A Cross-Entropy Method for Solving Passenger Flow Routing Problem

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***Abstract*—In this paper, a new design optimization method-cross entropy methods for passenger flow routing in passenger hubs is employed, in order to develop rational and efficient passenger flow routing program, which will help improving the passenger flow organization. According to the description and characteristics of the problem, we transform the problem into a combinatorial optimization problem, so that it is convenient to explore the best solution. The numerical example declares that the method given above can obtain the optimal solution under the condition of fixed demand. Results show that the cross entropy method is effective, and can be well applied in passenger flow routing design problem.**

Systematic studying the passenger terminal passenger flow routing design optimization techniques and methods and developing rational and efficient passenger flow routing program will help improving the organization of passenger flow. According to the description of the problem and the characteristics of itself, the paper transforms the problem into a combinatorial optimization problem, so that it is convenient to explore the best solution. The paper also employs the cross entropy method to solve the problems. Numerical example declares that the method given above can obtain the optimal solution under the condition of fixed demand. Results show that the cross entropy method is effective, and this method can be well applied in passenger flow routing design problem.

Keywords-Optimization for passenger flow routing; combinatorial optimization; cross entropy method

I. INTRODUCTION

With the expansion of passenger hubs' construction building area, the internal structure constantly tends to be complicated. It brings much difficulty for hubs' organization and operation. In order to meet passengers' needs in hubs such as transfer demand, hubs are ought to be handled as a whole to ensure to provide safe, convenient , comfortable and quality service in hubs. Under the condition of network operation and seamless transfer, the truth of "Fast in the middle, slow at both ends" put forward high request to hub integration operation [1].

Researchers have gone deep into this topic. Zhu et al. [2] has discussed the key factor which effects arrival efficiency though analyzing the arrival flow line. The paper has proposed measures that optimizing the facility layout and management measures of ticket entrance. The purpose of the research is to improve arrival efficiency. Tang et al.[3] has studied the mixed traffic flow line from the perspective of transfer interface. Fan has considered the transfer organization problem of multiple transportation modes (intercity highway, urban public traffic, railway and air transport), especially discussed flow line organization of transfer process.[4] Daamen has analysed the mass of data and established the utility model to describe passengers route choice behavior which is more accurate than traditional shortest path model[5]. Recently, Hu et al.[6] has introduced the concept of streamline design for the whole process of passenger transfer which expands the research scope of passenger flow optimization. The optimization procedure is regarded as a graph theory problem by abstracting the hub into a multiple stereo space connected graph.

II. PROBLEM DESCRIPTION AND METHOD DESIGN

A. *Problem description*

It is beneficial to research the route choice behavior of passengers in the hubs. With our research, service efficiency can be improved, and infrastructure constructions can be completed and passengers' arrival/departure procedures can be optimized. Many factors can influence the route choice behavior, including not only travel time and travel expenses, but also facilities service level causing psychological changes. Therefore, when the passengers are selecting the path, they are inclined to integrate the experience, all sorts of traffic information and traffic directions to make judgment. In the personal point of view, they are intended to choose the route which costs the least travel time.

The constitute of the structures in hubs has not only spatial attribute (spatial arrangement, size etc.), but also non-spatial attribute (hub level, traffic mode, rate of flow, flow directions and distance etc.). The functional planning and spatial arrangement in hubs are usually determined in the initial period of construction, so this paper focuses on the optimization of non-spatial attribute.

The method of graph theory is employed into this paper to solved flow line optimization. The nodes are active points such as the ticket gate, Channel crossing. The lines of the network are passageway between nodes. We define the links are directed arc, and the weight of them are travel time.

B. *METHOD DESIGN*

The network can be described as a Weighted directed graph $G=(V,E,W)$. V is the set of nodes in the network. E is the set of (directed) links in the network. W is the set of links weight. $R \subseteq V$ and $S \subseteq V$ are the sets of origin and destination respectively. The BPR function is employed to compute the link travel time, $t_e = t_0[1+\alpha(\frac{x_e}{Z_e})^{\beta}]$. In the function, t_e means the travel cost on link $e \in E$, x_e means the flow on e and Z_e indicates the capacity of link e. k is one route between (r,s), $k \in K^{rs}$.

The core methods of the problem can be concluded as three points:

1) *Firstly, the results of passenger choice behavior display the flow line in the hub, but it is not equal to the behavior on the urban transportation network completely. The routes which are serious round should be excluded. So* K *-short route is adopted between every OD (in/out/transfer). They are selected to be the optional flow line and* $\Omega = \{y_1,...,y_n\}$ *is the pool of optional flow line, where* $y_i \in K^{rs}$ *. The problem is corresponded to the really state, and The complexity of the calculation may be reduced.*

2) *Secondly, the paper assume that there is only one optimal route between one OD pair. So the flow line optimization problem is transferred to a combination optimization.*

3) *The link in the hub is exclusive for one path, and multiple paths can include the link simultaneously. So the flow on the link can be compute as follow function,* $x_e = \sum_r \sum_s \sum_{y_i} f_{y_i}^{rs} \delta_{e,y_i}^{rs}, y_i \subseteq \Omega^{rs}$ *where* $f_k^{rs} = q^{rs}$.

In conclusion, the optimization problem is summarized as follows. The set of flow lines $Y = \{y_1,..., y_m\}$ is selected from $\Omega = \{y_1,..., y_n\}$ to minimize $\sum_{y_i \in Y} C_{y_i}$. Here, $C_{y_i} = \sum c_e, e \in E$ and $Y \subseteq \Omega, m \le n$. m is the number of OD, and n is the total alternative routes.

III. SOLUTION ALGORITHM

This paper presents a new solution approach, called cross-entropy (CE), proposed by Rubinstein [7]. The CE algorithm is in the class of population-based heuristics such as GA, evolutionary strategies, scatter search, etc. The main idea of CE is related to the design of an effective learning mechanism throughout the search. The CE transforms the deterministic optimization problem into a stochastic one and then uses rare event simulation techniques to solve the problem.

The CE method, pioneered by Rubinstein in 1997 as a stochastic learning algorithm for estimating probabilities of rare events, has been broadened as a generic and efficient tool for solving difficult numerical and NP-hard combinatorial optimization problems. The CE method has been successfully applied to a variety of problems in combinatorial optimization. Applications areas include buffer allocation, queuing models of telecommunication systems, neural computation, control and navigation, DNA sequence alignment, signal processing, scheduling, vehicle routing problems [8], reinforcement learning, project management and reliable network design problem [9].

The cross-entropy (CE) method is inspired for estimating probabilities of rare events for stochastic networks. The main rationale of CE is the construction of a random sequence of solutions which converges probabilistically to the optimal or near-optimal solution in two iterative stages. In the first iteration, a sample of random data (e.g., a set of controller parameters) is generated according to a specified random mechanism. A better sample is produced in the next iteration and the parameters of the random mechanisms are updated with the corresponding data [10].

1) *Generate random network samples* $Y_1,...,Y_N$ *according to some specified random mechanism with Algorithm 1, and*

2) *Update the parameters of this mechanism to obtain better system objective in the next iteration with Algorithm 2.* [9]

A. **Algorithm 1** *[Generation Algorithm]*

a) Generate a uniform random streamline scheme $Y = \{y_1,..., y_m\}$ *with the probability of* a_t*, where* m *is the number of OD, i.e. random range of paths* $y_1,..., y_m$*. Set* $k = 1$ *(iteration counter);*

b) Calculate the cumulate total cost $C = \sum C_{y_i}, i \in m$*;*

c) If $k = N$*, then stop; otherwise, set* $k = k+1$*, and go to step a).*

B. **Algorithm 2** *[Main CE Algorithm]*

a) Initialize $a_t = \{0.5,...,0.5\}_{1\times n}$*. Set* $t = 1$ *(iteration counter). In the same OD, only one route can be selected, i.e.* $\sum_{i \in K_m} a_t^i = 0.5$ *, where* K_m *is the number of* K *-shortest paths between OD in* m*. So the elements in* a_t *is either 0 or 0.5;*

b) Generate a random sample $\{Y_1,...,Y_N\}$ *using Algorithm 1 with* a_t*. Compute the sample (*$1-\rho$*)-quantile of performance* $\gamma_t = C_{[(1-\rho)N]}$*;*

c) Use the same sample to update a_t

$$a_{t,i} = \frac{\sum_{j=1}^{N} H_{\{C(Y_j)\le\gamma_t\}} y_{t,i}}{\sum_{j=1}^{N} H_{\{C(Y_j)\le\gamma_t\}}}, i \in n$$

here $H_{\{C(Y_j)\leq\gamma_t\}}\in\{0,1\}$;

d) If $\max(\min(a_t,1-a_t))\leq\beta$, β *is a certain minimal value given at first, then stop; otherwise, set* $t=t+1$, *and go to step b).*

The core of the algorithm is to update a_t . In this process, the routes satisfying the optimized target are more inclined to be selected. The choice probabilities of the other routes are getting smaller and smaller, and finally satisfying the condition in *d)* in algorithm 2.

IV. NUMERICAL EXPERIMENT

An example in Figure 1(a) is employed to text and verify the method proposed in this paper. Figure 1(b) is the topology of the hub. Three entrances are in the hub, A、B、C in the Figure 1(a). The hub Is divided into two layers. Line-2 is operated on the Upper Infrastructure and Line-1 on the lower.

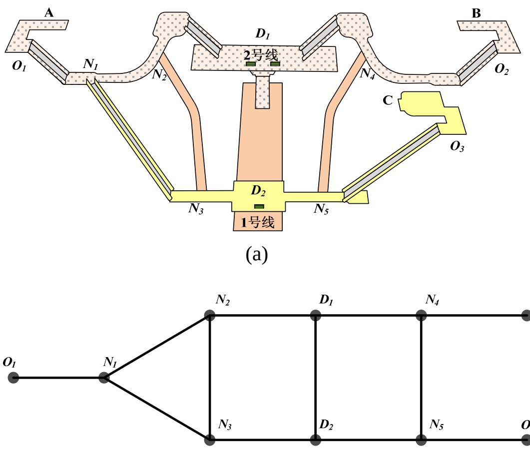

Figure 1. Test network of case study Example: (a) building structure of station;(b) abstract network

The network is constituted by 10 nodes and 12 lines. The demand between 14-OD is showed in Figure.2.

$$\begin{array}{c} \\ O_1 \\ O_2 \\ O_3 \\ D_1 \\ D_2 \end{array}\begin{array}{c} \begin{array}{ccccc} O_1 & O_2 & O_3 & D_1 & D_2 \end{array} \\ \begin{bmatrix} - & - & - & 1200 & 1040 \\ - & - & - & 800 & 960 \\ - & - & - & 720 & 880 \\ 2560 & 2400 & 2320 & - & 9600 \\ 2080 & 1920 & 2000 & 6240 & - \end{bmatrix} \end{array}$$

Figure 2. Passenger demand of the station

The length of all links and the free travel time on them are showed in TABLE I.

TABLE I. BASIC CIRCUMSTANCE OF THE TEST NETWORK

Num	*Link*	*Length(m)*	t_0 *(s)*	*Capacity (p/h)*
1	O1-N1	30.0	20.0	9420
2	N1-N2	45.0	30.0	9420
3	N1-N3	75.0	50.0	9420
4	N2-N3	225.0	150.0	14130
5	N2-D1	60.0	40.0	9420
6	N3-D2	60.0	40.0	9420
7	D1-D2	20.0	13.3	14130
8	D1-N4	60.0	40.0	9420
9	D2-N5	60.0	40.0	9420
10	N4-N5	225.0	150.0	14130
11	N4-O2	50.0	33.3	9420
12	N5-O3	70.0	46.7	9420

K-shortest path algorithm is employed here and the set of optional feasible flow line between every OD (r,s) is listed in TABLE II.

TABLE II. FEASIBLE PASSENGER ROUTE OF DIFFERENT OD DEMAND

OD	*#*	y_i	*OD*	*#*	y_i
O1-D1	1	O1-N1-N2-D1	D1-O1	13	D1-N2-N1-O1
	2	O1-N1-N3-N2-D1		14	D1-N2-N3-N1-O1
O1-D2	3	O1-N1-N2-N3-D2	D2-O1	15	D2-N3-N2-N1-O1
	4	O1-N1-N3-D2		16	D2-N3-N1-O1
O2-D1	5	O2-N4-D1	D1-O2	17	D1-N4-O2
O2-D2	6	O2-N4-N5-D2	D2-O2	18	D2-N5-N4-O2
O3-D1	7	O3-N5-N4-D1	D1-O3	19	D1-N4-N5-O3
O3-D2	8	O3-N5-D2	D2-O3	20	D2-N5-O3
D1-D2	9	D1-N2-N3-D2	D2 -D1	21	D2-N3-N2- D1
	10	D1-N2-N1-N3-D2		22	D2-N3-N1-N2- D1
	11	D1-D2		23	D2- D1
	12	D1-N4-N5-D2		24	D2-N5-N4- D1

The Cross-Entropy method is employed. On the basis of calculating the travel time on all optional feasible flow lines, C++ language assists operation. The convergence value $\beta=0.05$. The optimum proposal is showed in Fig.3.

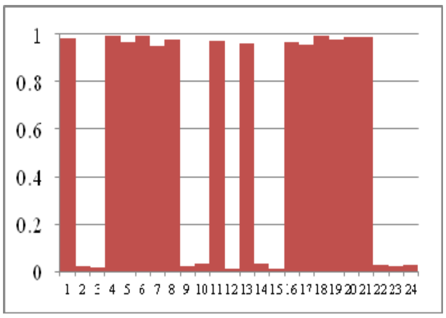

Figure 3. Route choice probability

The result shows that the optimal program in the hub including 14 paths (Figure.3). The number of the routes in Fig.3 are corresponding to TABLE II。

V. CONCLUSIONS

Passenger flow routing problem is transformed into a combinatorial optimization problem in this paper. The elements of the problem are all optional feasible flow lines. The optimal solution stands for the streamline organization scheme.

The Cross-Entropy method is employed in this paper. The results indicate that the algorithm is simple and easy to realize. The another advantage of the method is that the results are stable in the repeated calculation.

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