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An location-routing problem with simultaneous pickup and delivery in urban-rural dual-directions logistics network

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***Abstract*—The design and optimization of urban-rural dual-directions logistics network is a substantial important issue, which will directly affect the development of the urban-rural integration in China. A reasonable scheme of logistics network will contribute to supply efficient logistics services to customers scattering in urban and rural areas. In this paper, we consider a variant of the Location-Routing-Problem (LRP), namely the LRP with simultaneous pickup and delivery in specially background (LRPSB). The objective of LRPSB is to minimize the total system cost, including depot location cost and vehicle routing cost, and implement and control the effective dual-direct commodity flow to meet customers' requirement by simultaneously locating the depots and designing the vehicle routes that satisfy pickup and delivery demand of customer at the same time. A nonlinear mixed integrated programming model is formulated for the problem. Since such integrated logistics network design problems belong to a class of NP-hard problems, we propose a two-phase heuristic approach based on Tabu Search, tp-TS, to solve the large size problem and an initialization procedure to generate an initial solution for the tp-TS. We then empirically evaluate the strengths of the proposed formulations with respect to their ability to find optimal solutions or strong lower bounds, and investigate the effectiveness of the proposed heuristic approach. Computational results show that the proposed heuristic approach is computationally efficient in finding good quality solutions for the LRPSB.**

Keywords-urban-rural; dual-direct; location-routing problem; simultaneous pickup and delivery

I. INTRODUCTION

In order to realize economic and society integration between urban and rural areas in China through dual-directions logistics, three-party logistics enterprises should make strategic and operational decisions adapted to complex environmental factors existing in urban and rural areas. One of the most important strategic decision concerns the design of logistics networks since it offers great potential to reduce costs and to improve service level. The main elements in designing a logistics network are location and routing decisions. Such two elements are interdependent in the operation of logistics system, so the overall system cost can increase if routing decisions are ignored when locating facilities [1]. The location-routing problem (LRP) overcomes this drawback by simultaneously dealing with location and routing decisions. The LRP includes two fundamental problems: the facility location problem and the vehicle routing problem, and can be defined as follow: The problem is to determine the optimal number and location of depots simultaneously with finding distribution routes.

There are several surveys on location-routing problems presented in literature. Min et al. [2] and Nagy and Salhi [3] review the LRP literature based on the solution methods and problem characteristics and their application areas. Because of the complexity of LRP, some mathematical models and exact solution procedures have been developed for a small number of LRP models. Laporte et al. [4] made a series of significant contributions in the presentation of exact methods. Because exact approaches can consistently solve to proven optimality instances with less than 100 customers, heuristic algorithms have been proven to be the only viable alternative to solve large LRP instances. Different heuristic approaches can be generally classified into four basic types: sequential, iterative, hierarchical, and clustering based methods. Recent many heuristic approaches successfully combined different heuristic approaches, and implemented meta-heuristic approaches to solve more complex problems: Tabu search(TA), combined TS and simulated annealing (SA); and threshold accepting TA and SA[5,6]. In the studies of the general LRP cited above, only classical VRP is considered in problems, in which the customers have only delivery demand, and each vehicle delivers goods to customers and returns to the depot.

In this paper, we consider LRP with simultaneous pickup and delivery (LRPSB) which is a general case of the LRP by considering simultaneously pickup and delivery demands of each customer. Taking account of complex environment: different vehicle types and delivery modes are required to achieve efficient logistics operation and low costs; the location distribution and demand of customers in rural area are fairly disperse; the delivery vehicles used in rural area are relative larger than those used in city.

To the best of our knowledge, the LRPSB has received litter attention from researches so far, and only two papers on similar problem, LRP with simultaneous pickup and delivery (LRPSPD), proposed in the literature. In LRPSPD, customers have pickup and delivery demand, and they request that both demand should be met at the same time. Karaoglan et al. [7] proposed two MIP formulations, which are two-index node-based and flow-based formulations, for the LRPSPD problem and presented several polynomial-size valid inequalities adapted to strengthen the formulations. The

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branch-and-cut algorithm, proposed in the paper, can solve some instances with up to 88 customers and 8 potential depots in a reasonable computation time. Karaoglan et al. [8] proposed a two-phase heuristic approach based on simulated annealing to efficiently solve the large-size LRPSPD.

We extend the LRPSPD model proposed by Karaoglan et al., to a three-index node-based formulation for LRPSB, and presented several polynomial-size valid inequalities adapted from literature to strengthen the formulation. The objective of LRPSPD is to minimize the total system cost, including depot location cost and vehicle routing cost, and implement and control the effective dual-direct commodity flow to meet customers' requirement by simultaneously locating the depots and designing the vehicle routes that satisfy pickup and delivery demand of customer at the same time. Furthermore, we propose a two-phase heuristic approach based on (Tabu Search, TS), called tp-TS to solve large-size problems.

The rest of the paper is organized as follows: In Section 2, we formulate the mathematic model of LRPSB. The proposed tp-TS heuristics is described in Section 3. In Section 4, numerical analysis of case study is carried out to show the effectiveness of the proposed approach. Finally, this paper give the conclusion follows in Section 5.

II. PROBLEM DEFINITION AND MATHEMATICAL FORMULATION

The LRPSB is first defined and a MIP formulation for the problem is proposed in this section. Then, we further introduce valid inequalities used to strengthen the formulations. We assume that the number, location, and demand of customers, the location of all potential depots, as well as the vehicle type and size are given.

The LRPSB can be defined as follows: Consider $J_1 = \{1,...,n_1\}$ the set of customers in urban area, $J_2 = \{1,...,n_2\}$ the set of customers in rural area, and $I = (1,...m)$ the set of potential depots. Let $J = J_1 \cup J_2$ be the set of all customers in urban and rural areas, and $O = I \cup J$ be the set of all customers and potential depots. Each depot $i \in I$ is characterized by a limited capacity V_i, a fixed cost FD_i of establishment and a handle cost unit commodity DT_i. Each customer $j \in J$ has pickup (p_j) and delivery (d_j) demands, with $0 < d_j, p_j \le Q_k$. $K = K_1 \cup K_2$ is a set of vehicles or transportation routes in which K_1 and K_2 represent the vehicles or routes serving urban and rural customers, respectively, and a capacity Q_k and fixed operating cost FV_k including the costs of acquiring the vehicles used in the routing is available to serve the customers. Let $c_{ij}\,(i, j \in O)$ be the traveling cost between i and j, and r_{ijk} be the load of vehicle k in the arc $(i, j)(i, j \in O, k \in K)$. U_{jk} is subtour breaking constraint variable, represent the visited order of customers j in route k.

The LRPSPD consists of determine the locations of depots, the assignment of customers to opened depots and elaborate vehicle tours to visit the set of customer in order to minimize the total cost of location and delivery under following constraints:

- Each vehicle is used at most one route
- Each customer is served by exactly one vehicle and the demand of each customer can be satisfied. The location and number of customers are already determined.
- The total vehicle load at any point of the routes does not exceed the vehicle capacity.
- The total pickup and total delivery load of the customers assigned to an opened depot do not exceed the capacity of the depot.
- Each route begins and ends at the same depot.
- Two fleet types are used in rural and urban areas respectively.

To formulate the LRPSPD, following decision variables are used:

$$x_{ijk} = \begin{cases} 1 & \text{if a vehicle travels directly from node } j \text{ to node } i \\ & (i, j \in O, i \ne j, k \in K) \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if customer } j \text{ is assigned to distribution center } i \\ & (i \in I, j \in J) \\ 0 & \text{otherwise} \end{cases}$$

$$z_i = \begin{cases} 1 & \text{if distribution center } i \text{ is opened } (i \in I) \\ 0 & \text{otherwise} \end{cases}$$

Additional variables:

CU_j : Delivery load on vehicle just before having serviced customer $j\,(\forall j \in J)$.

CV_j : Pickup load on vehicle just after having serviced customer $j\,(\forall j \in J)$.

The proposed node-based formulation F-node, is as follows:

$$\begin{aligned} \min & \sum_{i \in I}\sum_{j \in J} DT_i (d_j + p_j) y_{ij} + \sum_{i \in I} FD_i z_i \\ & + \sum_{k \in K_2}\sum_{i \in I}\sum_{j \in J_2} FV_k x_{ijk} + \sum_{k \in K_1}\sum_{i \in I}\sum_{j \in J_1} FV_k x_{ijk} \\ & + \sum_{k \in K_1}\sum_{i \in J_1 \cup I}\sum_{j \in J_1 \cup I} c_{ij} x_{ijk} + \sum_{k \in K_2}\sum_{i \in J_2 \cup I}\sum_{j \in J_2 \cup I} c_{ij} x_{ijk} \end{aligned} \quad (1)$$

$$\text{s.t.} \quad \sum_{k \in K_1}\sum_{i \in I \cup J_1} x_{ijk} = 1 \quad (\forall j \in J_1) \quad (2)$$

$$\sum_{k \in K_2}\sum_{i \in I \cup J_2} x_{ijk} = 1 \quad (\forall j \in J_2) \quad (3)$$

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$$\sum_{k\in K}\sum_{i\in I\cup J} x_{ijk} = 1 \quad (\forall j \in J) \tag{4}$$

$$\sum_{k\in K}\sum_{j\in J} x_{ijk} \ge z_i \quad (\forall i \in I) \tag{5}$$

$$\sum_{j\in J}(d_j + p_j)\, y_{ij} \le V_i z_i \quad (\forall i \in I) \tag{6}$$

$$r_{ijk} \le Q_k x_{ijk} \quad (\forall i, j \in O, k \in K) \tag{7}$$

$$\sum_{j\in J} r_{ijk} = \sum_{i\in I\cup J}\sum_{j\in I\cup J} d_j x_{ijk} \quad (\forall i \in I, k \in K) \tag{8}$$

$$\sum_{i\in I\cup J} x_{ijk} = \sum_{i\in I\cup J} x_{jik} \quad (\forall j \in J, k \in K) \tag{9}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le 1 \quad (\forall k \in K) \tag{10}$$

$$\sum_{k\in K}\sum_{i\in I\cup J} r_{ijk} - p_j = \sum_{k\in K}\sum_{i\in I\cup J} r_{jik} - d_j \quad (\forall j \in J) \tag{11}$$

$$-y_{ij} + \sum_{u\in I\cup J}(x_{iuk} + x_{ujk}) \le 1 \quad (\forall i, j \in J, k \in K) \tag{12}$$

$$r_{ijk} \le x_{ijk}\sum_{j\in J}(d_j + p_j) \quad (\forall i, j \in I\cup J, k \in K) \tag{13}$$

$$U_{ij} - U_{jk} + (N_1 + N_2)\, x_{ijk} \le (N_1 + N_2) - 1 \quad (\forall i, j \in J, k \in K) \tag{14}$$

$$U_{jk} \ge 0 \quad (\forall j \in J, k \in K) \tag{15}$$

$$x_{ijk} = 0 \text{ or } 1 \quad (\forall i, j \in O, k \in K, i \ne j) \tag{16}$$

$$y_{ij} = 0 \text{ or } 1 \quad (\forall i \in I, j \in J) \tag{17}$$

$$z_i = 0 \text{ or } 1 \quad (\forall i \in I) \tag{18}$$

$$r_{ijk} \ge 0 \quad (\forall i, j \in O, k \in K) \tag{19}$$

In this formulation, objective function (1) minimizes the total system cost including location and operation cost of depot, transportation, and vehicle acquired costs in rural and urban area. Constraint sets include the constraints of location-allocation problem, multiple depot vehicle routing problem and jointing constraints of two problems. Constraints (2), (3) and (4) ensure that each urban customer must be visited by the vehicle in urban exactly once, and each rural customer must be visited by the vehicle in rural exactly once. Constraints (5) describe that the opened depots at least supply one vehicle routes. Capacity constraints for the depots are given in (6). Constrains (7) imply that the total load on any arc does not exceed the vehicle capacity. Constraints (8) ensure that the delivery load of vehicle dispatching from depot equals to total delivery demand of customers which are assigned to the corresponding vehicle. Constraints (9) guarantee that entering and leaving arcs to each node are equal. Constraints (10) assure that each route can be served at most once. Flow conservation constraints are expressed in constraint (11). Constraints (12) specify that a customer can be assigned to a depot only if there is a route from the depot going through that customer. Constraints (13) and (14) forbid the illegal routes, i.e. the routes, which do not start and end at the same depot. The auxiliary variables U_{jk} taking positive values are declared in (15). Constraints (16), (17), and (18) are the binary requirements on the decision variables. Constraints (19) are the integrality constraints which define the nature of the decision variables.

In the given formulation, any integer solution does not contain illegal routes because of the constraint sets (2)-(4), (14) together with (15). The validity of these constraints can be proven by contradiction.

This formulation includes $O\left((|I|+|J_1|+|J_2|)^2\right)$ binary variables $O\left((|I|+|J_1|+|J_2|)^2\right)$ additional variables and $O\left(|I|^2|J|\right)$ constraints.

In this paper, we utilize four polynomial size valid inequalities, where were developed for the VRP and FLP in the literature, in our algorithm. The inequalities adapted to the LRPSB are based feasible requirement, that is, any feasible solution must satisfy these constraints. Karaoglan et al. also employed this practical way to eliminate some fractional solutions from the solution space such that a stronger lower bound can be obtained, and its effectiveness is distinct in computational results.

First simple and efficient polynomial-size valid inequality that has used by Labbe et al. [7]for plant cycle location problem is given below:

$$y_{ij} \le z_i \quad \forall i \in I, j \in J \tag{20}$$

This inequality imposes that customer $j \in J$ cannot be assigned to the depot $i \in I$ if depot i is not open.

Other polynomial-size valid inequalities which bounds below the number of routes originating from depots are given as follows:

$$\sum_{i\in I}\sum_{j\in J_1} x_{ijk} \ge r_{LRPSB}(J_1) \tag{21}$$

$$\sum_{i\in I}\sum_{j\in J_2} x_{ijk} \ge r_{LRPSB}(J_2) \tag{22}$$

Where $r_{LRPSPB}(J) = \left\lceil \max\left(\sum_{j\in J} d_j ; \sum_{j\in J} p_j\right)/CV \right\rceil$ and $\lceil \bullet \rceil$ is the smallest integer bigger than .Validation of this inequality for the LRPSPD is given in Karaglan et al. [7].

Last polynomial size valid inequality is given as follow:

$$x_{ijk} + x_{jik} \le 1 \quad \forall i, j \in J; \forall k \in K \tag{23}$$

Constraints (23) ensure that any feasible route cannot contain subtour with only two customers. This constraint is a special case of following exponential-size constraints which are derived from capacity and sub-tour elimination constraints of the VRP.

III. A HEURISTIC APPROACH FOR THE LRPSB

The proposed mathematical model of the LRPSB in this paper cannot be directly solved to find optimal solutions for medium- and large-size problems. So we propose a two-phase heuristic algorithm to quickly obtain solutions for the problem. This two-phase approach offers a simple and natural representation of LRPSB, and the solution obtained in location phase is used as an input to the vehicle routing phase. In the location phase of the algorithm, a heuristic

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approach is performed on the location variables to determine a good configuration of depots to be used in the network. For each of the location configurations visited during the location phase, TS algorithm is run on the routing variables in order to obtain a good routing for the given configuration. These two phases are coordinated in such way that the solution space is searched efficiently. Each time a move is performed on the location phase, the routing phase is started in order to update the routing according to the new configuration. In the literature, different hierarchical heuristics based on TS have been proposed for the variants of LRP. According to classification of the LRP heuristics given in reference [3], the proposed heuristic approach belongs to the class of hierarchical heuristics, but differs from the previous ones in terms of the problem background and its complexity.

In the following subsections, we give the details of the proposed heuristic algorithm, called tp-TS, including the generation of an initial solution, the routing and location phases.

A. Initial solution

The effectiveness of heuristic approach relies on the initial solution that can be generated randomly or using heuristic algorithms developed for the problem. In this section, in order to speed up computation time, we design a heuristic algorithm, in which the location problem is solved optimally, and then the routing problem can be solved heuristically, to generate initial solution for the LRPSDP. Brief descriptions about the heuristic approaches are given below. In order to decrease the search space of solution in location decision, we can first determine the lower boundary of the number of depot which will be located. The number of depot can be estimated as follows:

$$n_f = \left\lceil \frac{\sum_{j\in J}\left(d_j + p_j\right)}{V_i} \right\rceil \tag{24}$$

Then n_f locations will be randomly chosen to construct initial facility set. The customers can be assigned in the facility set, in the condition of the total demand of customers less than the capacity of depot, in the light of the distance between customer and depot. So the problem is reduced to a capacitated facility location-allocation problem where each customer is directly served from a single capacitated depot. The objective of the problem is to minimize the total cost to assign the opened depots while considering its capacity constraints. The calculation of customer assignment cost is based on the direct distance between customer and depot, and the total delivery and pickup loads on any depot must not exceed the corresponding depot capacity.

This problem can be solved optimally up to practical scale case using the general purpose optimization software package CPLEX within a reasonable computation time. So the optimal solution of customer assignment in depots can be obtained and the routes in each depot can be constructed using the original savings algorithm followed by a simple 2-opt procedure to obtain the initial routing for the open facility.

B. Routing phase

After the solution is obtained by the initial procedure and the location procedure, the routing phase is started from the best routing found for the previous facility configuration in order to modify the routing according the current facility configuration. We implement a TS algorithm as a local search to improve the solution in the routing phase. Two set of moves are performed sequentially in the routing phase: insert moves, and swap moves. In insert move, one customer is inserted to a new position on a route originating from its current facility, or any other open facility that is close enough to the customer. Insert moves are terminated after a given number of iterations are preformed with improvement over the best solution found for the current facility configuration. In swap move, any two customers that are currently assigned to a depot can swap theirs positions. After the swap move is done, swapping these two customers is declared tabu for a number of iterations. Swap moves are also terminated after a given number of routing iterations are preformed without improvement. At the each iteration of the TS, the neighbors of the current solution are generated using two moving strategies and the best one among them is chosen as a new solution for the problem. In TS algorithm, a candidate list strategy is implemented in generating the neighbors, thereby reducing the computation time consumed in the process of neighborhood search. At each iteration, if the best solution is better than the current solution, then it is accepted as the current solution, otherwise it is declared tabu for a given tabu duration.

C. Location phase

In the location phase, we apply two different type of moves to search different facility configuration: swap moves and add moves, while the computation results in the routing phase can rarely change the status of the opened depots in the current solution. For a given number of facilities, swap moves close one of the open facilities, and open one that is selected from the set of closed depots, simultaneously. The number of open facilities in the solution keep constant, and the routes belonging to the closed depot are reassigned to the nearest opened depot if it can satisfy problem constraints. Until swap moves explored the configurations with the current number of facilities for a given number of non-profitable moves, add moves opens one of the closed facilities in the current solution. In this moving strategy, the facility to be added is the one whose addition yields the minimum estimated cost. So the some of the routes in the opened depots are reassigned to the recently opened depot in terms of the capacity constraint and the total cost saving, and the routing cost is again estimated using the direct distance between customer and depot. In iterations of the location phase, both moving strategy can be randomly selected to determine a new set of the opened depots. The location phase is terminated when a given number of moves are preformed without any improvement over the best objective function value.

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IV. COMPUTATIONAL RESULTS

In order to evaluate the performance of our two-phase heuristic algorithm, we can investigate the results form our computational experiment and compare it to one of the stage-of- the-art MIP solve CPLEX used to solve the formulation. The proposed heuristic algorithm was coded in C and was run on a PC with Inter 2.4GHZ processor. The total 10 test problems of varying size where constructed in computation time and the data requirements. Test problems for the LRPSB are drawn on the experience of reference [7]. The dates in these instances of the test problem, including the demand of customer, depot capacities, vehicle capacity levels and the distance between customers, are derived by similar methods adopted by reference [7]. The delivery and pickup demands of customers in each test instances can be generated through the well-known demand separation approach of Salhi and Nagy [9].

Based on our preliminary experiments, we implement the following parameter values in TS algorithm: the threshold parameters are set as max-add=2 and max-swap=5. The tuba durations were generated uniformly form intervals [3,6] and [11,15] for the location and routing attributes respectively. As to each instance, heuristic approaches are run ten times with different initial solution, and the best of ten runs for each instance is considered as current solution.

We report our computational results in Table 1 in terms of both solution quality and CPU time. The first two columns of the table are the number of customers and the number of depots, respectively. In the next two columns, five statistics for TS are presented: the average percentage gap, the maximum percentage gap, the average CPU time in seconds and the number of optimal solutions obtained over all instances for each test problem. The average and maximum percentage gaps of the heuristic solutions are calculated by considering optimal solutions, which are obtained by solving the SLP relaxation of the formulation with CPLEX for a maximum of 1h.

TABLE I. COMPUTATIONAL RESULTS OF THE TEXT PROBLEMS

$\|I\|$	$\|J_1\|$	$\|J_2\|$	GAP(%)		CPU(s)	Opt
			Avg	Max		
3	30	30	1.37	6.38	113.5	3
5	30	30	1.93	9.31	143.3	3
3	50	50	0.78	10.5	95.1	5
5	50	50	0.53	11.9	124.1	5
5	60	60	1.36	8.6	173.6	4
10	60	60	0.72	10.32	200.7	2
5	80	80	1.73	9.73	189.7	2
10	80	80	0.85	5.32	273.3	1
5	100	100	1.37	13.27	283.4	5
10	100	100	1.58	8.32	178.9	3

From Table 1, it is seen that the proposed heuristic approach, tp-TS, obtains good results (at most 5% from the best lower bound). Concerning the computational burden, it is observed that the tp-TS can solve the test problem in reasonable CPU times. So we can conclude that the proposed heuristic approach is a computationally efficient algorithm for solving the LRPSB.

V. CONCLUSION

In this study, a variant of the LRP called the location-routing problem with simultaneous pickup and delivery, LRPSB, is considered. We show that this problem can be formulated as a nonlinear mixed integrated programming model, for which we proposed a heuristic algorithm based on TS, to solve the medium and large size problems. The results indicate that this method performs well in terms of the solution quality and run time consumed. A future research extension is to solve LRPSB problem with time windows and further improve the computational efficiency of the algorithm.

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