"i"* needs to pass track section *"k"*.
d_{ik}	dwell time: the minimum dwell time of train *"i"* in station *"k"*
h_{ijk}	headway: minimum time interval between trains *"i"* and *j* to arrive/ depart in/ from track section *"k"*
ω_i	train importance weight
$Xa(i, k)$	the arrival time of train *"i"* to station *"k"*
$Xd(i, k)$	the departure time of train *"i"* from station *"k"*
L_k	the number of tracks in station *"k"*
$L_k(t)$	the number of occupied tracks in station "*k*" at time "*t*".
t_h	the start time of scheduling
t_d	the length of scheduling time

Then, we defined the dwell time for train *i* at station *k* as matrix *D* as in (3).

$$D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ d_{21} & d_{22} & \cdots & d_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & \cdots & d_{nm} \end{bmatrix} \tag{3}$$

B. Notation

For the later convenience, notations that will be used in the model are introduced as in Table 1.

C. Objective Function

As was mentioned before, the aim of this problem is to reasonably arrange the train dispatch sequence to minimize the total running time of the trains. So the objective function can be described as in (4).

$$\min Z = \sum_{i=1}^{n} \omega_i [X_d(i, m+1) - X_a(i, i)] \tag{4}$$

D. Constraints

Closely Preceding Train: the train which is in front most closely to the train at the behind.

Closely Preceding Section: the closest section before one section.

According to this, in the following model, train "*i*" is the *Closely Preceding Train* of train "*j*", and section "*k*-1" is the *Closely Preceding Section* of section "*k*".

Then, in this paper, take the above hypotheses as the prerequisite, 5 constraints are considered:

(1) *Constraints in sections*: constraint of the sequence of the train passing the sections.

As in (5), only when train "*i*" passes the section "*k*-1", it can pass the section "*k*".

$$X_d(i, k+1) - X_d(i, k) > 0 \tag{5}$$

(2) Constraint of time interval between departure and arrival

As in (6), train "*j*" can depart from station "*k*-1" only after a headway when train "*i*" arrives at station "*k*".

$$X_d(j, k-1) - X_a(i, k) > h_{ijk} \quad (i, j \in L, k \in S) \tag{6}$$

(3) *Constraints in stations*: constraint of time interval between arrival and departure.

As in (7), only when train "*i*" departs from station "*k*", train "*j*" can enter into station "*k*".

$$X_a(j, k) - X_d(i, k) > h_{ijk} \quad (i, j \in L, k \in S) \tag{7}$$

(4) Constraint on station tracks

As in (8) and (9), the total number of trains in station "*k*" at time "*t*" should not surpass the total number of tracks in station "*k*".

$$L_k(t) \le L_k,\ L_k(t) = \sum_{i=1}^{n} \Gamma(X_d(i,k), X_a(i,k)) \tag{8}$$

$$\Gamma(X_d(i,k), X_a(i,k)) = \begin{cases} 1 & X_a(i,k) \le t < X_d(i,k) \\ 0 & else \end{cases}, \forall t \in [t_b, t_b + t_d] \tag{9}$$

(5) The travel time in section "*k*":

$$p_{ik} = t_{ik} + O_{i,k} t_{ik}^d + O_{i,k+1} t_{i,k+1}^a \tag{10}$$

Where,

$$O_{i,k} = \begin{cases} 1 & \text{train "}i\text{" stops at station "}k\text{"} \\ 0 & \text{train "}i\text{" does not stop at station "}k\text{" but pass it} \end{cases} \tag{11}$$

t_{ik}^d 、 $t_{i,k+1}^a$ are the additional time for train to start and stop.

IV. Ant Colony Optimization

ACO was suggested as a new heuristic method to solve optimization problem by Dorigo and L. M. Gambardella [7]. This is reformed form of AS algorithm and functions as follows.

A. State Transition Rule

TABLE II. NOTATIONS IN THE MODEL

Notations	*Meanings*
q	a random number uniformly distributed in [0,…, 1]
q_0	a parameter between 0 and 1
$\tau_{ij}(t)$	the amount of pheromone in edge *ij*
$\eta_{ij}(t)$	the heuristic information of edge *ij*
α	the importance weight of τ
β	the importance weight of η
N_i^k	the remaining nodes set of ant *k* bases on moving from node *i* to build an feasible solution.

Each ant generates a complete solution by choosing the nodes according to a probabilistic state transition rule. The state transition rule given in (12) and (13) is called a pseudorandom-proportional rule:

$$s=\begin{cases}\arg[\max_{j\in N_i^k}\{[\tau_{ij}(t)][\eta_{ij}(t)]^{\beta}\}] & \text{if } q<q_0\\ S & \text{if } q\ge q_0\end{cases} \quad (12)$$

Where, S is a random variable selected according to the possibility distribution given in (13):

$$p_{ij}^k(t)=\begin{cases}\dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}(t)]^{\beta}}{\sum_{j\in N_i^k}[\tau_{ij}(t)][\eta_{ij}(t)]^{\beta}} & S\in N_i^k\\ 0 & \text{else}\end{cases} \quad (13)$$

Where, the meanings of the notations are shown in Table 2.

Here, we consider punctuality of the train as in (14), the heuristic information:

$$\eta_{ij}(t)=vt_{ij} \quad (14)$$

The punctuality of the train vt_{ij} can be identified as in (15)

$$vt_{ij}=\begin{cases}1 & z_{ik}=0\\ (t_d-z_{ik})/t_d & 0<z_{ik}<t_d\\ 0 & z_{ik}>t_d\end{cases} \quad (15)$$

Where, as show in (16), z_{ik} is the total delay time for train "i" depart from station "k".

$$z_{ik}=\sum_{g=1}^{k}[X_d(i,g)-X_a(i,g)-d_{ig}] \quad (16)$$

B. Global Updating Rule

In ACO, only globally best ant which has built the best solution, deposits pheromone in the graph. When all the ants built their solution, global updating rule of pheromone is applied, this rule is as in (17).

$$\tau_{ij}(t+n)\leftarrow(1-\rho)\tau_{ij}(t)+\rho\Delta\tau_{ij}(t) \quad (17)$$

Where $0<\rho<1$ is pheromone decay parameter and $\Delta\tau_{ij}$ equals to (18).

$$\Delta\tau_{ij}=\begin{cases}1/\cos\ t_{gb} & \text{if}(i,j)\in\psi^{gb}\\ 0 & \text{if}(i,j)\notin\psi^{gb}\end{cases} \quad (18)$$

ψ^{gb} is the best solution which was built and $\cos t_{gb}$ is the cost of the best solution.

In ACO, ants perform step-by-step pheromone updates using local updating rule of pheromone. These updates are performed to favor the emergence of other solutions than the best so far. The updates result in step-by-step reduction of the pheromone level of the visiting edges by each ant. The local updating rule of pheromone is performed by applying the rule as in (19).

$$\tau_{ij}\leftarrow(1-\xi)\tau_{ij}+\xi\tau_0 \quad (19)$$

τ_0 is a small fixed value and $0<\xi<1$ is the local evaporation coefficient of pheromone.

C. ACO Applied in TS

According to the above definition, selected path of each ant in the trains' network indicates the dispatching sequence of train.

In this algorithm, a colony consists of n ants where n is number of the nodes (trains) of the TSP. The ants are allocated in n groups. At first, both ants are placed at the figurative node of zero (the start node). Then, one of the ants is chosen from the group randomly. The first chosen ant chooses a train in group by using the pseudorandom-proportional rule as in (12) and (13). The arrival and departure times of the train from start station to the final station are calculated. Then another ant chooses its train randomly. The arrival and departure times of this train from each station are determined in regard to reconciliation of any collision incurred with the preceding train. In the case of collision, it is removed. This operation continues in the same way so that all the arrival and departure times from all stations are identified and there are not any collisions in the sections. Then the next train is chosen by other ants. The process of opposite trains is the same. This procedure continues until ants choose all the trains of the group.

V. The Case Study

According to the character of the model and algorithm, Matlab is decided to be used to program. At first, a set of values for parameters were adopted in the experiment:

q0=0.9, ρ=0.6, ζ=0.1, τ0=0.000005, α =0.1, β=0.7

The timetable of the experiment of 4 stations and 15 trains is shown in Fig.1.

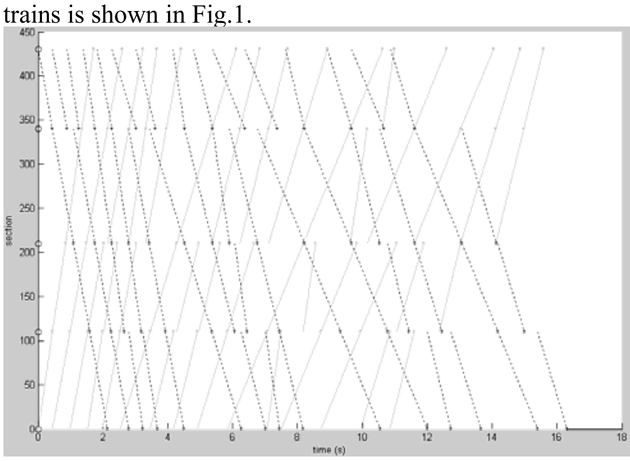

Figure 1. Timetable of the trains traveling for 4 stations and 15 trains

VI. Conclusion

This paper developed an algorithm for generating the timetable of the passenger and freight train on double-track railway line using the ant colony system. A mathematical model for a kind of train scheduling problem was developed and then the algorithm based on ACO was presented to solve the problem. ACO determined the dispatching sequence of trains on the graph of the TSP. Using the sequences obtained and removing for collisions incurred, railway timetable is generated. According to the case study, the ACO results in good quality in this particular problem.

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