The Inverse Solution to Eigenvalue Problem of Milling Machine Spindle based on Neural Network-Optimization Design

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Abstract—The design of milling machine spindle with respect to dynamic properties can be classified as the solution to inverse eigenvalue problem. In the paper, the solution to the inverse eigenvalue problem based on artificial neural network is studied, the research results show that the direct inverse model of ANN can not derive correct results while the optimization means based on artificial neural network is an effective one of solving inverse eigenvalue problem.

Key words-Neural network-optimization design; milling machine spindle; eigenvalue; inverse problem solution

I. INTRODUCTION

Milling spindle is a very important part in milling machine, the design of the part related to the overall performance of the milling machine. Therefore, in addition to consider its static properties in design, we still should consider its dynamic characteristics requirement.

The design of milling spindles' dynamic characteristic comes down to the inverse problem solving of eigenvalue. The so-called inverse problem solving of eigenvalue as that the eigenvalue of the structure is given in advance, and then according to the structural generalized characteristics equation solve the structure parameters. The solving methods of the inverse problem of eigenvalue including optimization method, structure matrix perturbation method and sensitivity analysis [1].

The application of the inverse problem solving method of dynamics based on neural network [2-3], the basic idea is using finite element analysis software or measured value attain the training sample that reflects the relationship between the structure and input (structure parameters) output (structural response), then respectively as the output-input of the neural network to train network, so as to realize the nonlinear inverse mapping from output parameters space to input parameter space. But this method can’t get the right result in the spindle contains multiple design variables, therefore the usable range is very narrow. Research shows that the neural network-optimization method is a universally applicable method of solving the inverse problem of eigenvalue.

II. THE NEURAL NETWORK MODEL OF THE INVERSE PROBLEM OF EIGENVALUE

A. The inverse problem of eigenvalue

The generalized characteristic equations of vibration for:

\[ [K] \{q_j\} = \lambda_j [M] \{q_j\}; \quad j = 1, 2, \ldots, N \quad (1) \]

Thereinto, [K] for stiffness matrix, [M] for quality array, \( \lambda_j \) for \( j \) order eigenvalue, \{q_j\} for the corresponding feature vector, N for free degrees. The inverse problem of eigenvalue, given the target eigenvalue \( \lambda_{obj} \) of the structure. According to (1) find out the structure size parameters \( X = \{x_i\}, (i = 1, \ldots, M) \), M as the design variables, contained in [K] and [M].

B. The neural network direct inverse model

In the basis that about the continuous function indicate law, Kolmogorov has proved a three orders feed forward networks have the ability with arbitrary precision approaching n sphere continuous function defined in tight subset of \( K^{3n} \). Thus multiorder feed forward neural network (BP network) is a kind of general function force implement, can realize the arbitrary the arbitrary nonlinear mapping from \( R^n \) to \( R^n \), this is neural network obverse model (as shown in figure 1). Neural network model is used to replace the finite element in proceeding structure similar analysis, realize the nonlinear mapping from structure size to structural response. The obverse model put the cart before the horse becomes direct inverse model (as shown in figure 2), realize the nonlinear mapping from \( R^n \) to \( R^n \). Neural network direct inverse model is used to realize the nonlinear mapping from the structure size to structural response, is not always able to get the right result.
C. Neural network-optimization model method

The eigenvalue inverse problem solving boiled down to a no constrained nonlinear programming problem.

Solving: \( X = \{x_i\} \quad (i = 1,2,...,m) \)

\[
\min_{X} (\sum_{j=1}^{n} (\lambda_j - y_j)^2)
\]

(2)

In the formula, \( \lambda_i \) (i = 1,..., n) is expected receive i order \( y_i \) indicate the actual eigenvalue of the structure. When the objective function \( E_r \) to the given convergence precision, the corresponding design variable value is the solution of the inverse problem's eigenvalue. For the function minimization, which of quadratic sum form, common optimization method can solve [5]. The mathematical model that formulas (2) defined may have a number of solutions, this is because the mapping from eigenvalue to the structure size is not one-to-one, it can be seen in the follow analysis. To make the results sole, mathematical model can be changed to the lightest weight design problem constrained by frequency [6].

\[
\min_{X} W(X) \quad s.t. \quad g_j(X) = \lambda_j - y_j = 0 \quad (j = 1,2,...,n)
\]

The lightest weight design constrained by frequency if criterion method is adopted to solve, the method will be into the local minimum [7]. In recent years, the good properties of genetic algorithm [8] that make it show a good application prospect in some continuous and discrete variable optimization design field. Its main advantage is the strong general optimization ability, do not need gradient information, the continuous function is not necessary, the optimal result is overall, so this paper using genetic algorithm. Genetic algorithm is a kind of probability search method, it needs a considerable number of chromosomes after generations breeding to search the optimal solution, a lot of heavy work for finite element structure analysis are very difficult calculation tasks, so using neural network obverse model instead of finite element for the structure similar analysis.

III. THE CALCULATION RESULTS AND ANALYSIS

A. An inverse eigenvalue problem in variables design

Figure 3 indicates horizontal milling spindle, it is known that \( p=7800 \text{kg/m}^3, E=206\times10^3 \text{Mpa} \). Now assume that all outside diameters indicated by a design variable, namely: \( D_1 = D_2 = D_3 = D_4 = X = \{x_1\} \), seek the section size \( x_1 \) of shaft when 1 order frequency \( \lambda_1 = 900, 1200, 1500, 1700 \) Hz. With neural network direct inverse model solving, through ANSYS gained five training samples shown in table 1, using a \( 1 \times 5 \times 1 \) three-layer neural network realized the mapping from \( \lambda_1 \) to \( x \). Mapping the results in table 2.

<table>
<thead>
<tr>
<th>( X_1 (\text{mm}) )</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_1 ) (Hz)</td>
<td>927.7</td>
<td>1138.8</td>
<td>1355.3</td>
<td>1569.4</td>
<td>1783.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \lambda_1 ) (Hz)</th>
<th>900</th>
<th>1200</th>
<th>1500</th>
<th>1700</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 ) (mm)</td>
<td>54.3</td>
<td>84.5</td>
<td>112.6</td>
<td>139</td>
</tr>
</tbody>
</table>

The corresponding \( \lambda_1 \) for \( X_1 \) solved by finite element

| Reverse error (%) | -3.4 | -1.1 | -0.4 | +4   |

From table 2 it is known when only a design variable, the results that neural network direct inverse model solve eigenvalue inverse problems are correct, the error generally is small, only in the sample points at both ends is larger.

B. The eigenvalue inverse problem of two design variables

Still take figure 3 for example, Take two design variables, namely \( x_1 = D_2, x_2 = D_3 \), the rest \( D_1 = D_4 = 60 \text{ mm} \), require 1 order frequency \( \lambda_1 = 1600 \) Hz, solve the section size of spindle \( X = \{x_1, x_2\} \).

(1) With the neural network direct inverse model solving, through ANSYS get 25 training samples as shown in table 3. To make the design point evenly distributed in design variables space, use the orthogonal table [9] setting points. Firstly tried to adopt a \( 1 \times 6 \times 2 \) single hidden layer BP
network to realize the mapping from $\lambda_1$ to $x_1$ and $x_2$, but BP network don't convergence. Figure 4 is training error curve. It can see from figure 4, from about 300 times to 20000 times, training error keep the same in 0.6, which indicates that the network can't convergence. When changing learning rate, the times of training the parameters still can't convergence. Of course, it can't come to a conclusion by a practice. Sometimes, the normal mapping occasionally appear the condition of figure 4, the training followed by can be normal; But when a mappings don't exist, no matter how to repeat training, will occur the condition shown in figure 4.

<table>
<thead>
<tr>
<th>TABLE 3</th>
<th>SECTION SIZE AND 1/2ORDER NATURAL FREQUENCY (HZ)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X_1$ (mm)</td>
</tr>
<tr>
<td></td>
<td>60</td>
</tr>
<tr>
<td>60</td>
<td>1027/1785</td>
</tr>
<tr>
<td>80</td>
<td>1371/1526</td>
</tr>
<tr>
<td>100</td>
<td>1572/1491</td>
</tr>
<tr>
<td>120</td>
<td>1820/1615</td>
</tr>
<tr>
<td>140</td>
<td>938/1677</td>
</tr>
</tbody>
</table>

Figure 4 Obverse model training error curve

Figure 5 Inverse model training error curve

This phenomenon can be seen from size-frequency grid figure (figure 6, figure 7), $\lambda_1$ and $\lambda_2$ are multi-modal function, therefore $x_1$ $x_2\rightarrow\lambda_1$ and $x_1$ $x_2\rightarrow\lambda_2$ are not one-to-one relationships. In other words, a $\lambda_1$ or $\lambda_2$ may correspond to multiple $x_1$, $x_2$. The contours of figure 6 or figure 7, are the combination of all $x_1$ $x_2$ in a frequency. From the function definition, $X = \{x_1, x_2\} = f (\lambda_i)$ is not a function, so it can't adopt neural network to approach to this relationship.

Figure 6 $X$-$f_1$ grid chart

Figure 7 $X$-$f_2$ grid chart

Therefore, when more than one design variable, the eigenvalue inverse problem of designed object horizontal
milling spindle can't use neural network direct inverse model solving.

Neural network-optimization model solving. Adopting a $2 \times 8 \times 1$ BP network to realize the mapping from $X = [x_1, x_2]$ to $\lambda$. Training network used the BP algorithm that join a momentum and learning rate self-adjusting, the largest number of training step $ep = 5000$, allow error $e = 0.001$, learning rate $\eta = 0.01$, incremental learning rate $\eta_1 = 1.05$ adopted, reducing learning rate $\eta_2 = 0.75$, momentum $\alpha = 0.7$, training error curve as shown in figure 5. In genetic algorithm, the initial population size is 80, cross rate $P_c = 0.7$, mutation rate $P_m = 0.01$, biggest genetic algebra epoch = 500. The results obtained by the neural network + genetic algorithm inverse model solving are given in table 4, the two are results are very close to, show that the genetic algorithm can converge to the most advantage points with a probably rule.

IV. CONCLUSION

The research in this paper shows that:

(1) In the solving method of milling spindle eigenvalue inverse problem, only in the condition eigenvalue and design variables have a one-to-one to mapping relationship, can it adopt neural network direct inverse model, and the using range is largely restricted;

(2) The solving model based on neural network and optimization method is a universally applicable method in eigenvalue inverse problem. To make the results sole, the solution of eigenvalue inverse problem can change to the lightest weight design problem constrained by frequency; In this model, the neural network is used as obverse model instead of finite element proceeding structure similar analysis;

(3) Using the genetic algorithm to solve the optimization mathematical model is expected to get the optimal solution and is a good optimization method.

REFERENCES


