Proceedings of the 2012 2nd International Conference on Computer and Information Application (ICCIA 2012)

Global Synchronization of Moving Agent Networks with Time-varying Topological Structure

Lifu Wang, Zhi Kong, Xingang Wang
College of Control Engineering, Northeastern University at Qinhuangdao, Qinhuangdao, China
e-mail: wlfkz@qq.com

Abstract—**The synchronization of moving agent networks with linear coupling in a two dimensional space is investigated. Base on the Lyapunov stability theory, a criterion for the synchronization is achieved via designed decentralized controllers. And, an example of typical moving agent network, having the Rössler system at each node, has been used to demonstrate and verify the design proposed. And, the numerical simulation results show the effectiveness of proposed synchronization approaches.**

Keywords-** **synchronization; complex network; controller design; moving agents

I. INTRODUCTION

Complex networks have been intensively studied in many fields, such as social, biological, mathematical, and engineering sciences. Generally, a complex network is made up of interconnected nodes in which a node is a basic unit with detailed contents. These interactions between nodes determine many basic properties of a network. To well understand the complex dynamical behaviors of many natural systems, we need to study their operating mechanism, dynamic behavior, synchronization, anti-jamming ability, and so on. Recently years, Synchronization of complex dynamical networks has received a great deal of attentions from various fields of science and engineering [1-3].

The synchronization properties of a complex network are mainly determined by its topological structures connections between nodes. In the current study of complex networks, most of the existing works on synchronization consider a static networks, that is, the topological structures of which do not change as time evolves[4-10]. However，numerous real-world networks such as biological, communication, social, and epidemiological networks generally evolve with time-varying topological structures. Henceforth, researchers have devoted more and more efforts to complex networks with time-varying topologies. Lu et al. [11] investigated the local synchronization in networks with time-varying coupling strengths. Stiwell et al. [12] prove that if the network of oscillators synchronizes for the static time-average of the topology, then the network will synchronize with the time-varying topology if the time-average is achieved sufficiently fast. At the same conditions, Lu et al. [13, 14] found that the directed network with switching topology can reach global synchronization for sufficiently large coupling strength if there exists a spanning directed tree in the network.

Inspired by the above discussions, in this paper we investigates the adaptive synchronizaiton control problem for a specific time-varying network model. The model arises from the interaction of mobile agents proposed by Frasca et al [15], and can be widely used to explore various practical problems, e.g., clock synchronization in mobile robots [16], synchronized bulk oscillations [17], and task coordination of swarming animals [18]. We adopt the constraint of fast switching to derive synchronization conditions. By using Lyapunov stability theory, adaptive controllers are designed for synchronization of moving agent network with time-varying topological structures. The adaptive controllers can ensure that the states of moving agent network fast synchronization, and are rather simple in form.

II. MOVING AGENT NETWORK AND PRELIMINARIES

We consider N moving agents distributed in a two-dimensional planar space of size L $\Gamma=\{(y_1,y_2)\in R^2: 0\le y_1\le L,\ 0\le y_2\le L\}$, with periodic boundary conditions. Each agent moves with velocity $v_i(t)$, and direction of motion $\theta_i(t)$. The velocity $v_i(t)$ is the same for all individuals, (denoted by v) and is updated in direction through the angle $\theta_i(t)$ for each time unit. The agents are considered as random walkers. Hence, the motion law of the ith agent is given as follows:

$$\begin{aligned} &y_{i1}(t+\Delta t_M)=y_{i1}(t)+v\cos\theta_i(t)\Delta t_M,\\ &y_{i2}(t+\Delta t_M)=y_{i2}(t)+v\sin\theta_i(t)\Delta t_M;\\ &\theta_i(t+\Delta t_M)=\eta_i(t+\Delta t_M), \end{aligned} \tag{1}$$

where $i=1,2,\cdots,N$, $y_i(t)=(y_1,y_2)\in\Gamma$ is the position of agent i in the plane at time t, $\eta_i(t)$ are N independent random variables chosen at each time unit with uniform probability in the interval $[0,2\pi]$, and Δt_M is the motion integration step size.

A dynamical system is associated to each agent. Each agent interacts at a given time with only those agents located within a neighborhood of an interaction radius, defined as R [15, 19, 20]. When two agents interact, the state equations of each agent are changed to include diffusive coupling with the neighboring agent. Under these hypotheses, the state dynamics of agent node i can be formulated as

$$\dot{x}_i(t)=f(x_i(t),t)-\sigma\sum_{j=1}^{N}g_{ij}(t)x_j(t)+u_i(t) \tag{2}$$

where $i=1,\cdots,N$, $x_i(t)=(x_{i1}(t),x_{i2}(t),\cdots,x_{in}(t))^T\in R^n$ are the state variables of the node *i*; $f:D\times R^+\to R^n$ is a smooth nonlinear vector-valued function governs the local dynamics of oscillator; σ is the coupling strength; $g_{ij}(t)$ are the elements of a time-varying Laplacian matrix: $G(t)=\left[g_{ij}(t)\right]\in R^{N\times N}$ which defines the neighborhood of agents at a given time *t* and depends on the trajectory of each agent. In detail, for arbitrary two agents *i*, *j* with distance d at time *t*, $g_{ij}(t)=-1$, $g_{ji}(t)=-1$ if $d<R$; $g_{ij}(t)=0$, $g_{ji}(t)=0$ if $d>R$; and $g_{ii}(t)=h$, where h is the number of neighbors of the *i*th agent at time *t*; $u_i(t)\in R^n$ are the control inputs.

In this study, the problem of controlling the state variables of network Eq. (2) towards the desired a priori solution *s*(t), i.e.

$$x_1(t)=x_2(t)=\cdots=x_N(t)=s(t) \quad (3)$$

where $s(t)$ is a solution of an isolated node, satisfying

$$\dot{s}(t)=f(s(t),t) \quad (4)$$

We assume that $s(t)$ is an arbitrary desired state which can be an equilibrium point, a periodic orbit, an aperiodic orbit, or even a chaotic orbit in the phase space.

Next, the rigorous mathematical definition of synchronization for dynamical network (2) is introduced.

Definition 1. Let $x_i(t,t_0,X_0)$ ($i=1,2,\cdots,N$) be a solution of dynamical network (2), where $X_0=(x_1^0,x_2^0\cdots x_N^0)$ are initial conditions, $f:D\times R^+\to R^n$ are continuously differentiable with $D\subseteq R^n$. If there is a nonempty subset $\Lambda\subseteq D$ with $x_i^0\in\Lambda$, $i=1,2,\cdots N$, for all $t\ge t_0$, and

$$\lim_{t\to\infty}\|x_i(t,t_0,X_0)-x_j(t,t_0,X_0)\|=0 \quad (5)$$

for $i,j=1,2,\cdots N$. (Hereafter, denote $\|\cdot\|$ as the Euclidean norm.) Then the dynamical network (2) is said to realize synchronization. And $\Lambda\times\cdots\times\Lambda$ is called the region of synchrony for dynamical network (2).

Network (2) is said to achieve local asymptotical synchronization if there exists a $\delta>0$, $\left\|x_i^0-x_j^0\right\|\le\delta$, $x_i^0,x_j^0\in\Lambda$ such that for (5). Moreover, network (2) is said to achieve global asymptotical synchronization if (5) are hold for all initial conditions $x_i^0,x_j^0\in\Lambda$.

III. Synchronization of Moving Agent Networks

In this section, we discuss the synchronization of moving agent network (2) by designing linear controllers for each agent node. Several network synchronization criteria are given.

In order to achieve the objective of synchronizaiton on the manifold (3), let us define the error vector

$$e_i(t)=x_i(t)-s(t),\ i=1,2,\cdots N \quad (6)$$

Subtracting (4) from (2) yields the error dynamical system

$$\dot{e}_i(t)=f(x_i(t),t)-f(s(t),t)-\sigma\sum_{j=1}^{N}g_{ij}(t)e_j(t)+u_i(t) \quad (7)$$

Then, synchronization problem of the dynamical network (2) is equivalent to the problem of stabilization of the error dynamical system (7).

Rewrite node dynamics $\dot{x}_i(t)=f(x_i(t),t)$ as $\dot{x}_i(t)=Bx_i(t)+h(x_i(t),t)$, where B is a constant matrix and h : $D\times R^+\to R^n$ is a smooth nonlinear function. Thus, network (2) is described by

$$\dot{x}_i(t)=Bx_i(t)+h(x_i(t),t)-\sigma\sum_{j=1}^{N}g_{ij}(t)x_j+u_i(t) \quad (8)$$

where $i=1,2,\cdots,N$. One can get the error system

$$\dot{e}_i(t)=Be_i(t)+\overline{h}(x_i,s,t)-\sigma\sum_{j=1}^{N}g_{ij}(t)e_j(t)+u_i(t)$$

where, $i=1,2,\cdots,N$ and $\overline{h}(x_i,s,t)=h(x_i,t)-h(s,t)$.

In the following, we give several useful hypotheses.

Assumption 1 (A1). Suppose there exists a nonnegative constant μ, satisfying

$$\left\|\overline{h}(x_i,s,t)\right\|\le\mu\|e_i\|$$

Assumption 2 (A2). Suppose there exists a constant *T* such that coupling matrix *G*(*t*) satisfies

$$\frac{1}{T}\int_t^{t+T}G(\tau)d\tau=\overline{G}$$

where, $\overline{G}$ is the time-average of the coupling matrix *G*(*t*).

This Assumption 2 implies that the switching between all the possible network configurations is sufficiently fast as defined in [12]. According to [12] the following Lemma can be given.

Lemma 1. Suppose a set of coupled oscillators network with fixed topology defined by

$$\dot{x}_i(t)=f(x_i(t),t)-\sigma\sum_{j=1}^{N}\overline{g}_{ij}x_j(t)+u_i(t)\ \ (\overline{G}=\left[\overline{g}_{ij}\right])$$

admits a stable synchronization manifold and if (A2) hold. Then the set of coupled oscillators with a time-variant network defined by (2) admits a stable synchronization manifold.

The proof of Lemma 1 can be obtained by main results of Reference [12]. Then the error dynamical system can be rewritten as follows

$$\dot{e}_i(t)=Be_i(t)+\overline{h}(x_i,s,t)-\sigma\sum_{j=1}^{N}\overline{g}_{ij}(t)e_j(t)+u_i(t) \quad (9)$$

According to analyze of Reference [15], under the constraint of fast switching, $\overline{G}=pG_A$, where p is the probability that a link is activated and thus $p=\pi R^2/L^2$, and G_A is the all-to-all coupling matrix with zero-row sum). That is

$$\bar{G} = \frac{\pi R^2}{L^2}\begin{bmatrix} N-1 & -1 & \cdots & -1 \\ -1 & N-1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & N-1 \end{bmatrix} \quad (10)$$

Theorem 1. Suppose that (A1) and (A2) hold. Then the dynamical moving agent network (2) is global asymptotically synchronized under the following sets of linear controllers

$$u_i(t) = -k_i e_i(t), \quad i = 1, 2, \cdots, N \quad (11)$$

Where, $k_i = \beta + \mu$, β is a nonnegative constant satisfying $\| B \| \le \beta$, μ is a nonnegative constant given in (A1).

Proof: Select a Lyapunov function as follows

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T e_i \quad (12)$$

Then the time derivative of $V(t)$ along the solution of the error system (9) is given as follows

$$\dot{V}(t) = \frac{1}{2}\sum_{i=1}^{N}(\dot{e}_i^T(t)e_i(t) + e_i^T(t)\dot{e}_i(t))$$

$$= \frac{1}{2}\sum_{i=1}^{N}(Be_i(t) + \bar{h}(x_i, s, t) - \sigma\sum_{j=1}^{N}\bar{g}_{ij}(t)e_j(t)$$

$$+ u_i(t))^T e_i(t) + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)(Be_i(t) + \bar{h}(x_i, s, t)$$

$$-\sigma\sum_{j=1}^{N}\bar{g}_{ij}(t)e_j(t) + u_i(t))$$

$$\le \sum_{i=1}^{N} e_i^T(t) \| \frac{B^T + B}{2} \| e_i(t) + \sum_{i=1}^{N}\bar{h}^T(x_i, s, t) \| e_i(t)$$

$$-\sigma\sum_{i=1}^{N}\sum_{j=1}^{N}(\bar{g}_{ij}(t)e_j(t))^T e_i(t) + \sum_{i=1}^{N} u_i(t)^T e_i(t)$$

B is a constant matrix, then there exists a constant β satisfying $\| B \| \le \beta$, that is $\| \frac{B^T + B}{2} \| \le \beta$. According to (A1), so

$$\dot{V}(t) \le \sum_{i=1}^{N} e_i^T(t) \| \frac{B^T + B}{2} \| e_i(t) + \sum_{i=1}^{N}\bar{h}^T(x_i, s, t)e_i(t)$$

$$-\sigma\sum_{i=1}^{N}\sum_{j=1}^{N}(\bar{g}_{ij}(t)e_j(t))^T e_i(t) + \sum_{i=1}^{N} u_i(t)^T e_i(t)$$

$$\le (\beta + \mu)\sum_{i=1}^{N}\| e_i(t) \|^2 - \sigma\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{g}_{ij}(t)e_j^T(t)e_i(t)$$

$$+ \sum_{i=1}^{N} u_i^T(t)e_i(t)$$

Since

$$-\sigma\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{g}_{ij}e_i^T e_j \le 0$$

The $\dot{V}(t)$ can be given as follows

$$\dot{V}(t) \le (\beta + \mu)\sum_{i=1}^{N}\| e_i(t) \|^2 + \sum_{i=1}^{N} u_i^T(t)e_i(t)$$

Substituting controller $u_i(t) = -(\beta + \mu)e_i(t)$ into the right side of the equation, we have $\dot{V}(t) \le 0$. Thus it follows that error that the error vector $E = [e_1^T, e_2^T, \cdots, e_N^T]^T \to 0$ as $t \to \infty$. That is, the error system (9) admits a stable synchronization manifold. According to Lemma 1, the synchronous solution $S(t)$ of moving agent network (2) is globally asymptotically stable under the controllers (11).

The proof is thus completed.

IV. Simulations

In this section, one example is given for illustrating the proposed synchronization criteria. Consider a dynamical network consisting of 5 identical Rössler oscillators. Where, state dynamics of each agent is described by

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ 0 & 0 & -c \end{pmatrix}\begin{pmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ x_{i1}x_{i3} + b \end{pmatrix}, \quad (13)$$

where $x_i = (x_{i1} \quad x_{i2} \quad x_{i3})^T$. The following parameters have been used: a=0.2, b=0.2, c=7.

Each agent node interacts at a given time with only those agents located within a neighborhood of an interaction radius. Here, we let interaction radius R=50, and periodic boundary conditions size L=100. When two agents interact, the state equations of each agent are changed to include diffusive coupling with the neighboring agent, acting on the state variable x_{i1}. Based on these assumptions, the state dynamics of each agent can be described in terms of the following equations:

$$\dot{x}_i = f(x_i) - \sigma\sum_{j=1}^{N} g_{ij}(t)Ex_j + u_i \quad (14)$$

where, $i = 1, \cdots, N$, $f: R^3 \to R^3$ is given by the Rössler dynamics, $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $g_{ij}(t)$ are the elements of a time-varying matrix $G(t)$, and $u_i(t) = -(\beta + \mu)e_i(t)$.

Obviously, one gets

$$\bar{h}(x_i, s, t) = [0 \quad 0 \quad x_{i1}x_{i3} - s_1 s_3]^T.$$

Similar to refs. [21, 22], since Rössler chaotic system has a chaotic attractor which is confined to abounded region $\phi \subset R^n$, there exists a constant M satisfying $| x_{ij} |, | s_j | \le M$ for $i = 1, 2, \cdots, N$ and $j = 1, 2, 3$. Therefore

$$\| \bar{h}(x_i, s, t) \| = \sqrt{(x_{i1}x_{i3} - s_1 s_3)^2} \le \sqrt{2}M \| e_i \|.$$

M can be got from the method of simimar to reference [22]. Thus, (A1) hold.

Assume that $v_i(t) = 200$, $\Delta t_M = 0.1$ to guarantee the fast-switching condition (A2). According to Theorem 2, the synchronous solution $s(t)$ of dynamical moving agent network (14) is globally asymptotically stable. The other parameters are assigned as follows: $k_i = 6$,

$x_i(0) = (2+i, 3+i, 4+i)$, and $\sigma = 1$. The synchronous error e_i are shown in Figure 1-3. Obviously, the zero error is globally asymptotically stable for dynamical network (14).

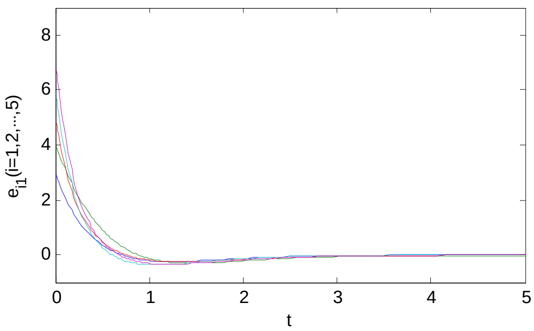

Figure 1. Synchronization errors of e_{i1} for the network.

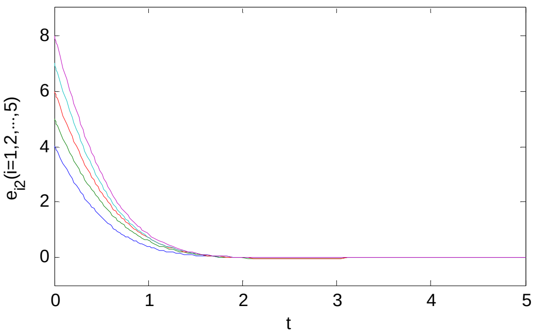

Figure 2. Synchronization errors of e_{i2} for the network.

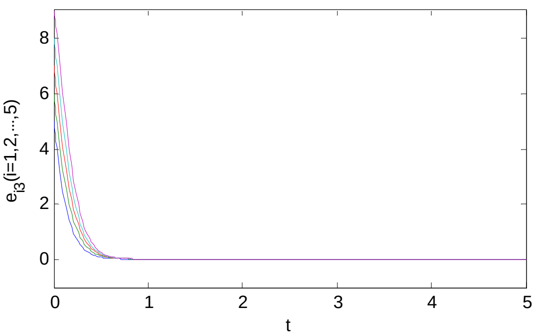

Figure 3. Synchronization errors of e_{i3} for the network.

V. CONCLUSIONS

Global synchronization for moving agent dynamical network is investigated. The moving agent network with decentralized controllers is considered as a large-scale nonlinear system. An adequate Lyapunov function is constructed to deal with the problem of controlled synchronization as to ensure the closed loop system stability. A novel network synchronization criterion has been proved by using Lyapunov stability theory. Decentralized adaptive controllers are designed to achieve synchronization for the moving agent networks. And a numerical simulation of coupled Rössler system network is given, which demonstrates the effectiveness of the proposed methods.

ACKNOWLEDGMENT

This work is supported by the Natural Science Foundation of Hebei under Grant No. F2012501030, the Fundamental Research Funds for the Central Universities under Grant No. N100323012, No. N100323011, and National Natural Science Foundation of China under Grant No. 51105068.

REFERENCES

[1] R. Albert and A. L. Barabasi, "Statistical mechanics of complex networks," Reviews of Modern Physics, vol. 74, pp. 47-97, Jan 2002.
[2] S. A. Pandit and R. E. Amritkar, "Characterization and control of small-world networks," Physical Review E, vol. 60, pp. R1119-R1122, Aug 1999.
[3] S. H. Strogatz, "Exploring complex networks," Nature, vol. 410, pp. 268-276, Mar 2001.
[4] M. Barahona and L. M. Pecora, "Synchronization in Small-World Systems," Physical Review Letters, vol. 89, p. 054101, 2002.
[5] X. F. Wang and G. R. Chen, "Synchronization in scale-free dynamical networks: Robustness and fragility," Ieee Transactions on Circuits and Systems I-Fundamental Theory and Applications, vol. 49, pp. 54-62, Jan 2002.
[6] S. Boccaletti, et al., "Complex networks: Structure and dynamics," Physics Reports-Review Section of Physics Letters, vol. 424, pp. 175-308, Feb 2006.
[7] J. G. Barajas-Ramírez, "Robust synchronization of a class of uncertain complex networks via discontinuous control," Computers and Mathematics with Applications, vol. 64, pp. 956-964, 2012.
[8] X. Guo and J. Li, "A new synchronization algorithm for delayed complex dynamical networks via adaptive control approach," Communications in Nonlinear Science and Numerical Simulation, vol. 17, pp. 4395-4403, 2012.
[9] W. Guo, "Lag synchronization of complex networks via pinning control," Nonlinear Analysis: Real World Applications, vol. 12, pp. 2579-2585, 2011.
[10] S. Zheng, et al., "Adaptive synchronization of two nonlinearly coupled complex dynamical networks with delayed coupling," Communications in Nonlinear Science and Numerical Simulation, vol. 17, pp. 284-291, 2012.
[11] J. H. Lu and G. R. Chen, "A time-varying complex dynamical network model and its controlled synchronization criteria," Ieee Transactions on Automatic Control, vol. 50, pp. 841-846, Jun 2005.
[12] D. J. Stilwell, et al., "Sufficient conditions for fast switching synchronization in time-varying network topologies," Siam Journal on Applied Dynamical Systems, vol. 5, pp. 140-156, 2006.
[13] X. B. Lu, et al., "Global Synchronization of Directed Networks with Fast Switching Topologies," Communications in Theoretical Physics, vol. 52, pp. 1019-1024, Dec 2009.
[14] B. Z. Qin and X. B. Lu, "Adaptive approach to global synchronization of directed networks with fast switching topologies," Physics Letters A, vol. 374, pp. 3942-3950, 2010.
[15] M. Frasca, et al., "Synchronization of Moving Chaotic Agents," Physical Review Letters, vol. 100, p. 044102, 2008.
[16] A. Buscarino, et al., "Dynamical network interactions in distributed control of robots," Chaos: An Interdisciplinary Journal of Nonlinear Science, vol. 16, pp. 015116-10, 2006.
[17] S. Dano, et al., "Sustained oscillations in living cells," Nature, vol. 402, pp. 320-322, 1999.
[18] L. Q. Peng, et al., "Consensus of self-driven agents with avoidance of collisions," Physical Review E, vol. 79, p. 026113, Feb 2009.
[19] M. Frasca, et al., "Dynamical network model of infective mobile agents," Physical Review E, vol. 74, p. 036110, 2006.
[20] L. Wang, et al., "Induced synchronization of a mobile agent network by phase locking," Physical Review E, vol. 82, p. 046222, 2010.
[21] K. Li and C. Lai, "Adaptive–impulsive synchronization of uncertain complex dynamical networks," Physics Letters A, vol. 372, pp. 1601-1606, 2008.
[22] D. Li, et al., "Estimating the ultimate bound and positively invariant set for the Lorenz system and a unified chaotic system," Journal of Mathematical Analysis and Applications, vol. 323, pp. 844-853, 2006.