The Improvement Algorithm of Wavelet in Spectral Analysis

CUI WEI PING, CAO ZHI WEN, LIU YONG

Department of Electronic and Information Engineering, North China Institute of Science and Technology, East Yanjiao,Beijing,China,101601

Department of Electronic and Information Engineering, North China Institute of Science and Technology, East Yanjiao,Beijing,China,101601

HANDAN IRON &STEEL GROUP CO.,LTD, Fuxing Road 232, Handan, Hebei,China,056000

E-MAIL: cuiweiping@ncist.edu.cn, czw72413@ncist.edu.cn, liuyongcwp@yahoo.cn

***Abstract*-This article is mainly about the application of wavelet in spectrum analysis. In signal processing, we usually need to know different frequency components in the signal. We use the frequency bands dividing characteristic of wavelet and FFT to process signals and pick up different frequency components. In processing, we find that there is frequency alias in sub-band reconstruction which is caused by Mallat algorithm. In this article, we offer a solution of this problem.**

Keywords-Wavelet Analysis, Spectrum Analysis, Mallat Algorithm, Frequency Alias

I. INTRODUCTION

Wavelet transform is a cogent tool of mathematics analysis, which attracts attention widely in recent years. The wavelet transform has character of multi-scale, multi-resolution analysis and time-frequency domain localize. The time and spectrum analysis technology that based on wavelet transform becomes an important tool in the analysis of non-stationary signal. However, in practice, there are some problems.

II. ISSUES ON THE WAVELET ANALYSIS IN THE SPECTRUM

Let the signal x by a 5Hz, 35Hz, 75Hz, 150Hz frequency sine wave four, namely

$$x=\sin(10\pi t)+\sin(70\pi t)+\sin(150\pi t)+\sin(300\pi t)$$

Based on the time-frequency characteristics of wavelet transform, the output signals are decomposed in three layers by adopting db10 wavelet. The signals reconstructed from each layer's wavelet coefficients, then will be transformed from time domain to frequency domain by Fast Fourier Transform. By Matlab program simulation, The results of the reconstructed signals are shown in Figure 1.

In theory, according to the Mallat algorithm, the frequency components of three layers should be included in Table1.

Table1

layer	1	2	3
low_frequency	5,35,75	5,35	5
high_frequency	150	75	35

But, the actual decomposition results are shown in Table 2.

Table2

layer	1	2	3
low_frequency	5,35,75，125	5,35,65	5,15
high_frequency	75,125,150	65，75,125	15，35,65

The result from Table 2 shows, using the Mallat algorithm for frequency division is not entirely feasible, because the resulting spectral components is not very accurate. Although the Main Frequency components in the diagram is obvious, but there are a lot of smaller amplitude frequency value can also be seen in Fig1.

The extra frequency components can be divided into two categories, of which one can be clearly seen that the value is the same frequency, but different amplitude. In Fig.1, the first layer of high-frequency reconstructed signal spectrum, there is obviously 75Hz, but it is much smaller amplitude, while the other is unlike any of the frequency, the amplitude is not small. In Fig.1, the first layer of high-frequency reconstructed signal, the middle of the protrusion is the case, its frequency is 125Hz, unlike any of the frequency.

III. THE REASON OF ERRORS

(1) The cause of the first error frequency

The first class of the excess frequency, its value is same, but the amplitude is small. The significant characteristic is that high frequency components appear in the reconstructed signal at low frequency, and low-frequency components appear in the high reconstructed signal. It is natural to think of frequency leakage. frequency leakage was mainly due to the filter.

In the Mallat algorithm, Wavelet filter is the half-band filter. In theory, Mallat algorithm requires that the filter should be the ideal cut-off characteristics. In fact, it is not ideal. The Daubechies wavelet filters, for example, take the sampling frequency is 400Hz, the frequency domain characteristics are compared respectively at N = 30,40. Fig.2 shows, the junction is not sharp in the low-pass and the high_pass, but extends into each other, the length of extending is increasing with the value of N decreasing.

As the filter is determined with the choice of wavelet, the error caused by the defect of the filter will not be able to avoid. To minimize such errors is to choose the larger N, but it is also accompanied by the increase of computation, so it is considered both the error and computation.

(2) The cause of the second error frequency

The second type of error frequency is not same with Main Frequency, next Mallat algorithm itself is analyzed.

First look at the formula of Mallat decomposition algorithm.

$$c_{j+1,k} = \sum_{n\in z} c_{j,n}\bar{h}_{n-2k} \quad k \in Z$$

$$d_{j+1,k} = \sum_{n\in z} c_{j,n}\bar{g}_{n-2k} \quad k \in Z$$

Mallat decomposition algorithm in the computer is actually the Interval Dot sample after the convolution process.

Next, look at the wavelet reconstruction formula.

$$c_{j,k} = \sum_{n\in z} c_{j+1,n}h_{k-2n} + \sum_{n\in z} d_{j+1,n}g_{k-2n} \qquad k \in Z$$

Reconstruction of wavelet coefficients is actually first plugging zero every other point and then convolution.

Then we analyze the reasons of the second error frequency from the mathematical point of view.

First analyze Interval Dot sample. Let the original signal is x (n), every other point sampled signal is v (n). There is the following relationship:

$$v(n) = x(2n) \quad n \in Z$$

Let Fourier transform of x (n) and v (n) respectively are X(w) and V(w),.

$$V(\omega) = \sum_{n\in Z} v(n)e^{-j\omega n} = \sum_{n\in Z} x(2n)e^{-j\omega n} = \left[\sum_{n\in Z} x(2n)e^{-j\omega n} + \sum_{n\in Z} x(2n)e^{-j\omega n}\right]/2$$
$$= \left[\sum_{n\in Z} x(2n)e^{-j\frac{\omega}{2}2n} + \sum_{n\in Z} x(2n)e^{-j\frac{\omega+2\pi}{2}2n}\right]\Big/2 = \left[X\left(\frac{\omega}{2}\right) + X\left(\frac{\omega}{2}+\pi\right)\right]\Big/2$$

It Shows that, the signal after every other point sample is composed of two parts, X(w/2) and X(w/2+π), X(w/2) is from the part of $f \le f_s/4$ of the original signal, while X(w/2) is consist of the part of $f_s/4 \le f \le f_s/2$ of the original signal.

In summation, both Interval Dot sample and plugging zero every other point, will produce the new frequencies.

IV. IMPROVED ALGORITHM

From the above analysis, the root causes of frequency alias is non-ideal cut-off characteristics of the wavelet filter and that sampling at every other point does not meet the sampling theorem..

If we can remove the excess sub-band frequency components, frequency alias will not exist in the reconstructed sub-band frequency signal. Specific approach is using Fourier transform and Fourier inverse transform to remove unwanted frequency components.

The improved algorithm of decomposition is as follows:

(1) The original signal a_{j-1}, after convolution with h, takes Fast Fourier Transform.

(2) It will be zero which is the spectrum of the frequency greater than $f_s/2^{j+1}$ in the results of FFT.

(3) After setting zero, the result takes Fourier inverse transform.

(4) After sampling of every other point, the sampled results as a_j, then proceed to the next step decomposition.

The improved method of reconstruction is as follows:

(1) Eliminate the approximate parts of sub-band frequency alias in the process of reconstruction.

(2) Eliminate the details of sub-band frequency alias in the process of reconstruction.

V. SIMULATION RESULTS AND SUMMARY

The improved algorithm is programmed and simulated in Matlab environment, simulation results are as follows.

The simulation results show that they are consistent with the theoretical value, this proves that the improved algorithm is effective. As the spectral analysis has a very important role in projects, the improved algorithm will contribute greatly to signal analysis, especially the extraction of specific frequency band signal.

ACKNOWLEDGMENT

Foundation item: Technology Fund of North China Institute of Science and Technology

REFERENCES

[1] Stephane Mallat, A wavelet Tour of Signal Processing, Academic Press 1998

[2] Stange G., Nguyen T, Wavelets and filter banks, Wellesley-Cam-Bridge Press 1996

[3] Mayer Y, Wavelet&Applications , Philadelphia: SIAM 1993

[4] Fliege N J, Multivariate Digital Signal Processing, New York: John Wiley&Sons Ltd 2000

[5] Marya M, Wang K, Gayden X H. Tensile-shear forces and fracture modes in single and multiple weld specimens in dual-phase steels[J].Journal of Manufacturing Science and Engineering, 2006, 128(1):287-298.

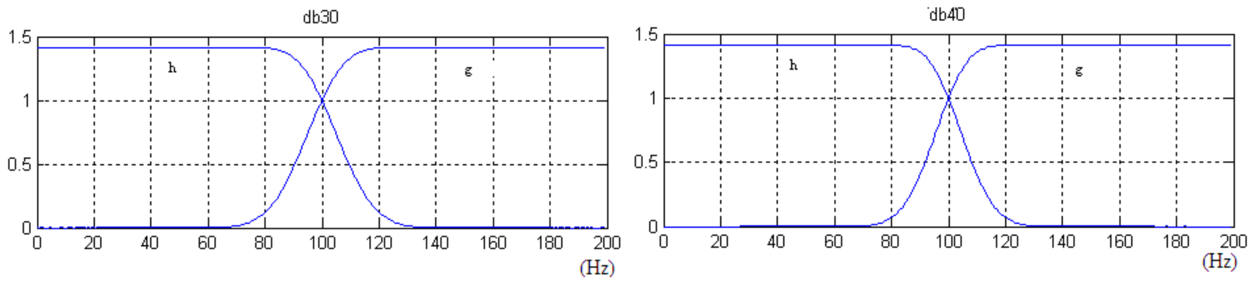

Fig.2 magnitude-frequency characteristics

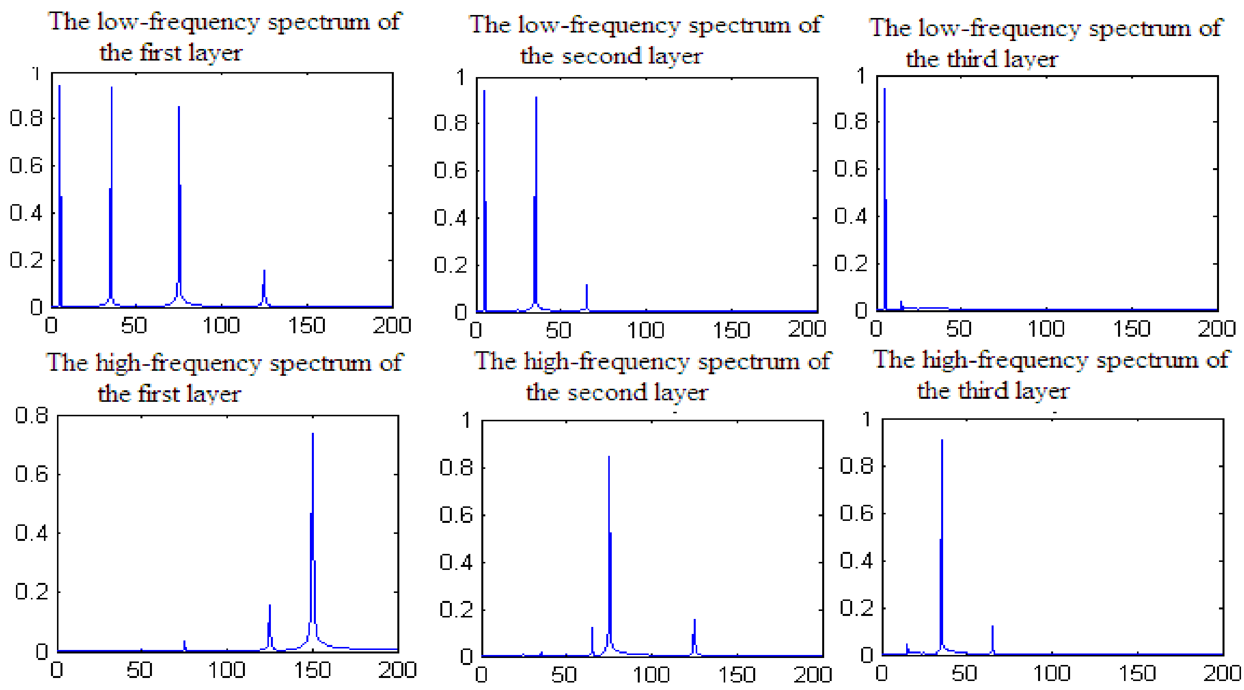
The low-frequency spectrum of the first layer
The low-frequency spectrum of the second layer
The low-frequency spectrum of the third layer
The high-frequency spectrum of the first layer
The high-frequency spectrum of the second layer
The high-frequency spectrum of the third layer

Fig.1

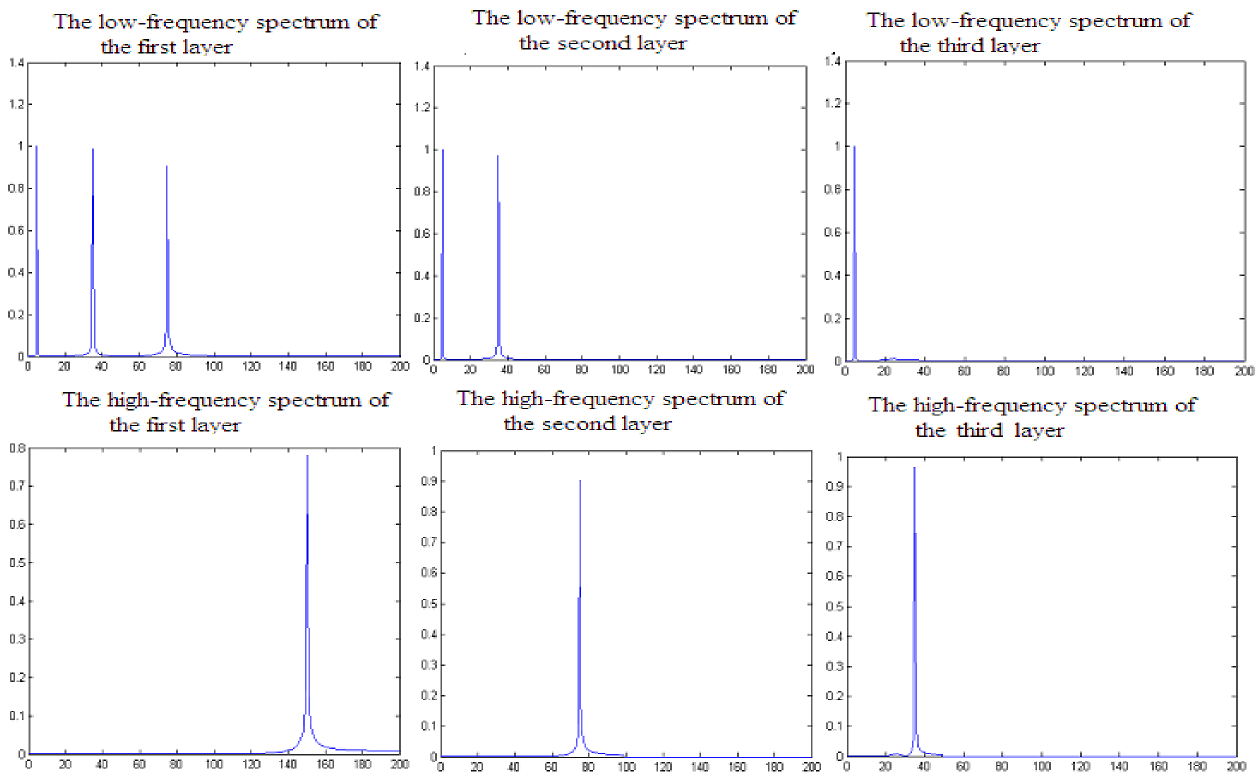
The low-frequency spectrum of the first layer
The low-frequency spectrum of the second layer
The low-frequency spectrum of the third layer
The high-frequency spectrum of the first layer
The high-frequency spectrum of the second layer
The high-frequency spectrum of the third layer

Fig.3