A Novel Simplified Limit-1 Polling Communication System

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***Abstract*—proposed a Limit-1 polling system model with simplified service process (SL-1), which was analyzed by the theory of imbedded Markov chain and the method of multidimensional probability generating function. The related characteristic of system was obtained based on the analysis of polling mechanism. The contrast of numerical curve and simulation experiment between SL-1 service and Limit-1 service showed that the SL-1 service polling model performed better in queuing performance and stability.**

Keywords- Polling system; SL-1; embedded Markov chain; probability generating function

I. INTRODUCTION

Polling system model is a significant queuing model of multiple queues that the server accesses each terminal station according to a certain rule and order. Polling system applied to many domains, such as computer communication, production, transportation and system maintenance [1].With the growth of communication business capacity and the diversification of data business, the demand of system resource and task scheduling are increasing highly. So it is essential to analyze the specific practical model.

Gated service [2], Exhaustive service [3] Limit service [4] were basic polling control models and provided research foundation for follow-up development. reference [5] discussed a polling model that customers leave queue according to waiting time and location of server based on Gated and Exhaustive service, and analyzed the system state of different arrival rates; references [6]-[8] analyzed the average queue length and the average delay of Limit *k* services in different system operation conditions.

This paper presented a SL-1 service polling system control model. Through the construction of embedded Markov chain and the probability generating function, the average queue length of SL-1 service polling system was accurately calculated. Numerical curve comparisons showed that SL-1 was superior to Limit-1 system in service fairness and transmission efficiency of information packet, and could handle burst traffic timely. Simulation experiments verified the comparison characteristics above.

II. ANALYSIS MODEL

A. Service Strategy

Information packets in each queue were served according to the rule of FCFS. If the queue is empty, the server would not stay and transfer directly to the next queue query; otherwise one information packet would be sent, the remnant packets would be sent in next cycle; there were no packets arrived when its queue was accessed.

B. Working Parameter

- The system worked in discrete time slots, and the system was statistically stationary;
- The process that information arrived in each queue obeyed by independent, identically distributed random process. The probability generating function, mean and variance of the distribution is $A(z)$ 、 $\lambda = A^{'}(1)$ and $\sigma_{\lambda}^{2} = A^{//}(1) + \lambda - \lambda^{2}$ respectively;
- When less than one packet was served, the random variable of service time obeyed by an independent, identically distributed probability distribution. The probability generating function, mean and variance of the distribution is $B(z)$ 、 $\beta = B^{'}(1)$ and $\sigma_{\beta}^{2} = B^{''}(1) + \beta - \beta^{2}$ respectively.
- The random variable of transfer time between adjacent queues obeyed by an independent, identically distributed probability distribution. The probability generating function, mean and variance of the distribution is $R(z)$ 、 $r = R^{'}(1)$ and $\sigma_{r}^{2} = R^{''}(1) + r - r^{2}$ respectively;
- Each queue buffer could accommodate infinite information packets.

C. Definition of Parameters and Function

N: Queue number to be accessed;

t_n : The polling moment of queue *i*;

$u_i(n)$: The transfer time from the queue *i* to the next;

$v_i(n)$: The service time for queue *i*;

$\mu_j(u_i)$: The packet number entered in queue *j* during $u_i(n)$;

$\eta_j(v_i)$: The packet number entered in queue *j* during $v_i(n)$;

$\xi_j(n)$: The packet number of queue *j* at the moment t_n.

$\pi_i(x_1, x_2, ..., x_N)$: The probability distribution under steady state, expressed as $\lim_{t\to\infty} P[\xi_j(n) = x_j,\ j = 1, ..., N]$;

$G_i(z_1, z_2, ..., z_N)$: Probability generating function of the state variables of the system at t_n while queue *i* was served, expressed as $\sum_{x_1}^{\infty} ... \sum_{x_N}^{\infty} \pi_i(x_1, ..., x_N) \bullet z_1^{x_1} ... z_N^{x_N}$;

$G_i(z_1, \cdots, z_{i-1}, 0, z_{i+1}, \cdots, z_N)$: The probability generating function when queue *i* was empty at t_n, expressed as $G_i(z_1, ..., z_{i-1}, z_{i+1}, ..., z_N)$.

III. Queueing Performance Analysis

A. Transfer Relation

According to the service rules of SL-1, the following equation could be derived:

$$\begin{cases} \xi_j(n+1) = \xi_j(n) + \mu_j(u_i) + \eta_j(v_i) & j \neq i \\ \xi_i(n+1) = \xi_i(n) - \min(1, \xi_i(n)) + \mu_i(u_i) \end{cases} \tag{1}$$

The relationship of probability generating functions between t_n and t_{n+1} should be as follows:

$$G_{i+1}(z_1, z_2, \cdots, z_i, \cdots, z_N) = R_i\left[\prod_{j=1}^{N} A_j(z_j)\right] \bullet \left\{ \begin{aligned} & B_i\left[\prod_{\substack{j=1 \\ \neq i}}^{N} A_j(z_j)\right] \frac{1}{z_i} \left[\begin{aligned} & G_i(z_1, \cdots, z_i, \cdots, z_N) - \\ & G_i(z_1, \cdots, z_{i-1}, 0, z_{i+1}, \cdots, z_N) \end{aligned}\right] \\ & + G_i(z_1, \cdots, z_{i-1}, 0, z_{i+1}, \cdots, z_N) \end{aligned} \right\} \tag{2}$$

B. First-order Property

The average queue length of system was defined that packets number of queue *j* when queue *i* began to accept service, could be expressed as follows:

$$g_i(j) = \lim_{z_1, z_2 \cdots, z_i \cdots, z_N \to 1} \frac{\partial G_i(z_1, z_2, \cdots, z_i, \cdots, z_N)}{\partial z_j} \tag{3}$$

From the derivation of Equation (1):

$$\begin{cases} g_{i+1}(j) = g_i(j) + \gamma\lambda + \rho C & i \neq j \\ g_{i+1}(i) = g_i(i) + \gamma\lambda - C \end{cases} \tag{4}$$

In which, $C = 1 - G_i(1, \cdots, 1, z_i, 1, \cdots, 1)\Big|_{z_i = 0}$.

From the summation calculation of equation (2), the following expressions could be obtained:

$$\theta = \frac{N\gamma\lambda(1+\rho)}{1 - N\rho + \rho} \tag{5}$$

$$g_i(j) = g_i(i) + \gamma\lambda - C + (\gamma\lambda + \rho C) \bullet \begin{cases} (i - j - 1), & i > j \\ (N + i - j - 1), & i \leq j \end{cases} \tag{6}$$

C. Average Queue Length

$g_i(j,k)$ was defined as the joint moment for random variables (x_j, x_k), could be expressed as $\lim_{z_1,...z_N \to 1} \frac{\partial^2 G_i(z_1,...z_N)}{\partial z_j \partial z_k}$.

$g_{i0}(j)$ indicated packet number of queue *j* when queue *i* was empty at polling moment, could be expressed as: $\lim_{z_1,...z_{i-1}, z_{i+1},...,z_N \to 1} \frac{\partial G_i(z_1,...,z_{i-1},0,z_{i+1},...z_N)}{\partial z_j}$.

From the two-order derivation of equation (2):

$$\begin{aligned} g_{i+1}(j,k) &= \lambda^2 R''(1) + \lambda^2\gamma - \rho[g_{i0}(k) + g_{i0}(j)] \\ &+ C\left[2\lambda\gamma\rho + \lambda\rho + \lambda^2 B''(1)\right] + g_i(j,k) \\ &+ (\lambda\gamma + \rho)[g_i(k) + g_i(j)] \\ & \qquad i \neq j \neq k \end{aligned} \tag{7}$$

$$\begin{aligned} g_{i+1}(j,i) &= R''(1)\lambda^2 + \gamma\lambda^2 + C(\gamma\lambda\rho - \gamma\lambda - \rho) \\ &+ (\gamma\lambda + \rho) g_i(i) + (\gamma\lambda - 1) g_i(j) - g_{i0}(j) + g_i(j,i) \\ & \qquad i \neq j \end{aligned} \tag{8}$$

Form the summation calculation by the form of $\sum_{\substack{j=1 \\ \neq k}}^{N} \sum_{k=1}^{N} \sum_{i=1}^{N} g_{i+1}(j,k)$:

$$
\begin{aligned}
&N^2(N-1)\left(\lambda^2R''(1)+\lambda^2\gamma\right)+N(N-1)C\bullet \\
&\left[(N-1)2\lambda\gamma\rho+(N-2)\left(\lambda\rho+\lambda^2B''(1)\right)-2\left(\gamma\lambda+\rho\right)\right] \\
&+2\left((N-2)\left(\lambda\gamma+\rho\right)+\gamma\lambda-1\right)\sum_{j=1}^{N}\sum_{\substack{k=1\\ \neq j}}^{N}g_j(k) \\
&-2\left(\rho(N-2)-1\right)\sum_{j=1}^{N}\sum_{\substack{k=1\\ \neq j}}^{N}g_{j0}(k)+ \\
&2N(N-1)\left(\gamma\lambda+\rho\right)g_i(i)=0
\end{aligned} \tag{9}
$$

From the two-order derivation of equation (2):

$$
\begin{aligned}
g_{i+1}(j,j)&=\lambda^2R''(1)+\gamma A''(1)+C\bullet \\
&[2\lambda\gamma\rho+\beta A''(1)+\lambda^2B''(1)]+ \\
&2(\lambda\gamma+\rho)g_i(j)-2\rho g_{i0}(j)+g_i(j,j) \\
&\qquad i\neq j
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
g_{i+1}(i,i)&=R''(1)\lambda^2+\gamma A''(1)+2\left(1-\gamma\lambda\right)C+ \\
&2\left(\gamma\lambda-1\right)g_i(i)+g_i(i,i)
\end{aligned} \tag{11}
$$

Form the summation calculation by the form of $\sum_{j=1}^{N}\sum_{i=1}^{N}g_{i+1}(j,j)$:

$$
\begin{aligned}
&N^2\left(\lambda^2R''(1)+\gamma A''(1)\right)+NC\bullet \\
&\left[(N-1)\left(2\lambda\gamma\rho+\beta A''(1)+\lambda^2B''(1)\right)+2\left(1-\gamma\lambda\right)\right]+ \\
&2(\lambda\gamma+\rho)\sum_{j=1}^{N}\sum_{\substack{i=1\\ \neq j}}^{N}g_i(j)-2\rho\sum_{j=1}^{N}\sum_{\substack{i=1\\ \neq j}}^{N}g_{i0}(j)+ \\
&2N\left(\gamma\lambda-1\right)g_i(i)=0
\end{aligned} \tag{12}
$$

According to equation (9) and (12), the average queue length could be obtained as follows:

$$
g_i(i)=\frac{N}{2(\rho+1)(1-N\rho+\rho)(1-N\rho+\rho-N\gamma\lambda)}\bullet
\left\{\begin{aligned}
&(N-1)\gamma\lambda^3B''(1)+(1-N\rho+\rho)(\rho+1)\lambda^2R''(1)+ \\
&(1-N\rho+2\gamma)\gamma A''(1)-\gamma\lambda(2N\rho-4\rho+\gamma\lambda+\lambda\rho \\
&+2N\rho^2-\lambda\rho^2-2\rho^2+\gamma\lambda\rho^2+N\gamma\lambda-N\lambda\rho+ \\
&2\gamma\lambda\rho+N\lambda\rho^2-N\gamma\lambda\rho^2-2)
\end{aligned}\right\} \tag{13}
$$

The input and output of system should be stable, and the service rate must be greater than arrival rate. So the system stability condition should be:

$$
\lambda\left((N-1)\beta+N\gamma\right)<1 \tag{14}
$$

IV. Experiments Analysis

A. *Theoretical Curve*

The theoretical curve and simulation results compared SL-1 and Limit-1 system on the MATLAB. The system satisfied the stability condition $\begin{cases}\lambda\left((N-1)\beta+N\gamma\right)<1\\ 1-N\lambda(\beta+\gamma)>0\end{cases}$, the four groups of curves reflected the influences of each parameter.

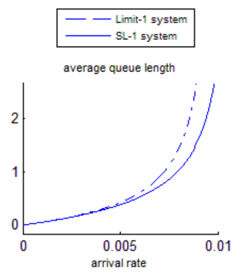

Figure 1. $\beta=\gamma=10, N=5$

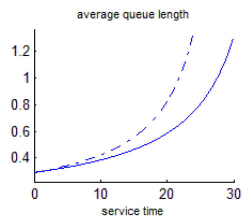

Figure 2. $\lambda=0.005, \gamma=10, N=5$

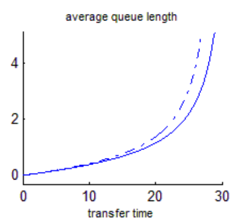

Figure 3. $\beta=10, \lambda=0.005, N=5$

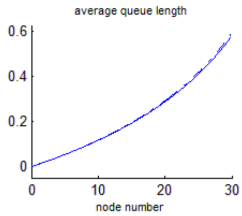

Figure 4. $\beta = \gamma = 10, \lambda = 0.005$

The system performance was evaluated as follows:

- From the expression of stability condition, SL-1 service system was looser than Limit-1 system and was more stable.
- The average queue length of SL-1 system maintain growth trend under each parameter. With stability decreased, the gap between SL-1 and Limit-1 was gradually obvious.
- No matter how each parameter changed, the average queue length of SL-1 system was always less than Limit-1 system, in which the effect of queue number was smallest for the simplified service.

B. *Simulation Experiment*

Experiments employed the function poissrnd (mu, V) to generate the Poisson distributed random sequence to obtain the arrival process of queue packet; the service and transfer process were constant; 95% confidence interval was selected, effective digit of experiments was 0.0001. After 1000000 slots, the results were obtained as follows:

TABLE I. AVERAGE WAITING TIME SIMULATION $\beta = 10, N = 5, \gamma = 10$

λ (10^{-3})	**Limit-1**			**SL-1**		
	Theo. value	***Expe. value***	***Polling times***	***Theo. value***	***Expe. value***	***Polling times***
1	0.0541	0.0543	18998	0.0535	0.0548	18991
2	0.1183	0.1147	18053	0.1154	0.1153	18033
3	0.1965	0.1969	16999	0.1884	0.1873	17102
4	0.2950	0.2941	16020	0.2764	0.2816	16112
5	0.4250	0.4250	15018	0.3862	0.3793	15292
6	0.6086	0.6026	14012	0.5293	0.5217	14378
7	0.8965	0.9083	12915	0.7281	0.7283	13472
8	1.4400	1.4172	12061	1.0333	1.0450	12599
9	2.9905	3.0405	11023	1.5908	1.5941	11747
10				3.0682	3.0667	10899

The effective digit of error was 0.01; with the incensement of polling times, the error would increase slightly. From the overall matching, theoretical calculation and simulation results were in good agreement.

V. CONCLUSION

This paper employed analytical method of multidimensional Markov chain and probability generating function to study SL-1 model newly proposed. The average queue length of the system was exactly solved. The numerical curves and simulation experiments under 95% probability confidence interval showed that the SL-1 service system worked in better queuing performance, which also verified the consistency of theoretical analysis and the simulation. Conclusion provided reliable method for the research of communications traffic theory.

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